ESI Erwin Schrödinger International Institute for Mathematics and Physics

universität wien

DVR 0065528

Programme on

"Optimal Transport"

April 15 – June 14, 2019

organized by

Mathias Beiglböck (U Vienna), Alessio Figalli (ETH Zürich), Jan Maas (IST Austria), Robert McCann (U Toronto), Justin Solomon (MIT, Cambridge)

Monday, May 27, 2019

14:30 – 15:15 **Bertram Tschiderer (U Vienna)**

Trajectorial Otto calculus

Joint work with Ioannis Karatzas and Walter Schachermayer
Abstract: We revisit the variational characterization of diffusion as entropic gradient flux, established by Jordan, Kinderlehrer, and Otto in [JKO98], and provide for it a probabilistic interpretation based on stochastic calculus. It was shown in [JKO98] that, for diffusions of Langevin-Smoluchowski type, the Fokker-Planck probability density flow minimizes the rate of relative entropy dissipation, as measured by the distance traveled in the ambient space of probability measures with finite second moments, in terms of the quadratic Wasserstein metric. We obtain novel, stochastic-process versions of these features, valid along almost every trajectory of the diffusive motion in both the forward and, most transparently, the backward, directions of time, using a very direct perturbation analysis. By averaging our trajectorial results with respect to the underlying measure on path space, we establish the minimum rate of entropy dissipation along the Fokker-Planck flow and measure exactly the deviation from this minimum that corresponds to any given perturbation.

15:45 – 16:30 **Malcolm Bowles (U of British Columbia)**

Linear mass transfers and ergodic properties of their Kantorovich operators

Abstract: In this talk, I will introduce the class of "linear mass transfers" between two probability distributions, and the dual class of nonlinear "Kantorovich" operators between function spaces. The class of linear mass transports can be seen as a natural extension of convex lower semi-continuous energies on Wasserstein space, and contains the cost minimising mass transports, as well many other functionals which correlate two probability measures, including the weak mass transports, martingale, and stochastic transports, among others. For many linear mass transfers, we show that there exists a critical constant, a fixed point for their dual Kantorovich operator, and an idempotent linear mass transfer, which can be seen as an extension of Mane's critical value, and Fathi's weak KAM solutions in Aubry-Mather theory. This amounts to the study of the asymptotic properties of the nonlinear Kantorovich operator, and allows an extension of the theory of deterministic Lagrangian dynamics to a stochastic counterpart. We also introduce the notion of convex transfers, which includes the logarithmic entropy, and exhibit duality formulae for convolutions of linear and convex mass transfers. This is joint work with Nassif Ghoussoub.

All talks take place at ESI, Boltzmann Lecture Hall!