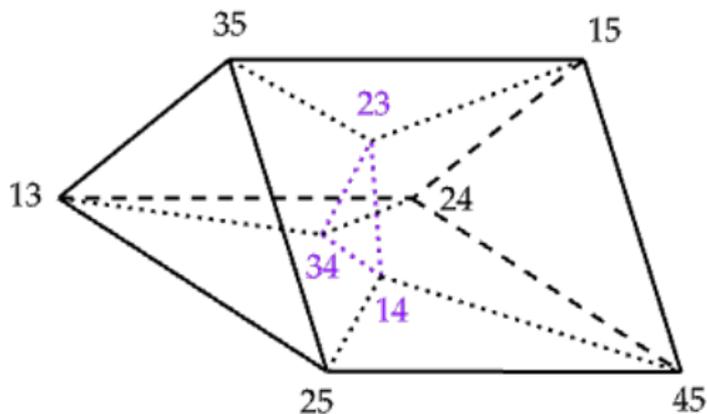# Kinematic Varieties III
## Let's Get Real

**Bernd Sturmfels**
MPI Leipzig



*Mini-Course at ESI Vienna, within*
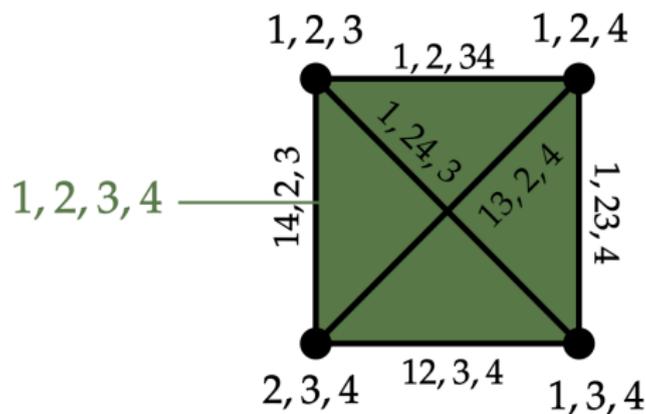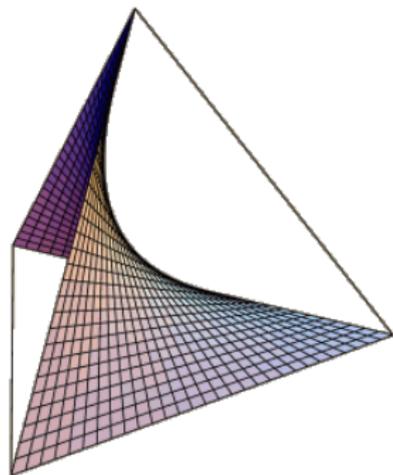*Amplitudes and Algebraic Geometry*
*February 18, 2026*

# Source

This lecture is based on the article
V. Calvo Cortes, H. Frost, BSt:
*Kinematic Stratifications*
arXiv:2503.09571

# One Particle

In physics, particles are represented by momentum vectors $p$ in *Minkowski space* $\mathbb{R}^{1+d}$, with Lorentzian inner product

$$p \cdot q = p_0 q_0 - p_1 q_1 - \cdots - p_d q_d.$$

The *universal speed limit* states that $p \cdot p \geq 0$ for each particle.

A particle is *massless* if the equality $p \cdot p = 0$ holds.



Positive Geometry in Particle Physics and Cosmology
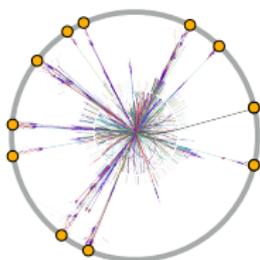
Massless: think photon
Massive: think proton

# The Lightcone

# Several Particles

Consider $n$ particles, with momenta $p^{(1)}, p^{(2)}, \ldots, p^{(n)} \in \mathbb{R}^{1+d}$.

The Lorentz group $\mathrm{SO}(1, d)$ acts on such configurations.
Kinematic data are invariant under this action.

The *Mandelstam invariants* are the entries in the symmetric matrix

$$
\begin{bmatrix}
s_{11} & s_{12} & \cdots & s_{1n} \\
s_{12} & s_{22} & \cdots & s_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
s_{1n} & s_{2n} & \cdots & s_{nn}
\end{bmatrix}
=
\begin{bmatrix}
- p^{(1)} - \\
- p^{(2)} - \\
\vdots \\
- p^{(n)} -
\end{bmatrix}
\begin{bmatrix}
+1 & 0 & \cdots & 0 \\
0 & -1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & -1
\end{bmatrix}
\begin{bmatrix}
| & | & \cdots & | \\
(p^{(1)})^T & (p^{(2)})^T & \cdots & (p^{(n)})^T \\
| & | & \cdots & |
\end{bmatrix}
$$

# Several Particles

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- & p^{(2)} & - \\
 & \vdots & \\
- & p^{(n)} & -
\end{bmatrix}
\begin{bmatrix}
+1 & 0 & \cdots & 0 \\
0 & -1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
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\begin{bmatrix}
| & | & \cdots & | \\
(p^{(1)})^T & (p^{(2)})^T & \cdots & (p^{(n)})^T \\
| & | & \cdots & |
\end{bmatrix}
$$

The *Mandelstam region* $\mathcal{M}_{n,r}$ is the semi-algebraic set of these matrices, for fixed rank $r \leq 1+d$. It has codimension $\binom{n-r-1}{2}$ in $\mathbb{R}^{\binom{n+1}{2}}$.

We examine the stratification of $\mathcal{M}_{n,r}$ by the signs of the $s_{ij}$.

# Being Lorentzian

The Mandelstam region $\mathcal{M}_{n,r}$ consists of matrices $S$ that satisfy:

- the $n$ diagonal entries $s_{ii}$ of $S$ are non-negative, and
- $S$ has one positive eigenvalue and $r - 1$ negative eigenvalues.

The *Lorentzian region* is the subset of nonnegative matrices

$$\mathcal{L}_{n,r} \;=\; \mathcal{M}_{n,r} \,\cap\, (\mathbb{R}_{\geq 0})^{\binom{n+1}{2}}.$$

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Points in $\mathcal{L}_{n,\leq n} = \sqcup_{r=1}^{n} \mathcal{L}_{n,r}$ are *Lorentzian polynomials* of degree two.

Petter Brändén and June Huh: *Lorentzian polynomials*, Ann. Math. (2020).

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<span style="font-size:small">Petter Brändén and June Huh: *Lorentzian polynomials*, Ann. Math. (2020).</span>

Brändén proved that $\mathcal{L}_{n,\leq n}$ is a topological ball of dimension $\binom{n+1}{2}$.

<span style="font-size:small">P. Brändén: *Spaces of Lorentzian and real stable polynomials are* **Euclidean balls**, Forum Math. Sigma (2021)</span>

Thus, $\mathcal{M}_{n,\leq n} = \sqcup_{r=1}^{n} \mathcal{M}_{n,r}$ is a disjoint union of $2^{n-1}$ such balls.

Our stratifications match those in

<span style="font-size:small">M. Baker, J. Huh, M. Kummer, O. Lorscheid: *Lorentzian polynomials and* **matroids** *over triangular hyperfields*.</span>

# Principal Minors

### Lemma
$S \in \mathcal{M}_{n,r}$ *if and only if the principal minors have* *alternating signs*:

$$(-1)^{|I|-1} \cdot \det(S_I) \ \geq \ 0 \qquad \textit{for all} \ \ I \subseteq [n].$$

For minors of size 2 and 3,

$$s_{ii}s_{jj} \leq s_{ij}^2 \quad \text{and} \quad 2s_{ij}s_{ik}s_{jk} + s_{ii}s_{jj}s_{kk} \geq s_{ii}s_{jk}^2 + s_{jj}s_{ik}^2 + s_{kk}s_{ij}^2.$$

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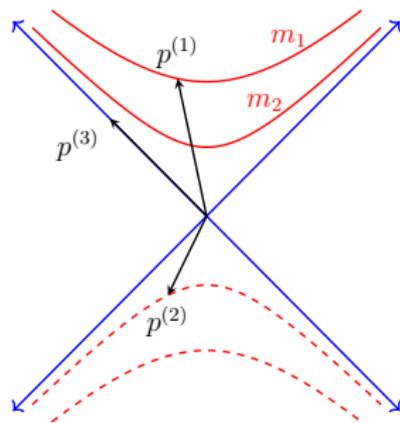
This implies

$$s_{ij}s_{ik}s_{jk} \ \geq \ 0.$$

Proposition

*If $S \in \mathcal{M}_{n,r}$ has nonzero entries then there exists $\sigma \in \{-,+\}^n$ such that $\mathrm{sgn}(s_{ij}) = \sigma_i\sigma_j$ for all $i, j$.*

# Causality



## Corollary

*The region $\mathcal{M}_{n,r}$ is the union of the $2^{n-1}$ signed Mandelstam regions $\mathcal{M}_{n,\sigma,r}$. Their relative interiors are pairwise disjoint:*

$$\mathcal{M}_{n,r} = \bigcup_\sigma \mathcal{M}_{n,\sigma,r}.$$

The sign vector $\sigma$ distinguishes the **future** from the **past**.

The *Lorentzian region* $\mathcal{L}_{n,r}$ is the closure of the region $\mathcal{M}_{n,\sigma,r}$ with $\sigma = (+,+,\dots,+)$. Everything lies in the future, or in the past.

## Massless Particles

From now on, all particles are massless:

$$s_{11} = s_{22} = \cdots = s_{nn} = 0.$$

Principal $4 \times 4$ minors of $S \in \mathcal{M}_{n,r}^0$ satisfy $\det(S_{\{i,j,k,l\}}) =$

$$s_{ij}^2 s_{kl}^2 + s_{ik}^2 s_{jl}^2 + s_{il}^2 s_{jk}^2 - 2 \cdot \left( s_{ij} s_{ik} s_{jl} s_{kl} + s_{ij} s_{il} s_{jk} s_{kl} + s_{ik} s_{il} s_{jk} s_{jl} \right) \leq 0.$$

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If we set $p_{ij} = \sqrt{s_{ij}}, \ldots, p_{kl} = \sqrt{s_{kl}}$, then this factors:

$$
\begin{aligned}
\det(S_{\{i,j,k,l\}}) =\ & (p_{ij} p_{kl} + p_{ik} p_{jl} + p_{il} p_{jk})(-p_{ij} p_{kl} - p_{ik} p_{jl} + p_{il} p_{jk}) \\
& (-p_{ij} p_{kl} + p_{ik} p_{jl} - p_{il} p_{jk})(p_{ij} p_{kl} - p_{ik} p_{jl} - p_{il} p_{jk}).
\end{aligned}
$$

Think: Plücker, Schouten, squared Grassmannian, …

H. Friedman: *Likelihood geometry of the squared Grassmannian*, Proceedings AMS (2025)

This guides us to **matroids**. In this talk, **all matroids have rank two**.

# Matroids          of rank two

A *matroid* is a partition $P = P_1 \sqcup P_2 \sqcup \cdots \sqcup P_m$ of a subset
of $[n] = \{1, \ldots, n\}$. The *bases* of $P$ are pairs $\{u, v\}$ where
$u \in P_i$ and $v \in P_j$ for $i \neq j$. Elements in $[n] \backslash P$ are *loops*.

The matroid $P$ has $m \geq 2$ parts and $l = n - |P|$ loops.

### Example

The *uniform matroid* $U_n$ is the partition
of $P = [n]$ into $n$ singletons $P_i = \{i\}$.

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of $P = [n]$ into $n$ singletons $P_i = \{i\}$.

For $\sigma \in \{-, +\}^P$, the pair $(P, \sigma)$ is a *signed matroid*.

## Definition

The stratum $\mathcal{M}^0_{P,\sigma,r}$ is the subset of the
massless Mandelstam region $\mathcal{M}^0_{n,r}$ defined by

$\mathrm{sign}(s_{ij}) = \sigma_i \sigma_j$ if $\{i, j\}$ is a basis of $P$, and $s_{ij} = 0$ otherwise.

# Stratification

**Theorem**
*Fix $r \geq 1$. The massless Mandelstam region equals*

$$\mathcal{M}^0_{n,r} = \bigsqcup_{(P,\sigma)} \mathcal{M}^0_{P,\sigma,r}.$$

*The disjoint union is over all signed matroids $(P,\sigma)$ on $[n]$.*

*The kinematic stratum $\mathcal{M}^0_{P,\sigma,r}$ is non-empty if and only if $3 \leq r \leq m$ or $r = m = 2$. If this holds, then its dimension is*

$$\dim\left(\mathcal{M}^0_{P,\sigma,r}\right) = m(r-2) + n - l - \binom{r}{2}.$$

*Recall:* The matroid $P$ has $m \geq 2$ parts and $l = n - |P|$ loops.

# Enumerative Combinatorics

We write $\left\{ {n-l \atop m} \right\}$ for the *Stirling number of second kind*. This is the number of partitions of the set $[n-l]$ into exactly $m$ parts.

## Corollary

*The number of kinematic strata $\mathcal{M}^0_{P,\sigma,r}$ of dimension $d$ in the Mandelstam region $\mathcal{M}^0_{n,r}$ is given, for a fixed sign vector $\sigma$ or for all possible sign vectors, respectively, by*

$$\sum_{m \geq r} \binom{n}{l} \left\{ {n-l \atop m} \right\} \quad \text{and} \quad \sum_{m \geq r} 2^{n-l-1} \binom{n}{l} \left\{ {n-l \atop m} \right\}.$$

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| d / r | 2 | | 3 | | 4 | |
|---|---|---|---|---|---|---|
| 1 | 6 | 12 | | | | |
| 2 | 12 | 48 | | | | |
| 3 | 7 | 56 | 4 | 16 | | |
| 4 | | | 6 | 48 | | |
| 5 | | | 1 | 8 | | |
| 6 | | | | | 1 | 8 |

(a) $n = 4$

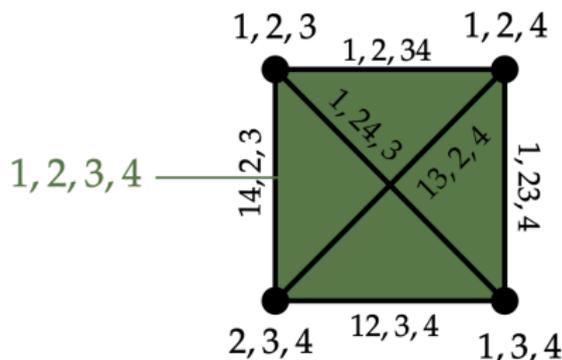| d / r | 2 | | 3 | | 4 | | 5 | |
|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 20 | | | | | | |
| 2 | 30 | 120 | | | | | | |
| 3 | 35 | 280 | 10 | 40 | | | | |
| 4 | 15 | 240 | 30 | 240 | | | | |
| 5 | | | 30 | 440 | | | | |
| 6 | | | 10 | 160 | 5 | 40 | | |
| 7 | | | 1 | 16 | 10 | 160 | | |
| 8 | | | | | | | | |
| 9 | | | | | 1 | 16 | | |
| 10 | | | | | | | 1 | 16 |

(b) $n = 5$

# Posets of Matroids

The strata of $\mathcal{L}^0_{n,r}$ form a poset: $P \leq P'$ if every loop of $P'$ is a loop in $P$, and the partition $P'$ refines the partition $P$.

Same as containment of matroid polytopes.

For $n = 4, r = 3$, there are $11 = 1 + 6 + 4$ strata.
The top stratum $\mathcal{L}^0_{U_4,3}$ has three connected components:



The strata $\mathcal{M}^0_{P,\sigma,r}$ of $\mathcal{M}^0_{n,r}$ form a poset:
$(P, \sigma) \leq (P', \sigma')$ if $P \leq P'$ and $\sigma = \sigma'$ for all non-loops of $P$.

# Inclusions and Topology

Our kinematic stratifications are nice: if a stratum intersects the closure of another stratum, then containment holds.

But, the topology of strata is quite interesting:

## Proposition

$\mathcal{M}_{P,\sigma,3}^0$ has $(m-1)!/2$ connected components.

# Inclusions and Topology

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But, the topology of strata is quite interesting:

## Proposition
$\mathcal{M}^0_{P,\sigma,3}$ has $(m-1)!/2$ connected components.

## Theorem
The kinematic stratum $\mathcal{M}^0_{P,\sigma,\leq r}$ is homotopic to the *configuration space* $F(\mathbb{S}^{r-2}, m)/\mathrm{SO}(r-1)$ for $m$ points on the sphere $\mathbb{S}^{r-2}$.

## Corollary
The stratum $\mathcal{M}^0_{P,\sigma,\leq 4}$ is homotopic to the moduli space $M_{0,m}(\mathbb{C})$, and hence to the complement of the affine braid arrangement.

E. Feichtner and G. Ziegler: *The integral cohomology algebras of ordered configuration spaces of spheres* (2000)
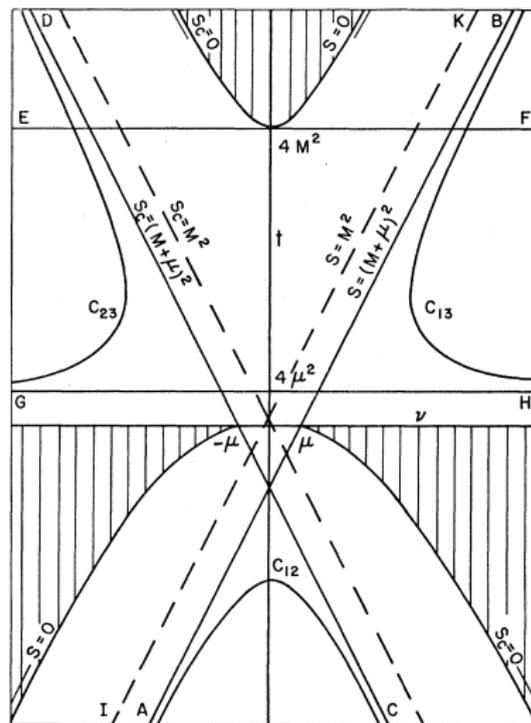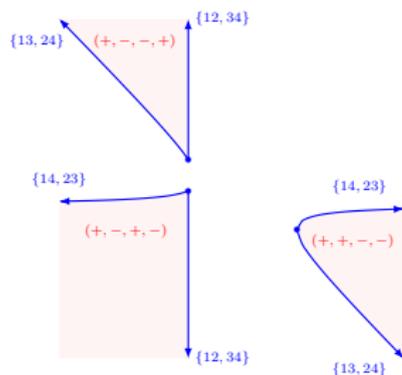
# What matters for physics?



Fig. 1. Kinematics of the reactions I, II, and III.

Stanley Mandelstam: *Determination of the pion-nucleon scattering amplitude from dispersion relations and unitarity. General theory*, Physical Review (1958).

## Momentum Conservation

The *massless momentum conserving (MMC) region* $\mathcal{C}_{n,r}^0$ is

$$\mathcal{C}_{n,r}^0 = \bigsqcup_{(P,\sigma)} \mathcal{C}_{P,\sigma,r}^0,$$

where $\mathcal{C}_{P,\sigma,r}^0$ is the intersection of $\mathcal{M}_{P,\sigma,r}^0$ with the subspace

$$\mathbb{R}^{n(n-3)/2} = \left\{ S : s_{i1} + s_{i2} + \cdots + s_{in} = 0 \quad \text{for } i = 1, 2, \ldots, n \right\}.$$

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$$\mathbb{R}^{n(n-3)/2} \;=\; \big\{ S \;:\; s_{i1}+s_{i2}+\cdots+s_{in} \;=\; 0 \quad \text{for } i=1,2,\ldots,n \big\}.$$

## Theorem
*The stratum $\mathcal{C}_{P,\sigma,r}^0$ is non-empty if and only if*

1. *For $3 \le r < m$: there exist $i,j,k,l$ in $[n]$, with $\sigma_i = \sigma_j = +$ and $\sigma_k = \sigma_l = -$, such that the restriction of the matroid $P$ to $\{i,j,k,l\}$ is either $U_4$ or $\{ik,jl\}$,*   **and**

2. *for $2 \le r = m$: each part of $P$ has elements with opposite signs.*

*In this case,* $\dim(\mathcal{C}_{P,\sigma,r}^0) \;=\; (m-1)(r-1) - \binom{r}{2} + (n-l-m) - 1.$

## Four Particles

Use the rank 3 matrix

$$S = \begin{bmatrix} 0 & x & -x-y & y \\ x & 0 & y & -x-y \\ -x-y & y & 0 & x \\ y & -x-y & x & 0 \end{bmatrix}.$$

The principal $3 \times 3$ minors are $\det(S_{ijk}) = -2xy(x+y) \geq 0$.

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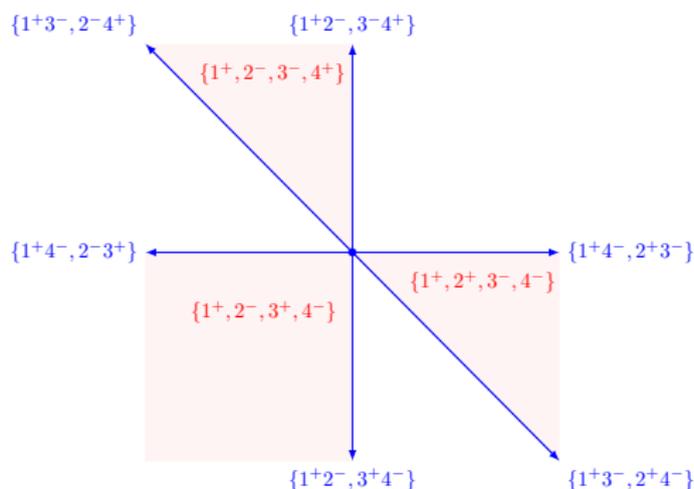The MMC region $\mathcal{C}^0_{4,\leq 3} = \mathcal{C}^0_{4,3} \cup \mathcal{C}^0_{4,2}$ has nine strata:



Figure 3: The $3 + 6$ MMC strata for $n = 4$.

## On-Shell

For $n = 4$ particles, fix masses $\mathbf{m} = (\mu, \mu, m, m)$ with $m > \mu > 0$.
We studied the MMC region $\mathcal{C}_{4,3}^{\mathbf{m}}$ by modifying our matrix:

$$
S = \begin{bmatrix}
\mu^2 & x & -x - y - \mu^2 & y \\
x & \mu^2 & y & -x - y - \mu^2 \\
-x - y - \mu^2 & y & m^2 & \mu^2 - m^2 + x \\
y & -x - y - \mu^2 & \mu^2 - m^2 + x & m^2
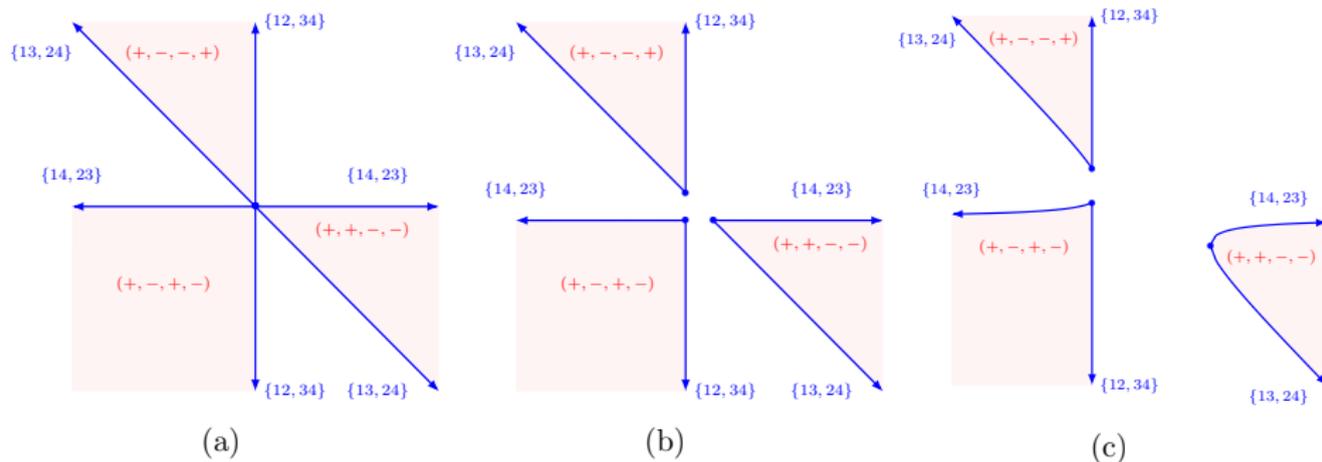\end{bmatrix}.
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Figure 4: Regions for (a) massless, (b) equal masses, and (c) two unequal masses.

# Back to 1958



{13, 24}   {12, 34}
(+, −, −, +)

{14, 23}
(+, −, +, −)
{12, 34}
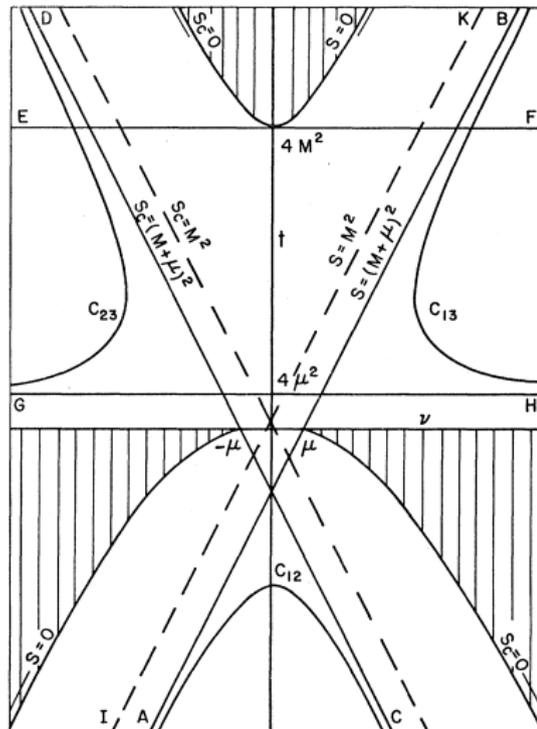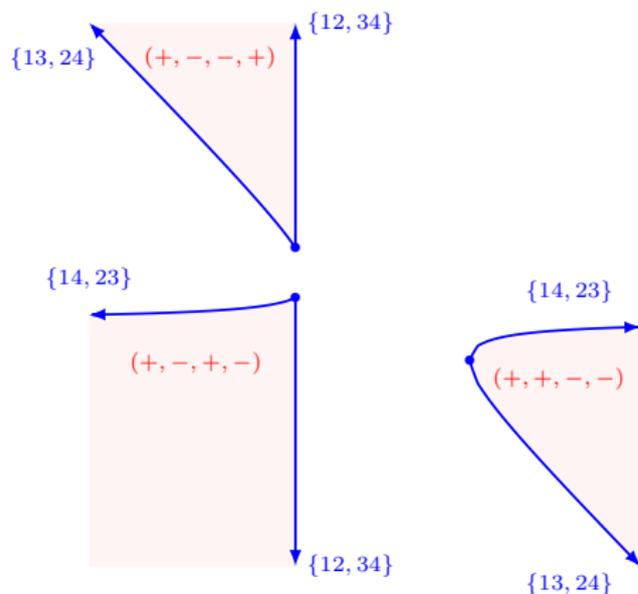
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{13, 24}

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Stanley Mandelstam: *Determination of the pion-nucleon scattering amplitude from dispersion relations and unitarity. General theory*, Physical Review (1958).

# Five Particles

| d / r | 2 | | 3 | | 4 | |
|---|---|---|---|---|---|---|
| 1 | 6 | 30 | | | | |
| 2 | 6 | 60 | 3 | 15 | | |
| 3 | | | 9 | 90 | | |
| 4 | | | 1 | 10 | | |
| 5 | | | | | 1 | 10 |

10 **cyclic polytopes** $C(4, 6)$
$f = (6, 15, 18, 9)$

$$S = \begin{bmatrix} 0 & a & -a-b+d & b-d-e & e \\ a & 0 & b & -b-c+e & -a+c-e \\ -a-b+d & b & 0 & c & a-c-d \\ b-d-e & -b-c+e & c & 0 & d \\ e & -a+c-e & a-c-d & d & 0 \end{bmatrix}.$$
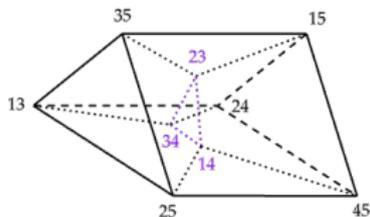
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| d / r | 2 | | 3 | | 4 | |
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| 1 | 6 | 30 | | | | |
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**Igusa quartic**
$$a^2b^2 + b^2c^2 + c^2d^2 + d^2e^2 + a^2e^2$$
$$+ 2abcd + 2abce + 2abde + 2acde + 2bcde$$
$$- 2ab^2c - 2bc^2d - 2cd^2e - 2ade^2 - 2a^2be \ < \ 0.$$



| $\sigma$ | $s_{12}$ | $s_{13}$ | $s_{14}$ | $s_{15}$ | $s_{23}$ | $s_{24}$ | $s_{25}$ | $s_{34}$ | $s_{35}$ | $s_{45}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $(-,-,+,+,+)$ | + | − | − | − | − | − | − | + | + | + |
| $(-,+,-,+,+)$ | − | + | − | − | − | + | + | − | − | + |
| $(-,+,+,-,+)$ | − | − | + | − | + | − | + | − | + | − |
| $(-,+,+,+,-)$ | − | − | − | + | + | + | − | + | − | − |
| $(+,-,-,+,+)$ | − | − | + | + | + | − | − | − | − | + |
| $(+,-,+,-,+)$ | − | + | − | + | − | − | + | − | + | − |
| $(+,-,+,+,-)$ | − | + | + | − | − | + | − | + | − | − |
| $(+,+,-,-,+)$ | + | − | − | + | − | − | + | + | − | − |
| $(+,+,-,+,-)$ | + | − | + | − | − | + | − | − | + | − |
| $(+,+,+,-,-)$ | + | + | − | − | + | − | − | − | − | + |

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Many thanks for listening