The (un?)reasonable effectiveness of mathematics in the natural sciences

Michael Miller · Department of Philosophy · University of Toronto

COMMUNICATIONS ON PURE AND APPLIED MATHEMATICS, VOL. XIII, 001–14 (1960)

The Unreasonable Effectiveness of Mathematics in the Natural Sciences

Richard Courant Lecture in Mathematical Sciences delivered at New York University, May 11, 1959

EUGENE P. WIGNER
Princeton University

"and it is probable that there is some secret here which remains to be discovered." (*C. S. Peirce*)

There is a story about two friends, who were classmates in high school, talking about their jobs. One of them became a statistician and was working on population trends. He showed a reprint to his former classmate. The reprint started, as usual, with the Gaussian distribution and the statistician explained to his former classmate the meaning of the symbols for the actual population, for the average population, and so on. His classmate was a bit incredulous and was not quite sure whether the statistician was pulling his leg. "How can you know that?" was his query. "And what is this symbol here?" "Oh," said the statistician, "this is π." "What is that?" "The ratio of the circumference of the circle to its diameter." "Well, now you are pushing your joke too far," said the classmate, "surely the population has nothing to do with the circumference of the circle."

> The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure even though perhaps also to our bafflement, to wide branches of learning. (Wigner 1960, p. 14)

“The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure even though perhaps also to our bafflement, to wide branches of learning. (Wigner 1960, p. 14)”

> The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure even though perhaps also to our bafflement, to wide branches of learning. (Wigner 1960, p. 14)

The applicability question: what explains the appropriateness of the language of mathematics for describing the physical world?

Here I am going to describe three approaches to facing up to the applicability question. I don't think that any of these approaches is ultimately entirely successful.

But I hope to use these accounts to argue that there is nothing fundamentally mysterious about the applicability of mathematics.

Claim: The applicability question is an ordinary scientific question in the sense that it admits of different theories which can fare better or worse at accounting for the phenomenon of interest.

We are warranted in holding a cautious optimism that we will eventually have a completely adequate answer to the applicability question.

What would it take to provide an adequate explanation of the applicability of mathematics to the natural world? That is, what are the data for which an adequate theory of applicability is responsible for explaining?

Wigner provides us with some such data, and I will fill in some more of my own as we go.

Data: we find unexpected mathematics in unexpected places.

Why does π which is about the ratio of the circumference of the circle to its diameter, appear in the gaussian distributions of population biology?

Data: many mathematical descriptions of physical phenomena are incredibly accurate.

E.g. The anomalous magnetic moment of the electron.

Data: mathematical structures which don't seem to correspond to anything in our experience are indispensable for describing some physical phenomena.

E.g. The complex numbers.

Data: sometimes just the right mathematical structure is waiting to be used to express a physical principle.

E.g. The group structure of the standard model.

Data: the same bits of mathematics are used to represent diverse physical situations.

Think, for example, of the harmonic oscillator. It describes physical systems as diverse as springs, pendula, circuits, and modes of quantum fields.

Data: we find unexpected mathematics in unexpected places.

Data: many mathematical descriptions of physical phenomena are incredibly accurate.

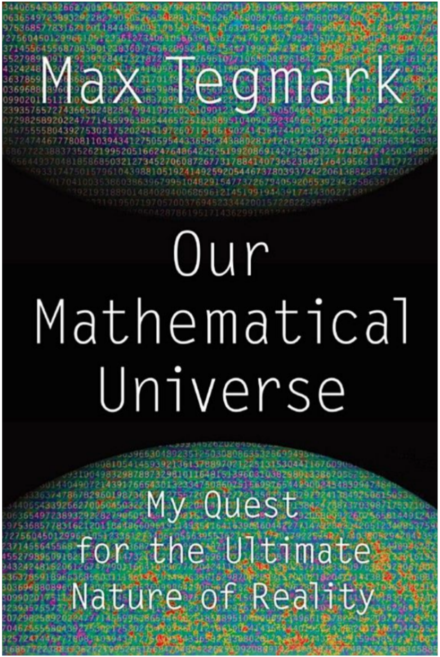
Data: mathematical structures which don't seem to correspond to anything in our experience are indispensable for describing some physical phenomena.

Data: sometimes just the right mathematical structure is waiting to be used to express a physical principle.

Data: the same bits of mathematics are used to represent diverse physical situations.

An adequate answer to the applicability question should be able to explain this data set.

Now let's consider some attempted solutions.


Max Tegmark
Our Mathematical Universe
My Quest for the Ultimate Nature of Reality

Pythagoreanism is the view that the physical realm is constituted of elements of the mathematical realm.

If this were true it would make answering the applicability question completely trivial.

On this view, saying that mathematics is applicable world is like saying that mathematics is applicable to mathematics.

That might seem like a virtue of the view, however, I think that it creates much deeper problems than this purported virtue is worth.

In particular, the statement of the view seems to me to involve a category mistake. Elements of the physical realm are enter into causal relationships and are spatiotemporally located.

The same is not true of elements of the mathematical realm. So mathematical objects seem like the wrong kind of thing to constitute the elements of the physical realm.

I think that this observation motivates a search for an alternative approach to facing up to the applicability question.

Before introducing you to another such attempt (the one endorsed by the majority of contemporary of philosophers), I need to introduce you to two very influential arguments.

There is a large debate in the philosophy of science concerning scientific realism and anti-realism. There are widely varied positions that fall under each of these headings.

But roughly, realists are those that maintain that our best scientific theories provide us with a literally true (or approximately true) account of what the world is like. Anti-realists deny this in one way or another.

The most widely advanced argument in favor of realism is called the no miracles argument.

It holds that if science didn't provide at least approximately true descriptions of reality, then its success would be a miracle.

And since we shouldn't believe in miracles, we should believe that science provides approximately true descriptions of reality.

The most widely advanced argument in favor of realism is called the pessimistic meta-induction.

It holds that in many instances of what were once regarded as successful scientific theories, we have come to learn that the central terms of those theories do not refer to anything real in the world.

Inductively we then conclude that we should think that we are in the same epistemic position as proponents of those past theories.

So the pessimistic meta-induction suggests that the central theoretical terms of our most successful theories probably do not refer to real physical things.

VII

STRUCTURAL REALISM: THE BEST OF BOTH WORLDS?

JOHN WORRALL

Presently accepted physical theories postulate a curved space-time structure, fundamental particles, and forces of various sorts. What we can know for sure on the basis of observation, at most, are only facts about the motions of macrosopic bodies, the tracks that appear in cloud chambers in certain circumstances, and so on. Most of the content of the basic theories in physics goes 'beyond' the 'directly observational'—no matter how liberal a conception of the 'directly observational' is adopted. What is the status of the genuinely theoretical, observation-transcendent content of our presently accepted theories? Most of us unreflectingly take it that the statements in this observation-transcendent part of the theory are attempted descriptions of a reality lying 'behind' the observable phenomena: that those theories really do straightforwardly assert that space-time is curved in the presence of matter, that electrons, neutrinos, and the rest exist and do various funny things. Furthermore, most of us unreflectingly take it that the enormous empirical success of these theories legitimizes the assumption that these descriptions of an underlying reality are accurate, or at any rate 'essentially' or 'approximately' accurate. The main problem of scientific realism, as I understand it, is that of whether or not there are, after reflection, good reasons for holding this view that most of us unreflectingly adopt.

Worall's stuctural realism is a view which intends to meet the challenge posed by both of these arguments.

It aims to do so by committing to the view that what is preserved on theory change is structure. It is fine that some of the objects that the past theory referred to did not exist because all we should have been committed to was the structure.

The view comes in two main strains:

- Epistemic structural realism: all we know is structure.
- Ontic structural realism: all there is is structure.

But what sort of thing is structure? It can't just be mathematical structure or we collapse into a view that exhibits the same problem as pythagoreanism did.

In some structural realist work the notion of structure is captured set theoretically, and physical systems are taken to exhibit set structure in the same way as mathematical objects.

To make sense of this view, we also need to be able to account for approximation and idealization.

Many of the scientific representations that we take to be successful do not perfectly mirror the structure of the system we are representing. So we need to retreat to partial isomorphism.

Another issue is that the ontic version of the view seems to commit to the existence of relations without relata.

Data: we find unexpected mathematics in unexpected places.

Data: many mathematical descriptions of physical phenomena are incredibly accurate.

Data: mathematical structures which don't seem to correspond to anything in our experience are indispensable for describing some physical phenomena.

Data: sometimes just the right mathematical structure is waiting to be used to express a physical principle.

Data: the same bits of mathematics are used to represent diverse physical situations.

Despite these problems, and despite not being designed to be an answer to the applicability question, the view fares reasonably well at accounting for our data.

*ERNAN McMULLIN**

GALILEAN IDEALIZATION

> Really powerful explanatory laws of the sort found in theoretical physics do not state the truth . . . We have detailed expertise for testing the claim of physics about what happens in concrete situations. When we look to the real implications of our fundamental laws, they do not meet these ordinary standards . . . We explain by *ceteris paribus* laws, by composition of causes, and by approximations that improve on what the fundamental laws dictate. In all of these cases, the fundamental laws patently do not get the facts right.[1]

IN GALILEO'S dialogue, *The New Sciences,* Simplicio, the spokesman for the Aristotelian tradition, objects strongly to the techniques of idealization that underlie the proposed 'new science' of mechanics. He urges that they tend to falsify the *real* world which is not neat and regular, as the idealized laws would make it seem, but complicated and messy. In a provocatively titled recent book, Nancy Cartwright argues a similar thesis, although on the basis of very different arguments to those of Simplicio. Her theme is that the theoretical laws of physics, despite their claims to be fundamental truths about the universe, are in fact false. They *do* have broad explanatory power, and therein lies their utility. But explanatory power (in Cartwright's view) has nothing to do with truth; indeed, the two tend to exclude one another. Idealization in physics, though permissible on pragmatic grounds, is thus not (as the Galilean tradition has unifor~~mly~~ assumed) truth-producing.

> “The Book of Nature is not written in the language of mathematics, strictly speaking. The syntax is mathematical, but the semantics is not. And both semantics and syntax are needed to constitute a language. The semantics of such terms as ‘mass’ and ‘energy’ is physical, even though m and E can be manipulated by an algebraic syntax. The Book of Nature, though it employs a mathematical grammar, is written in the language of physics (or chemistry or biology . . .).”

This is my preferred way to answer the applicability question. Unfortunately it too faces difficulties.

The most pressing of these is that in order to understand the inferential role that the mathematical semantics plays in deducing consequences from physically interpreted syntax we need to pass back and forth between the physical and mathematical semantics. And we do not yet have a systematic theory of such a mixed semantics.

Data: we find unexpected mathematics in unexpected places.

Data: many mathematical descriptions of physical phenomena are incredibly accurate.

Data: mathematical structures which don't seem to correspond to anything in our experience are indispensable for describing some physical phenomena.

Data: sometimes just the right mathematical structure is waiting to be used to express a physical principle.

Data: the same bits of mathematics are used to represent diverse physical situations.

In at least some specific cases, the physical semantics approach is able to account for significant portions of the data set.

Conclusion

The applicability question: what explains the appropriateness of the language of mathematics for describing the physical world?

Claim: The applicability question is an ordinary scientific question in the sense that it admits of different theories which can fare better or worse at accounting for the phenomenon of interest.

We are warranted in holding a cautious optimism that we will eventually have a completely adequate answer to the applicability question.