Asymptotic fields in QFT from the point of view of BV-BFV formalism

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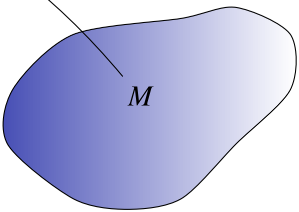
[1] joint work with Michele Schiavina, [arXiv:2002.09957]

Outline of the talk

BV-BFV data

- $(\mathfrak{F}, \Omega, S, Q)$



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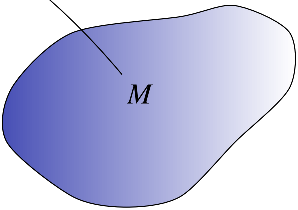

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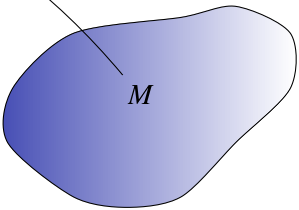

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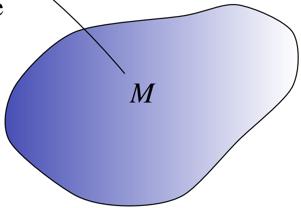

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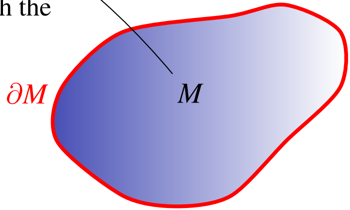

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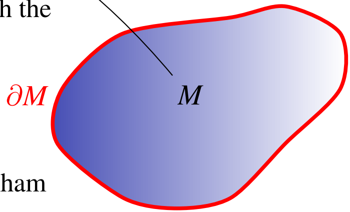

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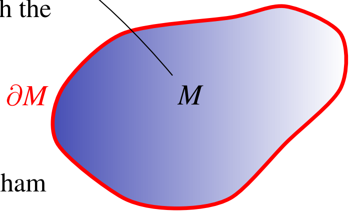

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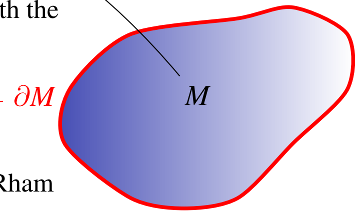

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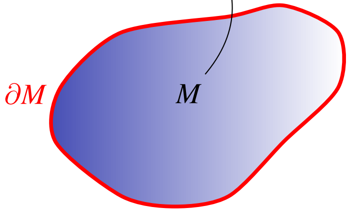

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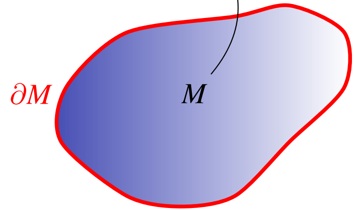

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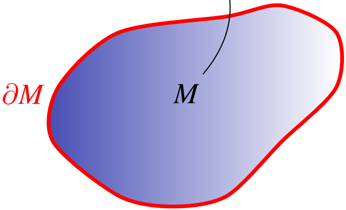

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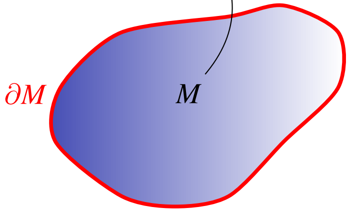

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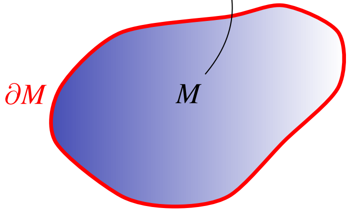

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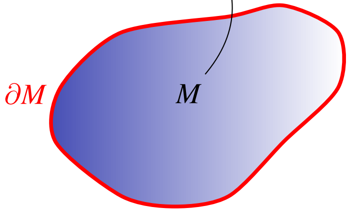

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We can generalize this and assign data to corners, etc.

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- Recent attention: works of Strominger et.al., including *New symmetries of QED* (2015), relate asymptotic charges to the *Weinberg soft photon theorem* and *memory effects*.

A little controversy…

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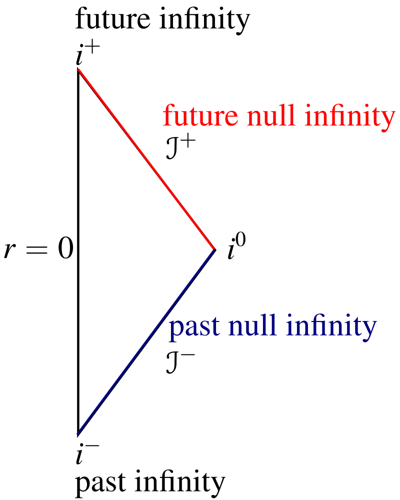
- So are LGTs symmetries or not?
- Depends… In fact, one needs to include higher BV-BFV data to answer this!

Asymptotic structure of (scalar) QED



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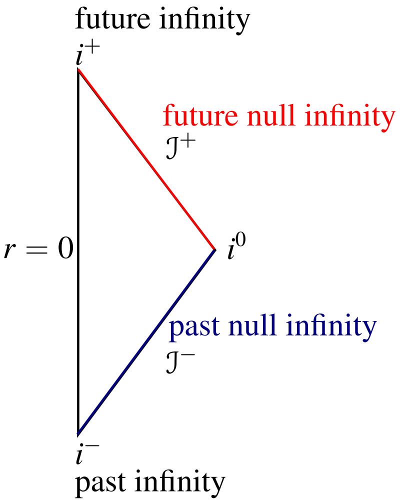

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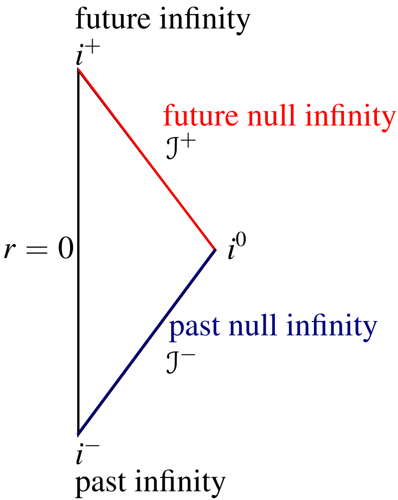

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- We focus on Minkowski spacetime M (see the diagram on the left).

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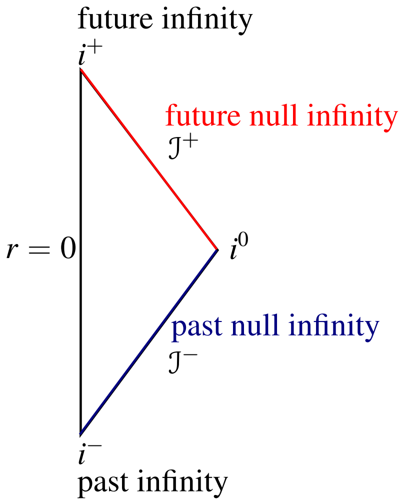

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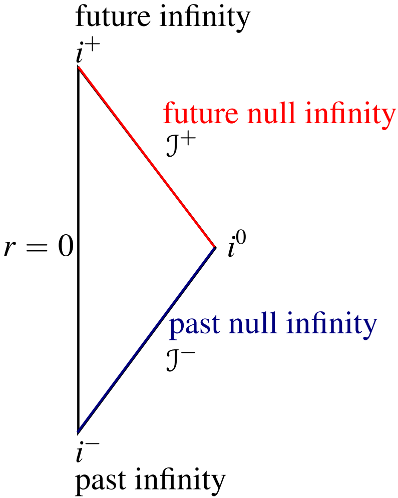

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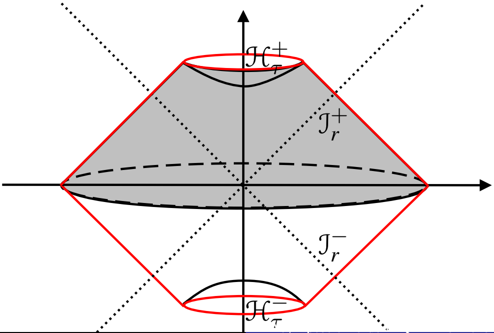
- Matter fields $\varphi \in \mathcal{C}^\infty(M, \mathbb{C})$ have asymptotes living on $i^\pm$.

Geometrical setup

Consider the theory of a finite region $\mathcal{W}_R \subset \mathbb{M}$, bounded by a piecewise-null and piecewise spacelike boundary

$$\partial\mathcal{W}_R := \mathcal{I}_r^+ \cup \mathcal{I}_r^- \cup \mathcal{H}_\tau^+ \cup \mathcal{H}_\tau^-$$

with r (radial coordinate) and τ (hyperbolic time $\tau = \sqrt{t^2 - r^2}$) get scaled by a parameter R that we send to $R \to \infty$.

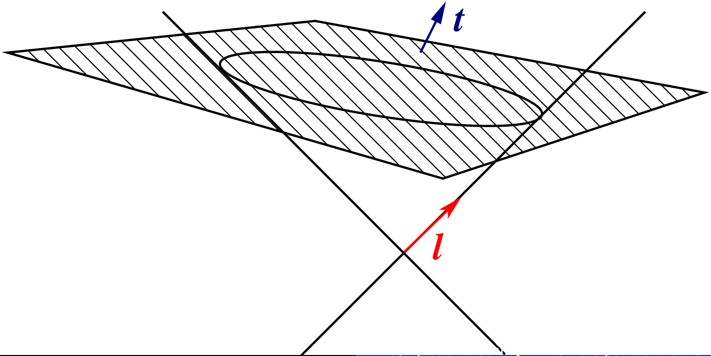


Asymptotic fields I

- Following Herdegen (JMP 95), we introduce variables $R, s, \boldsymbol{l}$:

$$x = R\boldsymbol{l} + s\frac{\boldsymbol{t}}{\boldsymbol{t}\cdot\boldsymbol{l}}\,,$$

for a fixed time-like vector $\boldsymbol{t}$.



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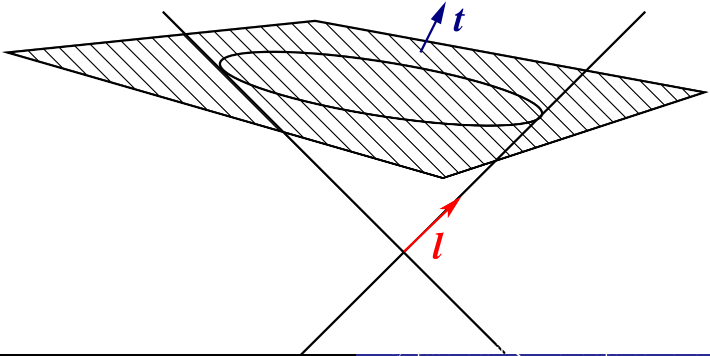
- Null asymptotics is obtained by taking $R \to \infty$.



BV data for scalar QED I

- The canonical shifted symplectic structure Ω on $\mathcal{F}$ is

$$\Omega = \int_M \delta A \delta A^\ddagger + \delta c \delta c^\ddagger + \delta \varphi \delta \varphi^\ddagger + \delta \overline{\varphi} \delta \overline{\varphi}^\ddagger,$$

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- The BV-extended action functional is given by:

$$S = \int_M \left(-\tfrac{1}{8\pi} F_A \wedge \star F_A + \tfrac{1}{2}\left(d_A\overline{\varphi} \wedge \star d_A\varphi + m^2\overline{\varphi}\varphi\right)\right) + \text{antifield terms}$$

where $d_A\varphi = d\varphi + iqA\varphi$ and $d_A\overline{\varphi} = d\overline{\varphi} - iqA\overline{\varphi}$.

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- The BV operator Q is given by

$$\begin{aligned}
QA &= d_A c & QA^\ddagger &= -\frac{1}{4\pi} d_A \star F_A - iq\,\overline{\varphi} \star d_A\varphi + iq \star d_A\overline{\varphi}\varphi \\
Q\varphi &= c\varphi & Q\varphi^\ddagger &= (-d_A \star d_A + m^2)\overline{\varphi} + \varphi^\ddagger c \\
Q\overline{\varphi} &= -c\overline{\varphi} & Q\overline{\varphi}^\ddagger &= (-d_A \star d_A + m^2)\varphi - \overline{\varphi}^\ddagger c \\
Qc &= 0 & Qc^\ddagger &= 0
\end{aligned}$$

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- We assume that $J \sim \tau^{-3}$.

BV data for scalar QED II

- The boundary action is given by

$$S^{\partial} = \frac{1}{2}\iota_Q\iota_Q\Omega = \int_M d\left[c\left(-\frac{1}{4\pi}d_A \star F_A + J\right)\right], \qquad (*)$$

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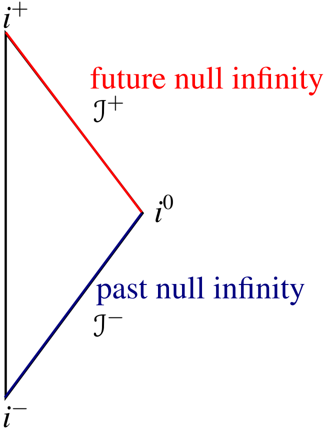
- The BFV operator Q^{∂} is the Hamiltonian vector field of S^{∂}, i.e.

$$\iota_{Q^{\partial}}\Omega^{\partial} = \delta S^{\partial}.$$

Asymptotic fields II

future infinity: $s = +\infty$



past infinity: $s = -\infty$

Asymptotic fields II

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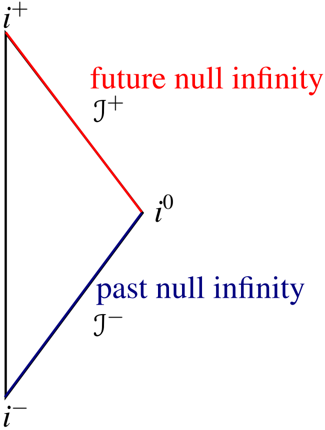

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Asymptotic variables:

Asymptotic fields II

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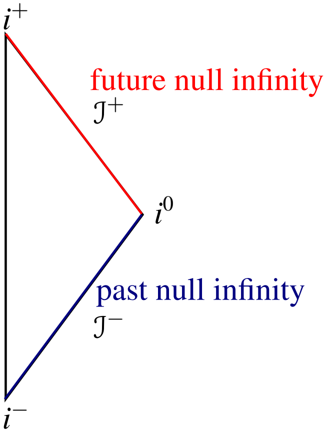

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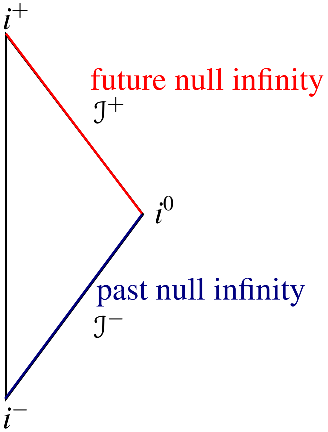

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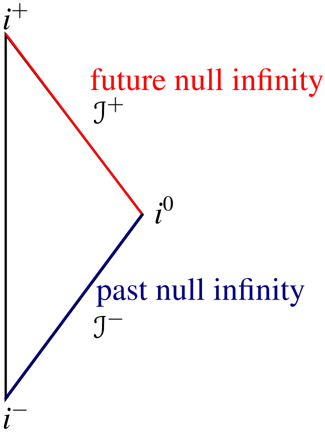

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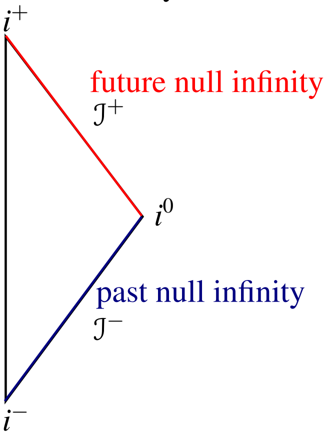
Asymptotic fields II

future infinity: $s = +\infty$



past infinity: $s = -\infty$

Asymptotic variables:

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- $\lim\limits_{R\to\infty} RA(x - R\boldsymbol{l}) = V'(x \cdot \boldsymbol{l}, \boldsymbol{l})$
- For $J = 0$, we have $V(+\infty, \boldsymbol{l}) = V'(-\infty, \boldsymbol{l}) = 0$. In general these asymptotes are given in terms of J.

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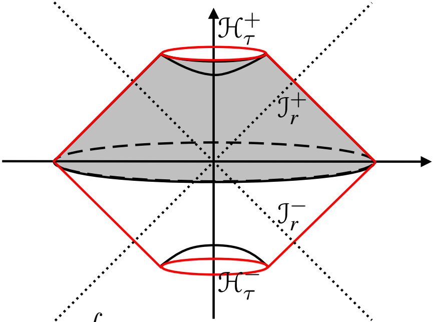
- Observation (due to Herdegen): for non-trivial asymptotic charge to exist, $\hat{A}$ cannot be in the Lorenz gauge.
- Such gauge transformations (often called large gauge transformations) do not preserve the symplectic structure on ∂M, but the failure to do so is governed by data assigned to the corner, i.e. $\partial\partial M$.

Calculating the charge from the boundary action

$$S^{\partial} = \lim_{R\to\infty} S^{\partial}_{\mathcal{W}_R} = \underbrace{-\frac{1}{4\pi}\lim_{R\to\infty}\int_{\partial\mathcal{W}_R} c d_A \star F_A}_{S^{\partial}_{\text{soft}}} + \underbrace{\lim_{R\to\infty}\int_{\partial\mathcal{W}_R} cJ}_{S^{\partial}_{\text{hard}}}\,,$$



(we used $\int_M d(c d_A \star F_A) = \int_M d(d_A c \wedge \star F_A)$ in (*) to rewrite the first term.)

Soft charge

- Recall that $\Box A = 4\pi J$ and there exist unique retarded/advanced Green functions $\Delta^{\mathrm{R/A}}$. Define $A^{\mathrm{R/A}} = 4\pi\Delta^{\mathrm{R/A}}J$ as the retarded/advanced solutions and split:

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- On-shell we find that:

$$S^{\partial}_{\mathrm{soft},\mathcal{I}^{+}} \approx \frac{1}{4\pi}\int_{S^2} V^{\epsilon^{+}}(\boldsymbol{l})V^{\mathrm{out}}(-\infty,\boldsymbol{l})d^2\boldsymbol{l} \equiv Q^{\mathrm{soft},+}_{\epsilon^{+}}$$

$$S^{\partial}_{\mathrm{soft},\mathcal{I}^{-}} \approx -\frac{1}{4\pi}\int_{S^2} V^{\epsilon^{-}}(\boldsymbol{l})V^{\mathrm{in}}(+\infty,\boldsymbol{l})d^2\boldsymbol{l} \equiv -Q^{\mathrm{soft},-}_{\epsilon^{-}}$$

Hard charge and the total charge

- The hard charge is computed in the similar way, by splititng

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- Definitng $Q_\epsilon^\pm \doteq Q_{\epsilon^\pm}^{\mathrm{soft},\pm} + Q_{\epsilon^\pm}^{\mathrm{hard},\pm}$, we conclude that

$$S^\partial \approx 0 \Rightarrow Q_\epsilon^+ \approx Q_\epsilon^- \,.$$

Higher BFV data saves the day!

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- The right hand side of the above formula can be interpreted as the canonical 1-form on $\mathcal{F}^{\partial\partial}_{\partial\partial M}$, denoted by $\Omega^{\partial\partial}$, so we have:

$$\mathcal{L}_{Q^{\partial}} \Omega^{\partial} = \pi^* \Omega^{\partial\partial} \,.$$

Thank you for your attention!