# Extracting bigravity from string amplitudes

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# Overview

**Spin-2 field theory**

**From fields to strings**

**String amplitudes**

**From strings back to fields**

# Gravity

**1.** field theory: Einstein's GR

- Lorentz invariance + locality $\overset{\nexists \text{ ghost}}{\Longrightarrow}$ diffeos

- unique kinematics of $g_{\mu\nu}$

  Gupta, Kraichnan, Weinberg, Boulware, Deser, ... '50s – '80s

**2.** string theory: closed string spectra

- massless level: $\alpha^i_{-1}\overline{\alpha}^j_{-1}|0\rangle \Rightarrow (g_{\mu\nu}, B_{\mu\nu}, \phi)$

  Scherk and Schwarz 1974

- effective Lagrangian from three– and four–point amplitudes

  Green, Schwarz, Brink 1982
  Gross, Sloan 1987

# Ghost–free bimetric theory

- two dynamical metrics:

$$\mathcal{L}_{\mathrm{HR}} = m_g^2 \sqrt{g} R(g) + m_f^2 \sqrt{f} R(f) - 2\, m_g^2 m_f^2 \sqrt{g}\, V(S;\beta_n)$$

$$V(S;\beta_n) = \sum_{n=0}^{4} \beta_n e_n(S) \quad , \quad S^{\mu}{}_{\nu} = \left(\sqrt{g^{-1} f}\right)^{\mu}{}_{\nu}\,,\, \alpha \equiv m_f/m_g$$

Hassan, Rosen '11

- inspiration: ghost–free massive gravity

$$\mathcal{K}^{\mu}_{\nu} \equiv \delta^{\mu}_{\nu} - \left(\sqrt{g^{-1} f}\right)^{\mu}_{\nu}$$

de Rham, Gabadadze, Tolley '10, '11

7 propagating d.o.f.

no ghosts; else: Boulware, Deser 1972

# String embedding

Idea: extract info on $V(S;\beta_n)$ from string scattering amplitudes

- Plan:
  1. expand: treat bimetric theory *peturbatively*
  2. identify: field $\leftrightarrow$ string state
  3. compute: string states as on–shell asymptotic states
  4. extract effective action and compare

1st non–trivial case: cubic interactions $\Rightarrow$ 3–point amplitudes

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# Bimetric expansion, quadratic level

- expand around proportional bkgs (we choose equal Minkowski)

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu} \quad , \quad f_{\mu\nu} = \eta_{\mu\nu} + \delta f_{\mu\nu}$$

- mass eigenstates:

$$G_{\mu\nu} \equiv m_g \left(\delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}\right) \quad , \quad M_{\mu\nu} \equiv \alpha m_g \left(\delta f_{\mu\nu} - \delta g_{\mu\nu}\right)$$

- quadratic level:

$$\begin{aligned}
\mathcal{L}^{(2)}(G) &= \tfrac{1}{2} G^{\mu\nu}\, \hat{\mathcal{E}}^{\rho\sigma}_{\mu\nu} G_{\rho\sigma} \\
\mathcal{L}^{(2)}(M) &= \tfrac{1}{2} M^{\mu\nu}\, \hat{\mathcal{E}}^{\rho\sigma}_{\mu\nu} M_{\rho\sigma} - \tfrac{m_{\mathrm{FP}}^2}{4} \left([M^2] - [M]^2\right)
\end{aligned}$$

Fierz, Pauli 1939

$$m_{\mathrm{FP}}^2 \equiv m_g^2 (1+\alpha^2) \left(\beta_1 + 2\beta_2 + \beta_3\right) \quad , \quad m_{\mathrm{Pl}}^2 \equiv m_g^2 (1+\alpha^2)$$

Hassan, Schmidt-May, von Strauss '12

- $M_{\mu\nu}$: viable DM candidate

Aoki, Mukohyama '16
Babichev, Marzola, Raidal, Schmidt-May, Urban, Veerm, von Strauss '16

# Bimetric expansion, cubic order

$$
\begin{aligned}
\mathcal{L}_{\mathrm{GM}^2} \quad = \quad & \frac{m_{\mathrm{Pl}}}{8}(\beta_1 + 2\beta_2 + \beta_3)\left[[G][M]^2 - 4[M][GM] - [G][M^2] + 4[GM^2]\right] \\
& + \frac{1}{4m_{\mathrm{Pl}}}\Big[G^{\mu\nu}\big(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\mu[M]\partial_\nu[M] + 2\partial_\nu[M]\partial_\rho M^\rho_\mu \\
& + 2\partial_\nu M^\rho_\mu \partial_\rho[M] - 2\partial_\rho[M]\partial^\rho M_{\mu\nu} + 2\partial_\rho M_{\mu\nu}\partial_\sigma M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu \\
& - 2\partial_\rho M_{\nu\sigma}\partial^\sigma M^\rho_\mu + 2\partial_\sigma M_{\nu\rho}\partial^\sigma M^\rho_\mu\big) + \frac{1}{2}[G]\big(\partial_\rho[M]\partial^\rho[M] \\
& - \partial_\rho M_{\mu\nu}\partial^\rho M^{\mu\nu} - 2\partial_\rho[M]\partial_\mu M^{\mu\rho} + 2\partial_\rho M_{\mu\nu}\partial^\nu M^{\mu\rho}\big)\Big] \\
& + \frac{1}{2m_{\mathrm{Pl}}}\Big[M^{\mu\nu}\big(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\mu[G]\partial_\nu[M] + \partial^\rho G_{\rho\mu}\partial_\nu[M] \\
& + \partial_\nu G_{\mu\rho}\partial^\rho[M] - \partial_\rho G_{\mu\nu}\partial^\rho[M] + \partial_\rho G^{\rho\sigma}\partial_\sigma M_{\mu\nu} - 2\partial_\mu G^{\rho\sigma}\partial_\sigma M_{\nu\rho} \\
& + \partial_\mu[G]\partial^\rho M_{\rho\nu} + \partial^\rho G_{\mu\nu}\partial^\sigma M_{\rho\sigma} - 2\partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma} - 2\partial^\rho G_{\mu\sigma}\partial^\sigma M_{\nu\rho} \\
& + 2\partial^\rho G_{\mu\sigma}\partial_\rho M^\sigma_\nu + \partial^\rho[G]\partial_\nu M_{\mu\rho} - \partial^\rho[G]\partial_\rho M_{\mu\nu}\big) + \frac{1}{2}[M]\big(\partial_\rho[G]\partial^\rho[M] \\
& - \partial_\rho G_{\mu\nu}\partial^\rho M^{\mu\nu} - \partial_\rho[G]\partial_\sigma M^{\rho\sigma} - \partial_\rho G^{\rho\sigma}\partial_\sigma[M] + 2\partial_\rho G_{\mu\nu}\partial^\nu M^{\mu\rho}\big)\Big]
\end{aligned}
$$

Babichev, Marzola, Raidal, Schmidt-May, Urban, Veerm, von Strauss '16

# Bimetric expansion, cubic order

| Vertex | 0–derivative | two–derivative |
|---|---|---|
| $G^3$ | no | yes (Ricci) |
| $M^3$ | yes | yes (Ricci) |
| $GM^2$ | yes | yes |

- $G_{\mu\nu}$ *is* the graviton also @ cubic level
  graviton self–interactions: strictly GR

  Boulanger, Damour, Gualtieri, Henneaux 2000

- couplings depend on $(m_g, m_f, \beta_n)$ nontrivially

- no $G^2M$ term

Babichev, Marzola, Raidal, Schmidt-May, Urban, Veerm, von Strauss '16

# On-shell conditions

- string theory: operator–state correspondence
  1. on–shell mass

  2. transversality and tracelessness

  on–shell asymptotic states ⇒ *on–shell* string amplitudes

- field theory:
  1. $\Box G_{\mu\nu} = 0 \quad , \quad \partial^\mu G_{\mu\nu} = 0 \quad , \quad [G] = 0 \qquad \Rightarrow 2$ d.o.f.

  2. $(\Box - m^2_{\rm FP})M_{\mu\nu} = 0\,,\, \partial^\mu M_{\mu\nu} = 0\,,\, [M] = 0 \qquad \Rightarrow 5$ d.o.f.

  we have to impose these on the cubic vertices

# On-shell bimetric cubic

On–shell conditions + partial integrations ⇒

$$\mathcal{L}_{\mathrm{G}^3} = \frac{1}{m_g\sqrt{1+\alpha^2}}\, G^{\mu\nu}\left(\partial_\mu G_{\rho\sigma}\partial_\nu G^{\rho\sigma} - 2\partial_\nu G_{\rho\sigma}\partial^\sigma G^\rho_\mu\right)$$

$$\mathcal{L}_{\mathrm{GM}^2} = \frac{1}{m_g\sqrt{1+\alpha^2}}\left[G^{\mu\nu}\left(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\right) + 2M^{\mu\nu}\left(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma}\right)\right]$$

$$\mathcal{L}_{\mathrm{M}^3} = \frac{(-\beta_1+\beta_3)\,(1+\alpha^2)^{3/2}m_g}{6\,\alpha}\,[M^3] + \frac{(1-\alpha^2)}{m_g\alpha\sqrt{1+\alpha^2}}\, M^{\mu\nu}\left(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 2\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\right)$$

Lüst, CM, Mazloumi, Stieberger ’21

# On-shell bimetric cubic

Observations:

- 4 free parameters: ($m_g$, $\alpha$, $\beta_1$, $\beta_3$)
- no–derivative terms: $[GM^2]$ disappears, $[M^3]$ survives
- derivative terms: they are *all* possible Lorentz invariant terms with TT rank–2 tensors (up to partial integrations), excluding $G^2M$

these terms are not unique to ghost–free bimetric theory, but the particular coefficients and couplings are

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# Identification of states

$M_{\mu\nu}$: closed or open string state? motivation from

Ferrara, Kehagias, Lüst '19

- *we choose* the first massive spin–2 state that appears on open string towers for the present work

$$b^i_{-3/2}|0\rangle \quad , \quad \alpha^i_{-1} b^j_{-1/2}|0\rangle$$

$$m_g^2(1+\alpha^2)\,(\beta_1 + 2\beta_2 + \beta_3) \stackrel{!}{=} \frac{1}{\alpha'}$$

- other possibilities under investigation

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# Scattering of open and closed strings

- tree–level disk amplitude:

$$\mathcal{A}(N_o, N_c) \quad = \quad \sum_\sigma \left( \int_{\mathcal{I}_\sigma} \prod_{j=1}^{N_o} \mathrm{d}x_j \prod_{i=1}^{N_c} \int_{\mathcal{H}_+} \mathrm{d}^2 z_i \right) V_{\mathrm{CKG}}^{-1}$$

$$\langle \prod_{j=1}^{N_o} : V_o(x_j) : \prod_{i=1}^{N_c} : V_c(z_i, \overline{z}_i) : \rangle_{\mathbb{D}_2}$$

Stieberger '09, Stieberger, Taylor '15

- massless external states: late '90s

Klebanov, Thorlacius
Gubser, Hashimoto, Klebanov, Maldacena
Garousi, Myers
Hashimoto, Klebanov

- 1 massive and other massless

Feng, Lüst, Schlotterer, Stieberger, Taylor '10

- our novelty:
  1. *all* external states are either helicity–2 or spin–2
  2. at least *two* external states are massive

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# Vertex operators

- Graviton $G_{\mu\nu}$:

$$V_G^{(0,0)}(z,\overline{z},\varepsilon,q) = -\tfrac{2g_c}{\alpha'}\,\varepsilon_{\mu\nu}\left[i\overline{\partial}X^\mu + \tfrac{\alpha'}{2}(q\widetilde{\psi})\widetilde{\psi}^\mu(\overline{z})\right] \times\left[i\partial X^\nu + \tfrac{\alpha'}{2}(q\psi)\psi^\nu(z)\right] e^{iqX(z,\overline{z})}$$

$$\varepsilon_{\mu\nu}q^\mu = \varepsilon_{\mu\nu}q^\nu = 0 \quad , \quad q^2 = 0 \quad , \quad \varepsilon_{\mu\nu} = \varepsilon_{\nu\mu} \quad , \quad \varepsilon_{\mu\nu}\eta^{\mu\nu} = 0$$

eg. Mayr, Stieberger '94

- massive spin–2 $M_{\mu\nu}$:

$$V_M^{(-1)}(x,\alpha,k) = \tfrac{g_o}{(2\alpha')^{1/2}} T^a\, e^{-\phi(x)}\,\alpha_{\mu\nu}\, i\partial X^\mu(x)\psi^\nu(x)\; e^{ikX(x)}$$

$$V_M^{(0)}(x,\alpha,k) = \tfrac{g_o}{(2\alpha')} T^a\,\alpha_{\mu\nu}\left[i\partial X^\mu(x)\partial X^\nu(x) - 2i\alpha'\partial\psi^\mu(x)\psi^\nu(x) +2\alpha'\,(k\psi)(x)\,\psi^\nu(x)\partial X^\mu(x)\right]e^{ikX(x)}$$

$$\alpha_{\mu\nu}k^\mu = 0 \quad , \quad k^2 = -\tfrac{1}{\alpha'} \quad , \quad \alpha_{\mu\nu} = \alpha_{\nu\mu} \quad , \quad \alpha_{\mu\nu}\eta^{\mu\nu} = 0$$

Koh, Troost, van Proeyen 1987
Feng, Lüst, Schlotterer, Stieberger, Taylor '10
Bianchi, Guerrieri '15

# One graviton and two massive spin–2 states

$$\mathcal{A}(2,1) = \int_{\mathcal{R}} \int_{\mathcal{H}_+} \frac{\mathrm{d}x_1 \mathrm{d}x_2 \, \mathrm{d}^2 z}{V_{\mathrm{CKG}}}$$

$$\langle : V_M^{(-1)}(x_1, \alpha_1, k_1) :: V_M^{(-1)}(x_2, \alpha_2, k_2) :: V_G^{(0,0)}(z, \overline{z}, \varepsilon, q) : \rangle_{\mathbb{D}_2}$$

- heavy brane $\Rightarrow$ momentum conservation along the brane

$$(k_1 + k_2 + q_{\|})^\mu = 0$$

$$q_{\|} = \tfrac{1}{2}(q + D \cdot q) \quad , \quad q_\perp = \tfrac{1}{2}(q - D \cdot q)$$

- can define Mandelstam (!)

$$s \equiv \alpha'(k_1 + k_2)^2 \, , \, t \equiv \alpha'(k_1 + q/2)^2 \, , \, u \equiv \alpha'(k_1 + Dq/2)^2$$

$\Rightarrow \exists$ a single kinematic invariant, eg. $s = -2 + 2\alpha' k_1 k_2$

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# One graviton and two massive spin–2 states

- we compute all contractions using correlators on $\mathbb{D}_2$
- $PSL(2,\mathbb{R})$: fix *three* vertex op. positions + insert c–ghosts
  $\Rightarrow$ integral over real line

$$\mathcal{A}(2,1) = \frac{g_c}{\alpha'^2}\ \mathrm{Tr}\left(T^a T^b\right) \sum_{i=1}^{4} \mathbf{A}_i$$

$$\mathbf{A}_4 \quad = \quad 4^s\, \alpha^1_{\kappa\lambda}\alpha^2_{\rho\sigma}\varepsilon_{\mu\nu}g^{\lambda\sigma} \int_{-\infty}^{\infty} dx\, |x|^{s+2}(x^2+1)^{-s}\, \tfrac{(x-i)(x+i)}{(2x)^4} \Big\{ A^{\mu\nu\kappa\rho} + \tfrac{B^{\mu\nu\kappa\rho}}{(x-i)(x+i)} + \tfrac{C^{\mu\nu\kappa\rho}}{(x+i)^2} + \tfrac{\widetilde{\Delta}^{\mu\nu\kappa\rho}}{(x-i)^2} + i\,\tfrac{E^{\mu\nu\kappa\rho}}{x+i} + i\,\tfrac{F^{\mu\nu\kappa\rho}}{x-i} \Big\}$$

$$A^{\mu\nu\kappa\rho} \quad = \quad -\tfrac{1}{16}\alpha'^3\, qDq\, D^{\mu\nu}k_2^\kappa k_1^\rho + \tfrac{1}{8}\alpha'^2\, D^{\mu\nu}\left(k_1^\rho k_2^\kappa + \tfrac{1}{4}g^{\kappa\rho}\, qDq\right) - \tfrac{1}{16}\alpha'\, D^{\mu\nu}g^{\kappa\rho}$$

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# One graviton and two massive spin–2 states

- we have computed all 36 kinematic packages
- exact expressions, terms organized in orders of $\alpha'$:
- partial fractioning, evaluation of integrals:

$$\mathbf{A}_4 \quad = \quad \frac{1}{16}\,4^s\left\{2A\,\frac{\sqrt{\pi}2^{-s}s\,\Gamma\left(\frac{s-1}{2}\right)}{\Gamma\left(\frac{s}{2}+1\right)} - (C+\widetilde{\Delta})\,\frac{\sqrt{\pi}2^{-s}\Gamma\left(\frac{s-1}{2}\right)}{\Gamma\left(\frac{s}{2}+1\right)}\right.$$

$$\left. +(E-F)\,\frac{(s-1)\left[\Gamma\left(\frac{s-1}{2}\right)\right]^2}{4\Gamma(s)}\right\},$$

$$E = \alpha^1_{\kappa\lambda}\alpha^2_{\rho\sigma}\varepsilon_{\mu\nu}\,g^{\lambda\sigma}\left[2E^{\mu\nu\kappa\rho}+B^{\mu\nu\kappa\rho}\right]$$

- schematically: $\mathbf{A}_i = \mathcal{K}(k_1, k_2, q; \alpha') \times \mathcal{I}(s)$

we have computed the *full* amplitude, valid in both 10D and 4D and for any D–brane dimension

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# Three massive spin–2 states

$$
\begin{aligned}
\mathcal{A}(3,0) \quad = \quad & \tfrac{g_o}{4\alpha'^3}\,\mathrm{Tr}(T^{a_1}\{T^{a_2},T^{a_3}\})\Big\{3\,(2\alpha')^2\,\mathrm{Tr}(\alpha^1\cdot\alpha^2\cdot\alpha^3)+(2\alpha')^3\times \\
& \Big[(k_1\cdot\alpha^2\cdot k_1)(\alpha^3\cdot\alpha^1)+(k_2\cdot\alpha^3\cdot k_2)(\alpha^2\cdot\alpha^1)+(k_3\cdot\alpha^1\cdot k_3)(\alpha^2\cdot\alpha^3) \\
& +3\,k_1\cdot\alpha^2\cdot\alpha^1\cdot\alpha^3\cdot k_2+3\,k_2\cdot\alpha^3\cdot\alpha^2\cdot\alpha^1\cdot k_3+3\,k_3\cdot\alpha^1\cdot\alpha^3\cdot\alpha^2\cdot k_1\Big] \\
& +(2\alpha')^4\Big[(k_1\cdot\alpha^2\cdot k_1)(k_2\cdot\alpha^3\cdot\alpha^1\cdot k_3)+(k_2\cdot\alpha^3\cdot k_2)(k_3\cdot\alpha^1\cdot\alpha^2\cdot k_1) \\
& +(k_3\cdot\alpha^1\cdot k_3)(k_1\cdot\alpha^2\cdot\alpha^3\cdot k_2)\Big]\Big\}
\end{aligned}
$$

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- no integrals ⇒ simpler extraction of Lagrangian
- compare with universal three–graviton amplitude !

Gross, Sloan 1987
Lüst, Theisen, Zoupanos 1988
Stieberger, Taylor '14 – '16

# Effective Lagrangians

- Simplifications:
  1. $D^{\mu}_{\nu} = \delta^{\mu}_{\nu} \quad , \quad D^{\mu\nu} = D^{\mu}_{\lambda} g^{\lambda\nu} = g^{\mu\nu}$
  2. brane gauge group: $U(1)$
- Replacements

$$\varepsilon_{\mu\nu} \to G_{\mu\nu} \quad , \quad \alpha^{1,2}_{\mu\nu} \to M_{\mu\nu} \quad , \quad k_\mu \, , q_\mu \to i\partial_\mu$$

- How to truncate? How to expand in $\alpha'$?

$$\alpha' \, k_1 \cdot k_2 \overset{\alpha' \to 0}{\longrightarrow} 1 \quad , \quad \alpha' \, k_{1,2} \cdot q \overset{\alpha' \to 0}{\longrightarrow} 0$$

$$s = -2 + 2\alpha' k_1 k_2 \overset{\alpha' \to 0}{\longrightarrow} 0$$

- expansion in small $s$ and substitution via $s = -2 + 2\alpha' k_1 k_2$

$\Rightarrow$ not meaningful truncation

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# Effective Lagrangians

1. *instead*: expand $\mathcal{I}(s)$ in small $s = -2\alpha' \, k_1 \cdot q$

2. simplify $\mathcal{K}(k_1, k_2, q; \alpha')$ for spacetime–filling D–branes

3. keep terms up to order $\alpha'^2$ in $\mathbf{A}_i = \mathcal{K}(k_1, k_2, q; \alpha') \times \mathcal{I}(s)$

$$
\begin{aligned}
\mathcal{A}(2,1) \quad = \quad & g_c \Big\{ -2\,\mathrm{Tr}(\alpha^1 \cdot \alpha^2)\, \varepsilon_{\mu\nu} k_1^\mu k_2^\nu + 2(\varepsilon \cdot \alpha^2 \cdot \alpha^1)_{\mu\nu} k_1^\mu k_2^\nu \\
& + 2(\varepsilon \cdot \alpha^1 \cdot \alpha^2)_{\mu\nu} k_1^\nu k_2^\mu + 2(\varepsilon \cdot \alpha^2 \cdot \alpha^1)_{\mu\nu}\, k_2^\mu q^\nu + 2(\varepsilon \cdot \alpha^1 \cdot \alpha^2)_{\mu\nu}\, k_1^\mu q^\nu \\
& + 2\,\mathrm{Tr}(\varepsilon \cdot \alpha^1 \cdot \alpha^2)\,(k_1 \cdot q) + \big[\,\mathrm{Tr}(\varepsilon \cdot \alpha^2)\alpha^1_{\mu\nu} - 2(\alpha^1 \cdot \varepsilon \cdot \alpha^2)_{\mu\nu} \\
& + \mathrm{Tr}(\varepsilon \cdot \alpha^1)\alpha^2_{\mu\nu}\big]\, q^\mu q^\nu \Big\} + \mathcal{O}\left(\alpha'^3\right) \,.
\end{aligned}
$$

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# Effective Lagrangians

- up to two–derivatives:

$$\mathcal{L}^{\text{eff}}_{\text{G}^3} \;=\; g_c\, G^{\mu\nu}\big[\partial_\mu G_{\rho\sigma}\partial_\nu G^{\rho\sigma} - 2\partial_\nu G_{\rho\sigma}\partial^\sigma G^\rho_\mu\big]$$

$$\begin{aligned}\mathcal{L}^{\text{eff}}_{\text{GM}^2} \;=\;& g_c\,\Big[G^{\mu\nu}\big(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\big)\\ &+M^{\mu\nu}\big(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma}\big)\Big]\end{aligned}$$

- can we set $\frac{1}{m_g\sqrt{1+\alpha^2}} \equiv g_c$ ?

$$\begin{aligned}\mathcal{L}^{\text{eff}}_{\text{M}^3} \;=\;& \frac{g_o}{\alpha'}\Big\{\big[M^3\big] + 2\alpha' M^{\mu\nu}\big[\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 3\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\big]\\ &+4\alpha'^2\,\partial^\mu\partial^\nu M_{\rho\sigma}\partial^\rho M^\kappa_\nu\partial^\sigma M_{\mu\kappa}\Big\}\end{aligned}$$

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# Conclusions

- We have computed the *full* tree–level disk amplitudes for $GM^2$ and $M^3$

- respective effective Lagrangians:
  1. $GM^2$: *partial* agreement between string and bimetric
  2. $M^3$: $M_{\mu\nu}$ kinematics not GR–like

  our 2 vs 3 discrepancy is strikingly reminiscent of but qualitatively different from the vDVZ discontinuity

$$\mathcal{D}^{M}_{\mu\nu,\kappa\lambda}(p) \xrightarrow{m_{\rm FP}\to 0} \frac{-i}{p^2}\left[\frac{1}{2}(\eta_{\mu\kappa}\eta_{\nu\lambda}+\eta_{\mu\lambda}\eta_{\nu\kappa})-\frac{1}{3}\eta_{\mu\nu}\eta_{\kappa\lambda}\right]+\text{singular terms}$$

van Dam, Veltman, Zakharov 1970

- open string massive spin–2 state not a suitable candidate for ghost–free bimetric theory, other possibilities currently under our investigation

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