Exploring the Landscape/Swampland of 6d supergravity

Yuta Hamada (KEK)

Based on the works
2311.00868 and 2404.08845 w/ Gregory J. Loges (KEK)

15/07/2024 The Landscape vs. the Swampland workshop at ESI

Swampland Approach to Quantum Gravity

This talk: Try to understand Swampland bound in controllable situation, and then reduce Q.

Controllability

Q=#SUSY

String constructions

Maximally supersymmetric — toroidal

Smaller supersymmetric theory — K3, Calabi-Yau, flux, …

KKLT, LVS, …

Our world (de Sitter, hierarchy small c.c., Higgs mass)

Current Status (asymptotically flat)

#(SUSY)

Dimension	8Q	16Q	32Q
11d	×	×	M-theory
10d	×	$SO(32)$, $E_8 \times E_8$	IIA/IIB
9d	×	rank $= 1,9,17$	
8d	×	rank $= 2,10,18$	
7d	×	rank $= 1,3,5,7,11,19$ (?)	
6d		rank $= 0,2,4,6,8,12,20$ (?) for $\mathcal{N} = (1,1)$, Unique EFT for $\mathcal{N} = (2,0)$.	

↓ $(S^1)^d$

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Dimension

32Q

Gravity multiplet

(Almost) unique

32Q → 16Q

Gravity multiplet

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Gravity multiplet | Gravity multiplet

+ Vector multiplet

(Almost) unique

32Q → 16Q

32Q	16Q
Gravity multiplet	Gravity multiplet
	+ Vector multiplet
(Almost) unique	Choice of G

32Q	→	16Q	→	8Q
Gravity multiplet		Gravity multiplet + Vector multiplet		
(Almost) unique		Choice of G		

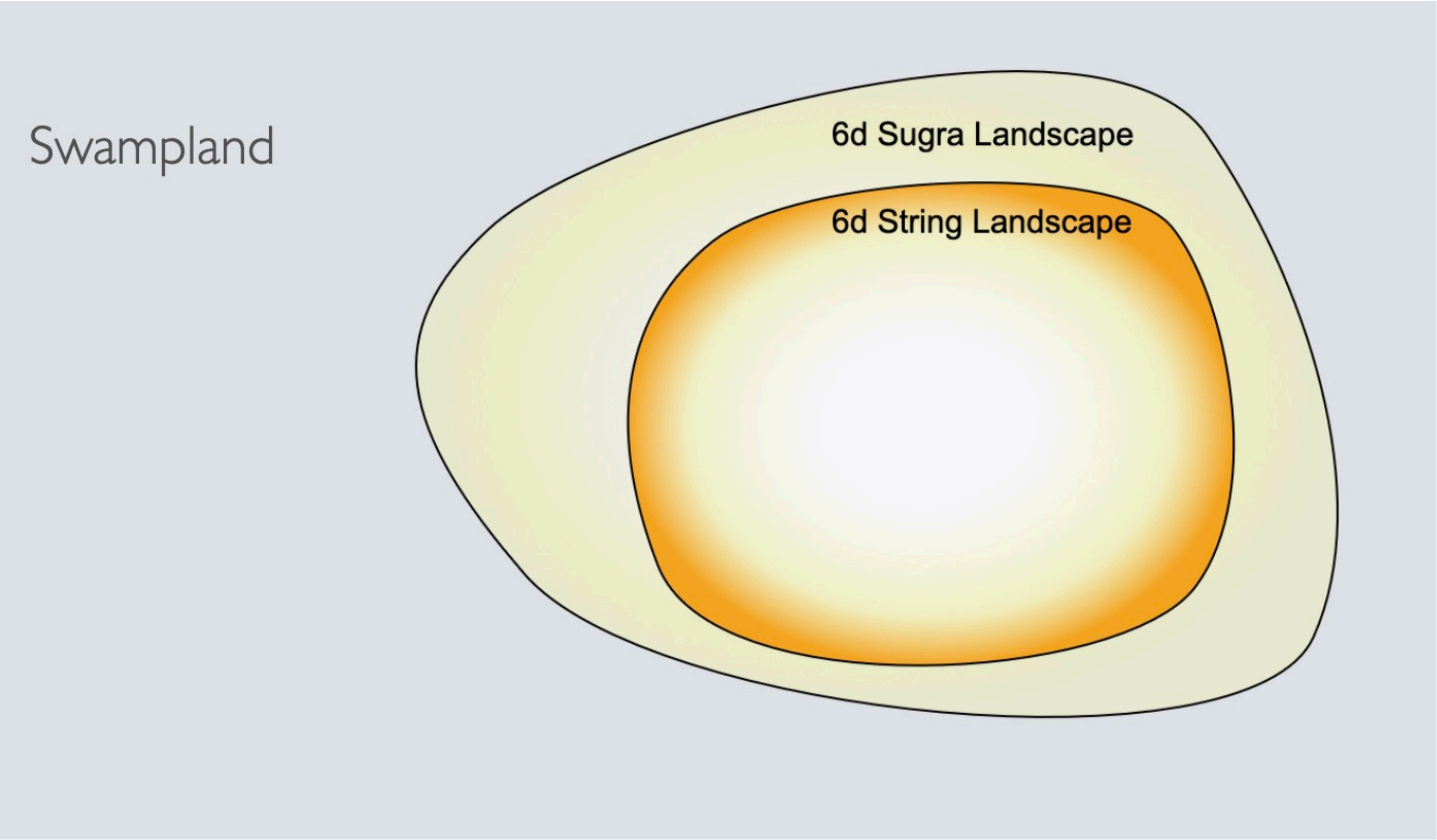
32Q → 16Q → 8Q

32Q	16Q	8Q
Gravity multiplet	Gravity multiplet	Gravity multiplet
	+ Vector multiplet	+ Vector multiplet
		+ Hypermultiplet
(Almost) unique	Choice of G	

32Q	16Q	8Q
Gravity multiplet	Gravity multiplet + Vector multiplet	Gravity multiplet + Vector multiplet + Hypermultiplet
(Almost) unique	Choice of G	Choice of G and Rep.

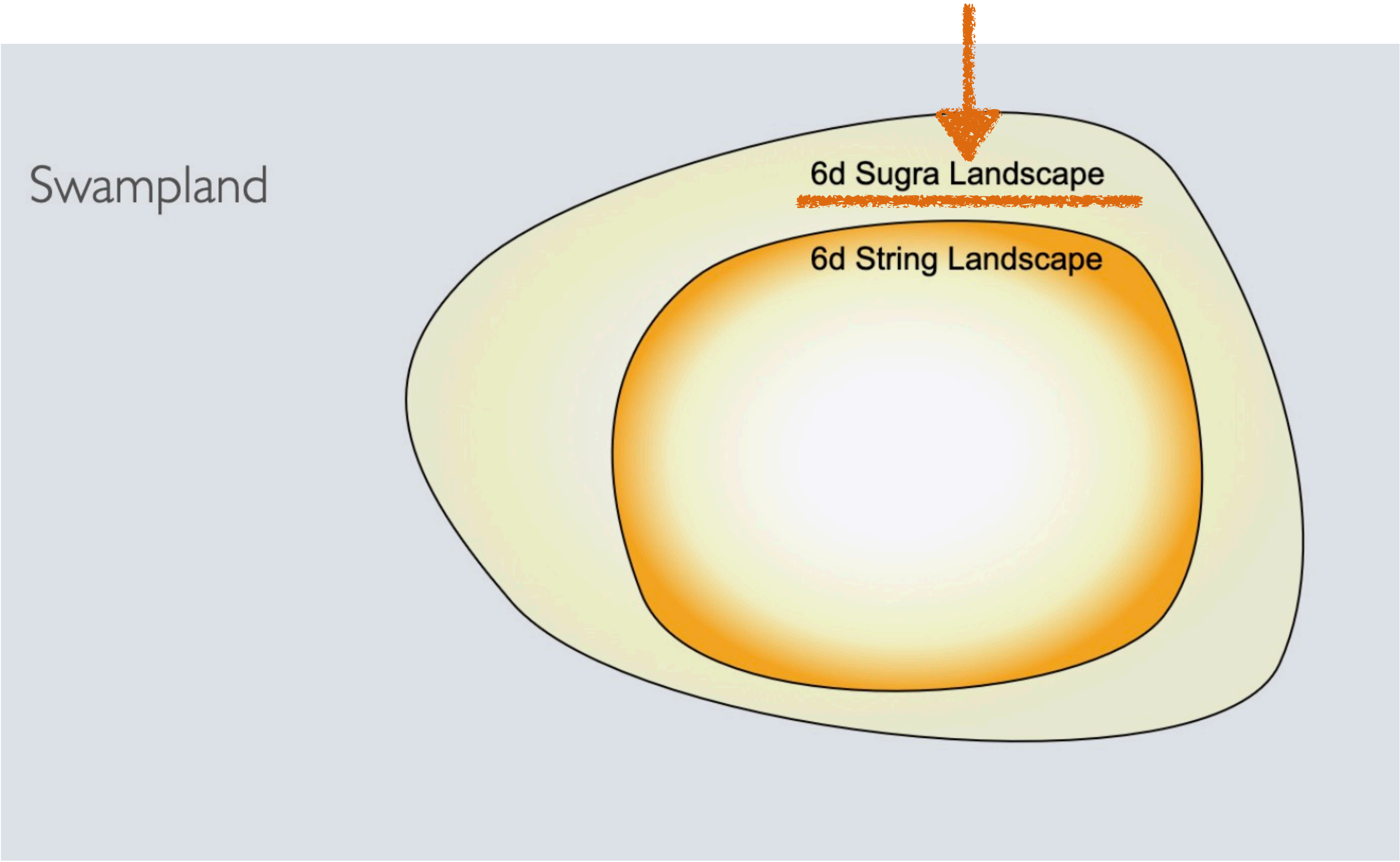
Goal

Understanding the Landscape/Swampland for 8Q theories.



Talk Plan

1: Identify theories here (non-trivial due to complicated consistency conditions)



2: Impose the QG bounds (from supergravity string [Kim, Shiu, Vafa '19, …]).

[YH, Loges '23, YH, Loges '24]

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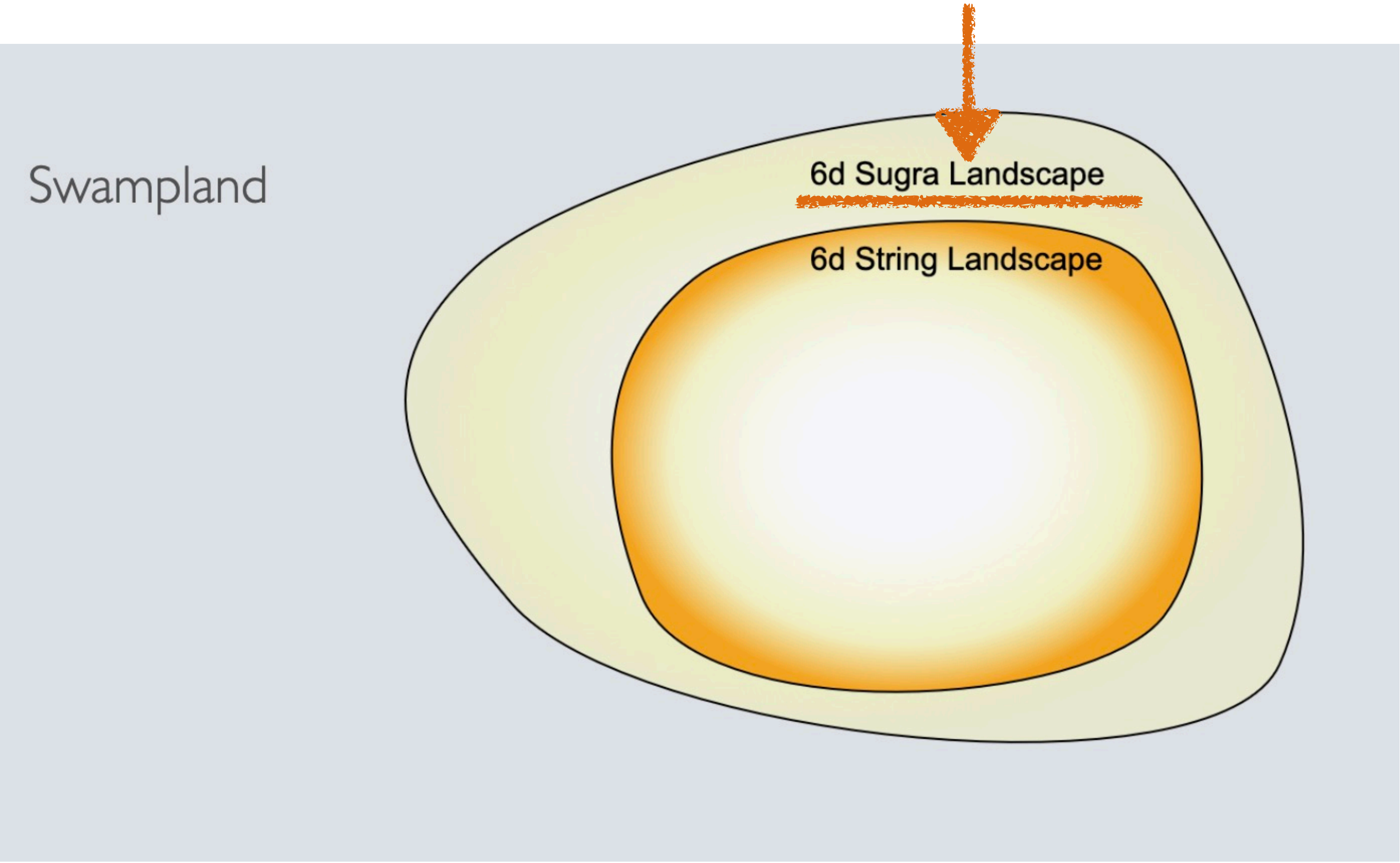

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6d Supergravity

6d gravity theories with minimal supersymmetry (8 SUSY).

Multiplets are

- Gravity multiplet $(g_{\mu\nu}, \psi_\mu, B^-_{\mu\nu})$.
- Tensor multiplet $(B^+_{\mu\nu}, \phi, \psi)$.
- Vector multiplet (A_μ, λ).
- Hyper multiplet (Φ, Ψ).

I will denote #(tensor), #(vector), #(hyper) by T, V, H.

There are $(T+1)$ B-fields, B_α, $\alpha = 0,1,\cdots,T$.

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- Vector multiplet (A_μ, λ).
- Hyper multiplet $(\underline{\Phi}, \Psi)$. ← Higgs branch.

Moduli

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There are $(T+1)$ B-fields, B_α, $\alpha = 0,1,\cdots,T$.

Three SUGRA consistency conditions

1: Anomaly cancellation

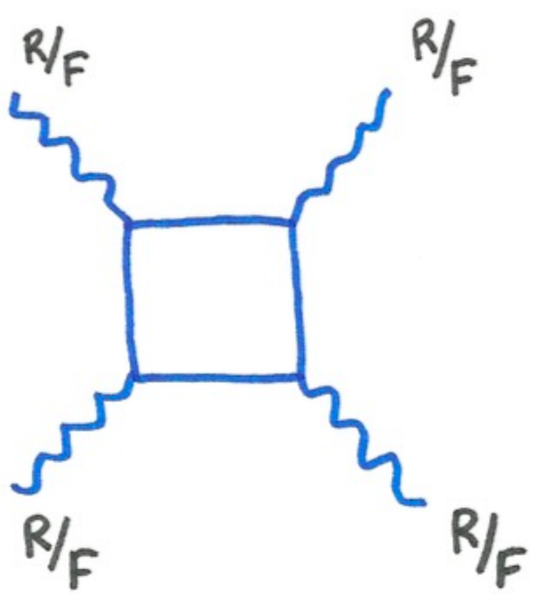
2: Positivity of gauge kinetic term

3: Unimodularity

1:Anomaly Cancellation

$$I_8 = \#\mathrm{Tr}\, R^4 + \#\mathrm{Tr}\, F^4 + \#\left(\mathrm{Tr}\, R^2\right)^2 + \#\left(\mathrm{Tr}\, F^2\right)^2 + \#\mathrm{Tr}\, R^2 \mathrm{Tr}\, F^2.$$

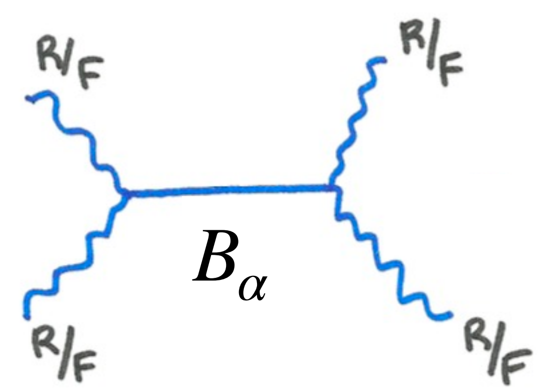
1: Fermions/B fields (loop)



$+$

2: B fields (tree)

$$I_8 \sim \sum Y_4^2, \qquad Y_4 \sim b_0^\alpha\, \mathrm{Tr} R^2 + b_F^\alpha\, \mathrm{Tr} F^2$$



$= 0$

$$\mathscr{L}_{\mathrm{int}} \sim b_0^\alpha B_\alpha R^2 + b_F^\alpha B_\alpha F^2.$$

Couplings

Formula

$b_{F_i} = b_i,$

n_R: #(Rep. R hyper),

$\mathrm{Tr}_R F^2 = A_R \mathrm{Tr} F^2,$

$\mathrm{Tr}_R F^4 = B_R \mathrm{Tr} F^4 + C_R \left(\mathrm{Tr} F^2\right)^2.$

$$H - V = 273 - 29T,$$ ← $\mathrm{Tr}(R^4)$

$$0 = \sum_R n_R^i B_R^i - B_{\mathrm{Adj}}^i,$$ ← $\mathrm{Tr}(F_i^4)$

$$b_0 \cdot b_0 = 9 - T, \qquad b_0 \cdot b_i = \frac{1}{6}\left(\sum_R n_R^i A_R^i - A_{\mathrm{Adj}}^i\right),$$

$$b_i \cdot b_i = \frac{1}{3}\left(\sum_R n_R^i C_R^i - C_{\mathrm{Adj}}^i\right), \qquad b_i \cdot b_j = \sum_{R,S} n_{(R,S)}^{i,j} A_R^i A_S^j, \quad (i \neq j).$$

Green-Schwarz

$i = 1, \cdots, k.$ $\quad b_0, b_i \in \mathbb{R}^{1,T},$ No anomaly $\leftrightarrow {}^\exists b_0, b_i.$

The cancellation of local anomaly implies the absence of the global anomaly as

$\Omega_7^{\mathrm{spin}}(pt) = \Omega_7^{\mathrm{spin}}(BSU(2)) = \Omega_7^{\mathrm{spin}}(BSU(3)) = \Omega_7^{\mathrm{spin}}(BG_2) = 0.$ (cf. $\pi_6(SU(2)) = \mathbb{Z}_{12}, \pi_6(SU(3)) = \mathbb{Z}_6, \pi_6(G_2) = \mathbb{Z}_3$)

[Lee, Tachikawa '20; Davighi, Lohitsiri '20]. (See however [Basile, Leone '23] for twisted string structure)

Other consistency conditions

2: Positivity of gauge kinetic term

Gauge kinetic terms $-j \cdot b_i \,\mathrm{Tr}(F_i^2) \implies \quad {}^{\exists}j \in \mathbb{R}^{1,T}$ such that $j \cdot b_i > 0$.

3: Unimodularity

Consider the lattice $\Lambda = \bigoplus_I \mathbb{Z}\, b_I \subset \mathbb{R}^{1,T}$ [Kumar, Morrison, Taylor '10].

Then, there must be unimodular lattice Γ s.t. $\Lambda \subseteq \Gamma \subset \mathbb{R}^{1,T}$ [Seiberg, Taylor '11].

$$W_{v,\tilde{v}_r} = \exp\left(i \oint_{\Sigma_2} \left(v\, B^{+} + \tilde{v}_r B^{-r} \right) \right), \qquad r = 1, \cdots, T, \quad v, \tilde{v}_r\text{: charges.}$$

T^2 compactification: $W_{v_a,\tilde{v}_r} \rightarrow$ Wilson and 't Hooft loops. $\rightarrow$ Dirac quantization.

Toward a classification

Consistency conditions on theories put strong constraints.

Classification of consistent supergravities is not yet done.

Previously:

- With $T < 9$, the number of anomaly-free 6d, $\mathcal{N} = (1{,}0)$ supergravities are known to be finite [Kumar, Morrison, Taylor '10].
- Given specific gauge groups, enumeration for $T = 0{,}1$ theories [Avramis, Kehagias '05, Kumar, Park, Taylor '10].

What's new

New in [YH, Loges '23, YH, Loges '24]:

- Gauge groups with any number of simple factors.
- Any Representation.

Technical limitations [YH, Loges '23, YH, Loges '24]:

- no $U(1)$, $SU(2)$ or $SU(3)$ simple factors.
- Some problematic choices of G and $\mathscr{H}$ are removed (for $T \geq 2$)

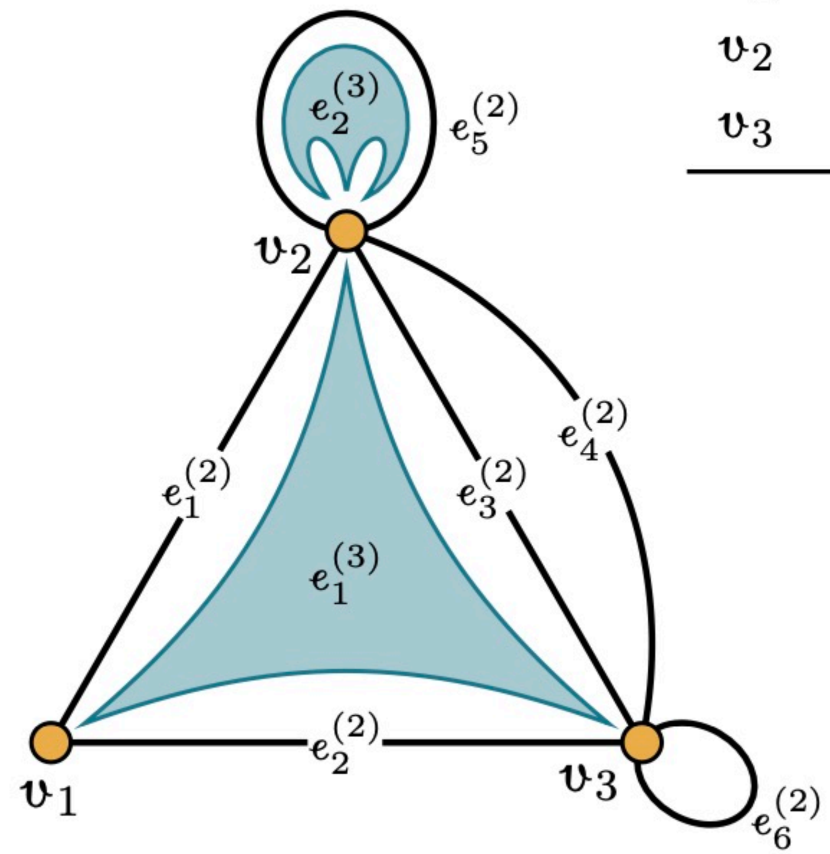
[YH, Loges '23, YH, Loges '24]

Idea

Express theories as hypergraphs (vertices, edges, …).

Vertices: gauge group G_i & charged matter H_i

Example



v_i	$G(v_i)$	H_i	$\Delta(v_i)$
v_1	SU(4)	$56 \times \underline{\mathbf{4}} \oplus \underline{\mathbf{15}} \oplus \underline{\mathbf{20}'}$	244
v_2	Sp(3)	$28 \times \underline{\mathbf{6}} \oplus 4 \times \underline{\mathbf{14}'} \oplus \underline{\mathbf{21}}$	224
v_3	G_2	$24 \times \underline{\mathbf{7}} \oplus \underline{\mathbf{14}}$	168

$:= H_i - V_i$

hyperedges: bi-charged matter H_{ij}, 3-charged H_{ijk}

$e_i^{(r)}$	$\iota(e_i^{(r)})$	H_{ij}, H_{ijk}
$e_1^{(2)}$	(v_1, v_2)	$7(\underline{\mathbf{4}}, \underline{\mathbf{6}}) \oplus (\underline{\mathbf{4}}, \underline{\mathbf{14}'})$
$e_2^{(2)}$	(v_1, v_3)	$6(\underline{\mathbf{4}}, \underline{\mathbf{7}})$
$e_3^{(2)}$	(v_2, v_3)	$4(\underline{\mathbf{6}}, \underline{\mathbf{7}})$
$e_4^{(2)}$	(v_2, v_3)	$\frac{3}{2}(\underline{\mathbf{6}}, \underline{\mathbf{7}}) \oplus \frac{1}{2}(\underline{\mathbf{14}'}, \underline{\mathbf{7}})$
$e_5^{(2)}$	(v_2, v_2)	$3(\underline{\mathbf{6}}, \underline{\mathbf{6}}) \oplus \frac{1}{2}(\underline{\mathbf{6}}, \underline{\mathbf{14}'}) \oplus \frac{1}{2}(\underline{\mathbf{14}'}, \underline{\mathbf{6}})$
$e_6^{(2)}$	(v_3, v_3)	$2(\underline{\mathbf{7}}, \underline{\mathbf{7}})$
$e_1^{(3)}$	(v_1, v_2, v_3)	$(\underline{\mathbf{4}}, \underline{\mathbf{6}}, \underline{\mathbf{7}})$
$e_2^{(3)}$	(v_2, v_2, v_2)	$\frac{1}{2}(\underline{\mathbf{6}}, \underline{\mathbf{6}}, \underline{\mathbf{6}})$

[YH, Loges '23, YH, Loges '24]

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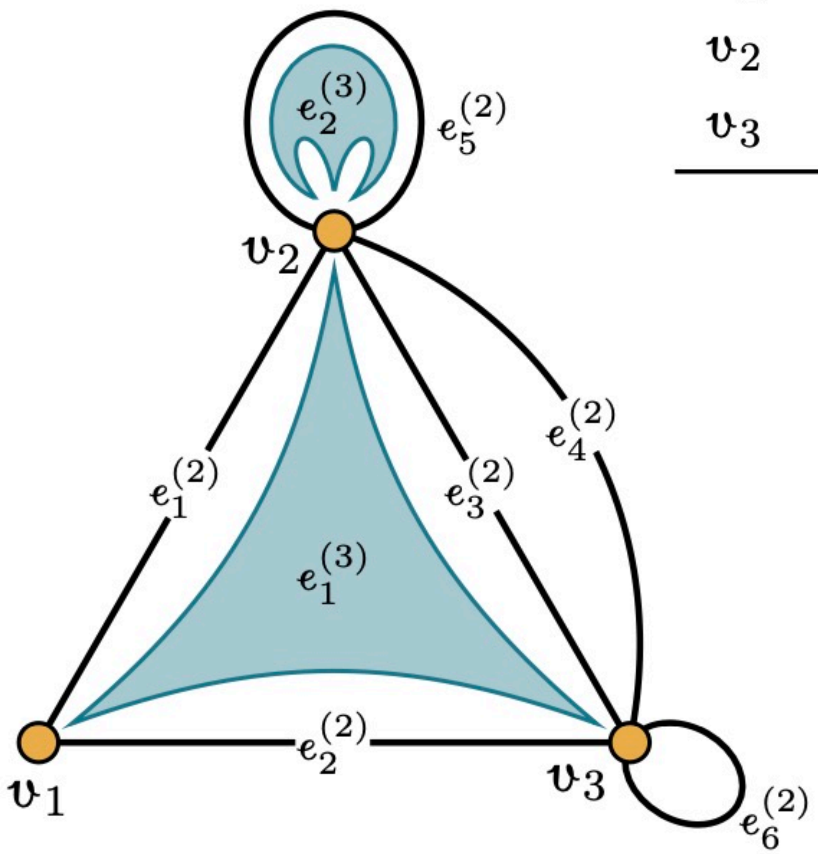

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$:= H_i - V_i \lesssim 273 - 29T$

$\mathrm{Tr}(R^4)$ anomaly

hyperedges: bi-charged matter H_{ij}, 3-charged H_{ijk}

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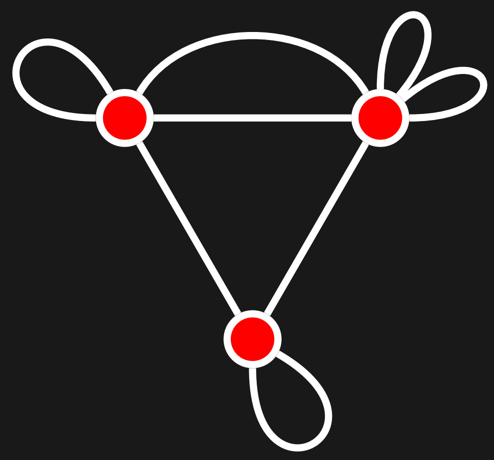
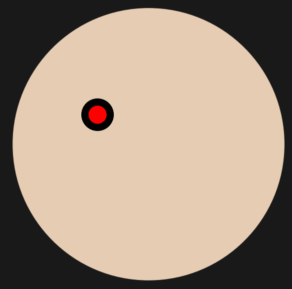
$e_2^{(3)}$	(v_2, v_2, v_2)	$\frac{1}{2}(\underline{\mathbf{6}},\underline{\mathbf{6}},\underline{\mathbf{6}})$

Clique construction: branch & prune





[Slide by G. Loges]

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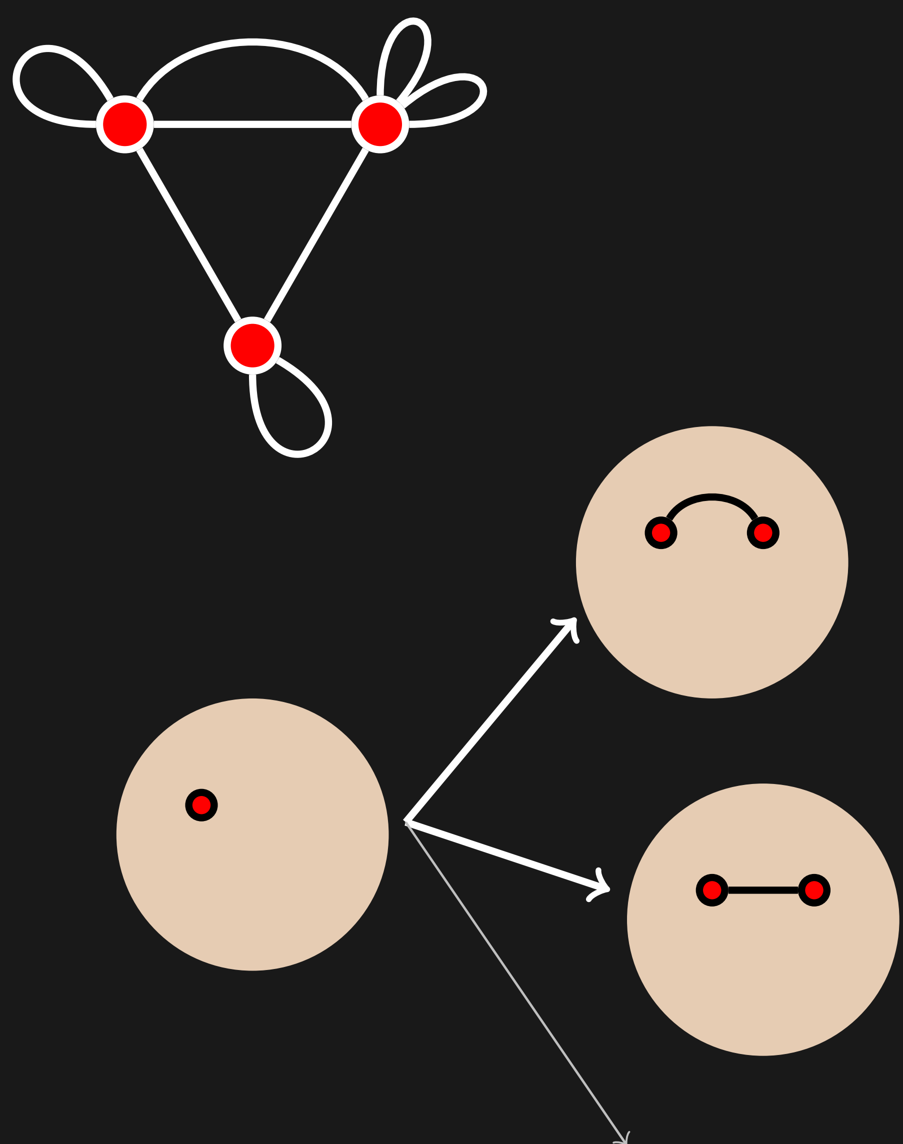

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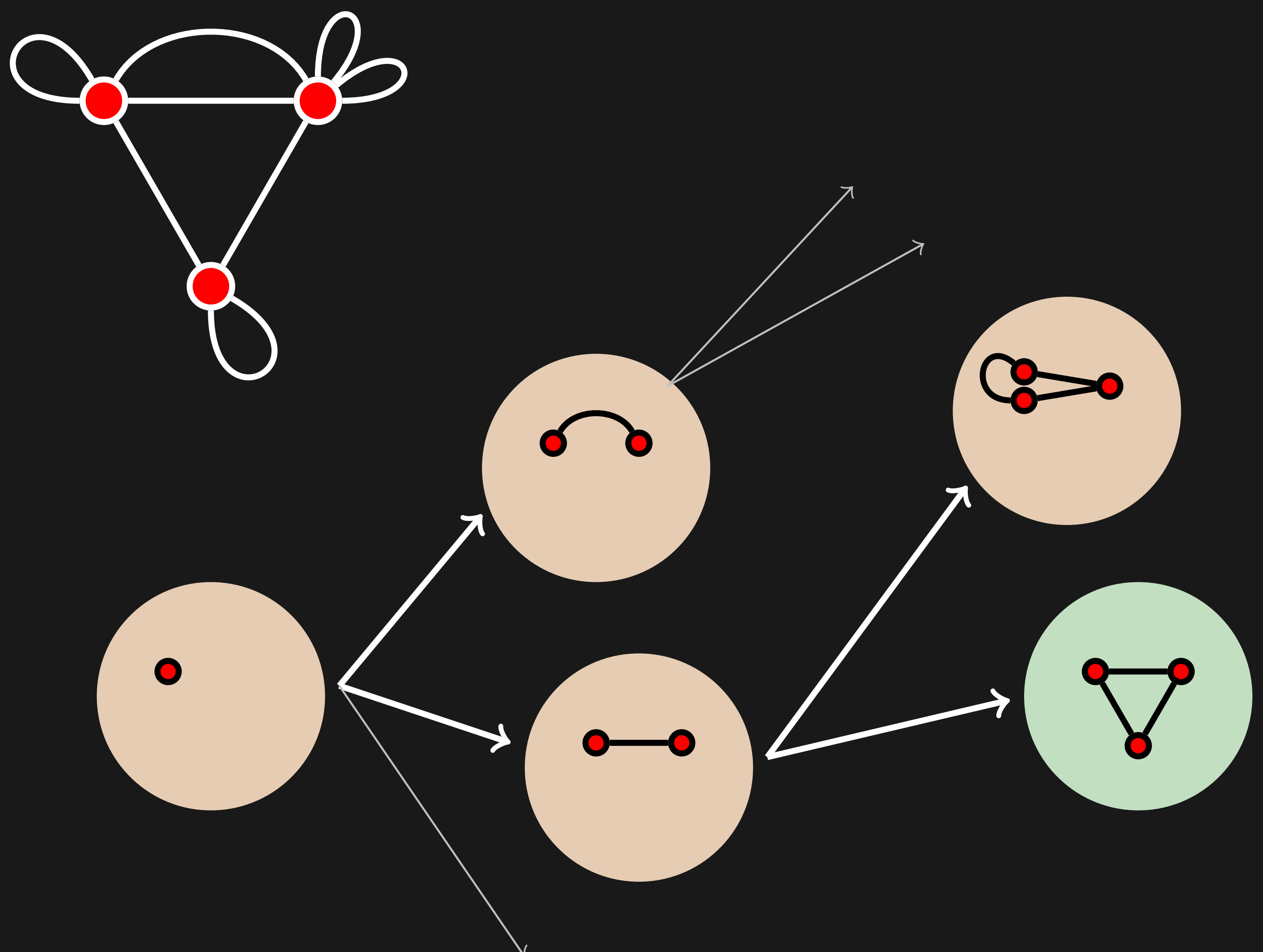

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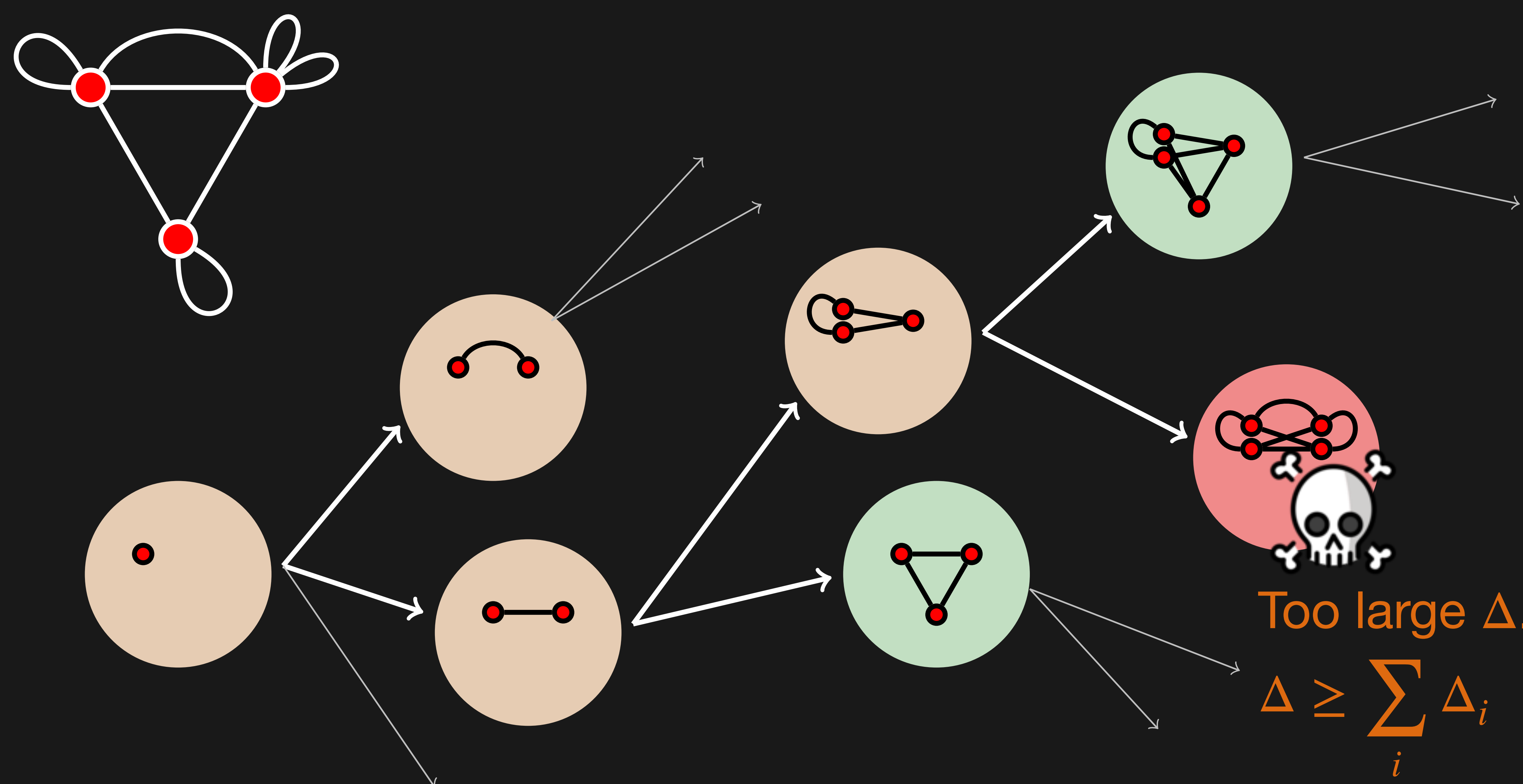


[Slide by G. Loges]

In the following, I focus on T=1.

[YH, Loges '23, YH, Loges '24] for other T.

$T = 1$ Results:

608,355 theories

$T = 1$: Largest G

- Largest $\dim G$

$G = SU(16) \times SU(32)$ $\quad$ $(\dim G = 1278)$

$\mathcal{H} = 18 \times (\bullet, \bullet) + (\mathbf{16}, \mathbf{32}) + 2 \times (\bullet, \mathbf{496})$

$$b_I \cdot b_J = \begin{pmatrix} 8 & 0 & 2 \\ 0 & -2 & 1 \\ 2 & 1 & 0 \end{pmatrix} \Longleftrightarrow b_0 = (2,2)\,,\, b_1 = (1,-1)\,,\, b_2 = (0,1).$$

- Largest $\mathrm{rank}\, G$

$G = SU(18) \times SU(6)^6$ $\quad$ $(\mathrm{rank}\, G = 47)$

$\mathcal{H} = 9 \times (\bullet; \bullet^6) + (\mathbf{18}; \underline{\mathbf{6}, \bullet^5}) + (\bullet; \underline{\mathbf{20}, \bullet^5})$

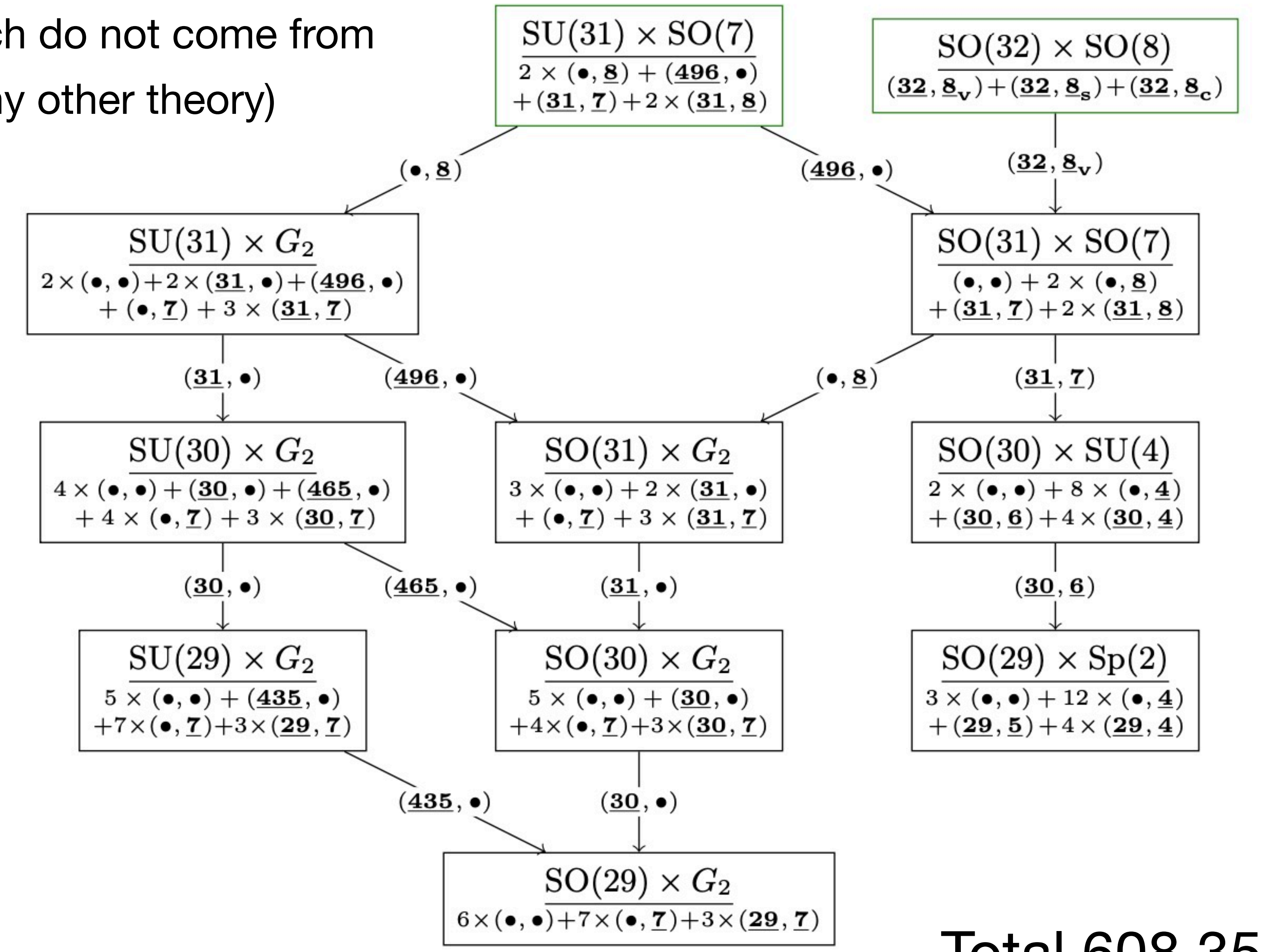
$$b_I \cdot b_J = \begin{pmatrix} 8 & 0 & 2_6 \\ 0 & -2 & 1_6 \\ 2_6 & 1_6 & 0_{6\times 6} \end{pmatrix} \Longleftrightarrow b_0 = (2,2)\,,\, b_1 = (1,-1)\,,\, b_{2\to 7} = (0,1).$$

[YH, Loges '24]

Higgsing

~ 10,000 source theories

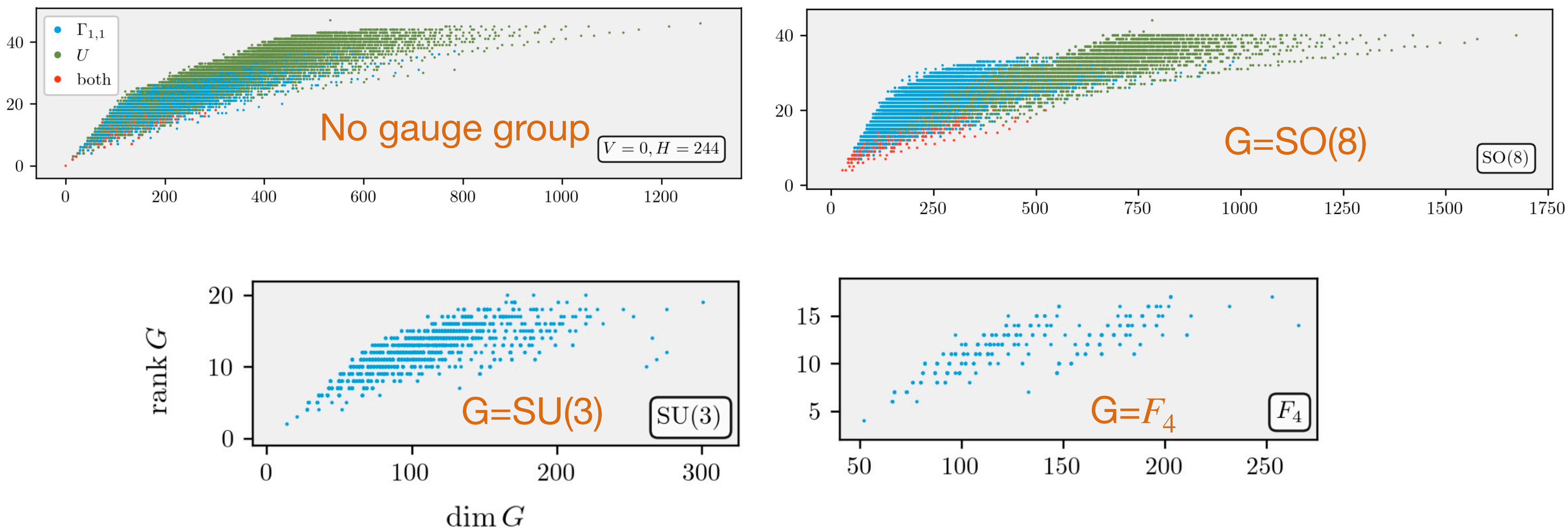
(Theories which do not come from Higgsing of any other theory)



Total 608,355 theories

Maximally Higgsed Phase

There are 8 maximally Higgsed phase (matches w/ F-theory result [Morrison, Taylor '12]).



and others

[YH, Loges '23, YH, Loges '24]

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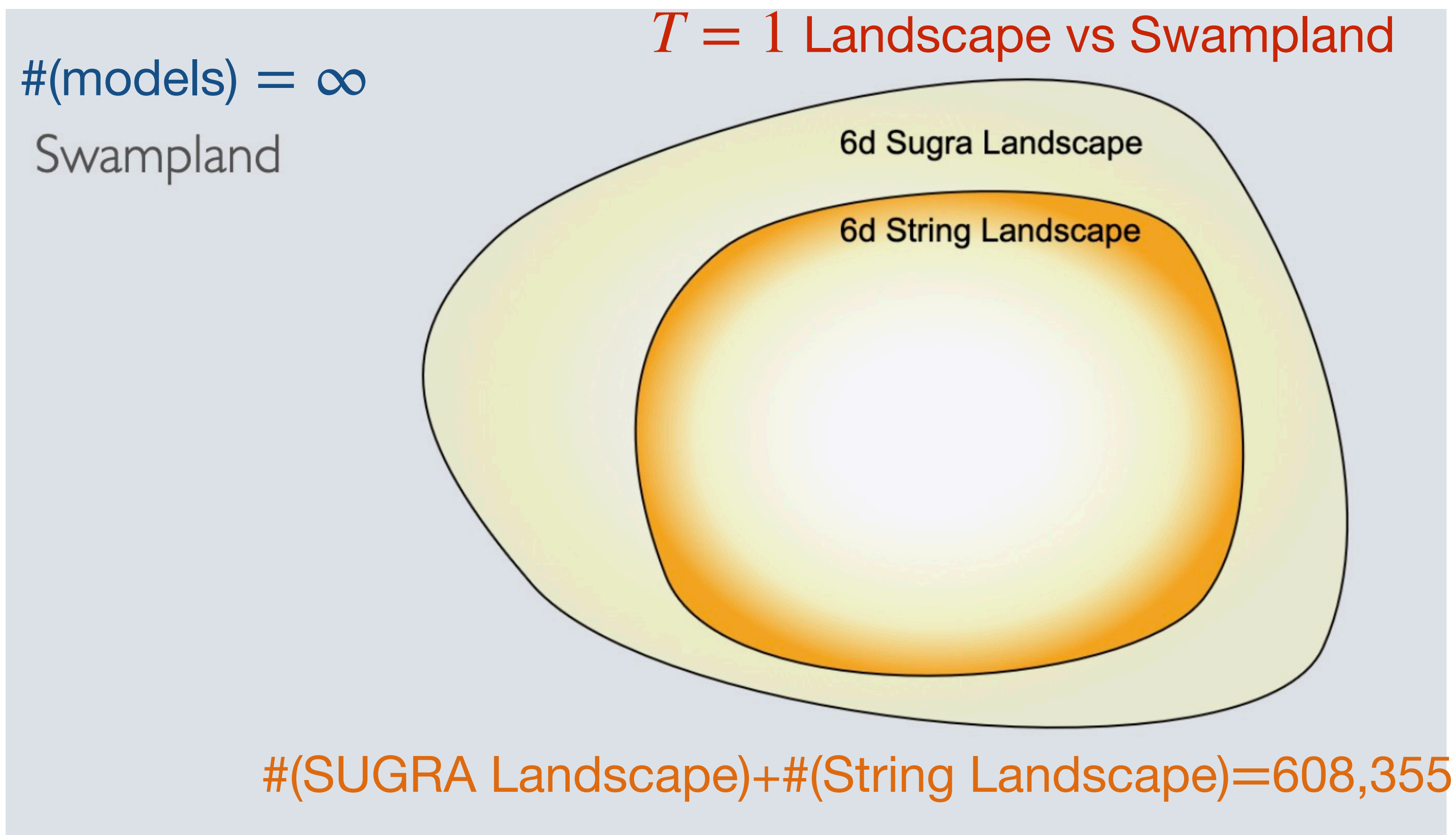


Note: no $U(1)$, $SU(2)$, $SU(3)$, $Sp(2)$.

2: Impose the QG bounds (from supergravity string [Kim-Shiu-Vafa '19, …]).

All 608,355 model can couple to QG?

String Bounds

[Kim-Shiu-Vafa ’19, …]

Supergravity String

$$\int_{\text{string}} B_2$$

Anomaly Inflow

(B_2 is not gauge inv.)

Inflow determines central charges of 2d theory on string, c_{inflow}.

$$c_G + \cdots = c_{\text{inflow}} \rightarrow c_G \leq c_{\text{inflow}}$$

Current algebra

$$\sum_{i=1}^{\kappa} \frac{k_i \dim G_i}{k_i + h_i^\vee} \leq c_L.$$

k_i: level of current algebra

$h_i^\vee$: dual Coxeter number

Bounds on Irrep

Bulk: Turning on VEV to Hypers.

Supergravity String: relevant/marginal deformation.

Conformal weight of vertex op. $$\Delta_R := \frac{C_2(R)}{2(k_i + h_i^\vee)} \leq 1,$$

Existence of Irrep R primary $$\sum_{m=1}^{r} a_m^\vee \lambda_m(R) \leq k_i.$$

$C_2(R)$: second Casimir of irrep R

$a_m^\vee$: comark

λ_m : Dynkin label

Theories w/ exotic representation tend to be excluded.

$T = 1$

Roughly 1/4 models are excluded by the brane probe bounds.

- $\sum_{m=1}^{r} a_m^\vee \lambda_m \leq k_i$ excludes

  models w/ $SU(N)$ gauge algebra and $(N-8)\times\mathbf{N}\oplus\mathbf{N(N+1)/2}$.

  81,484 models are excluded (≈13.4%).

Consistent w/ top-down picture:

No F-theory realizations [Kumar, Morrison, Taylor '10].

- $\Delta_R := \dfrac{C_2(R)}{2(k_i + h_i^\vee)} \leq 1$ excludes $SU(6)\ 15\times\mathbf{6}\oplus\frac{1}{2}\mathbf{20}$.

173,843 theories

Many models are excluded (173,843 theories). E.g.

$G = SU(5),$

$\mathscr{H}_{\text{ch}} = 13 \times \mathbf{5} + 11 \times \mathbf{10} + 3 \times \mathbf{24}.$

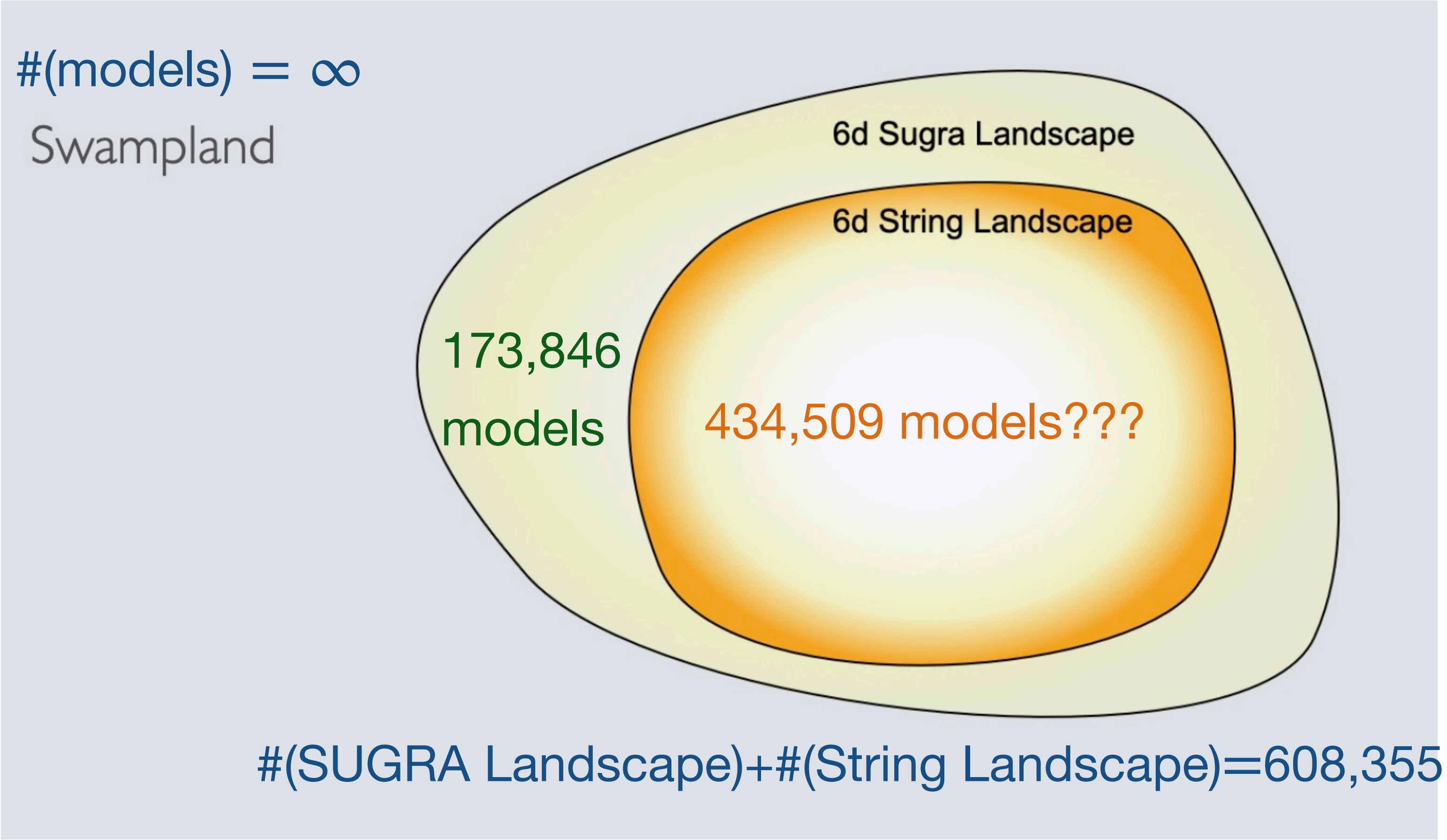
$G = SO(10),$

$\mathscr{H}_{\text{ch}} = 3 \times \mathbf{10} + 7 \times \mathbf{16} + 3 \times \mathbf{45}.$

$G = Sp(6) \times Sp(4) \times SO(10)^4,$

$\mathscr{H}_{\text{ch}} = (\mathbf{1}, \mathbf{42}, \mathbf{1}^4) + \frac{1}{2}(\mathbf{12}, \mathbf{1}, \underline{\mathbf{10}, \mathbf{1}^3}) + \frac{1}{2}(\mathbf{1}, \mathbf{8}, \underline{\mathbf{16}, \mathbf{1}^3})$

$T = 1$ Landscape vs Swampland



Note: no $U(1)$, $SU(2)$, $SU(3)$, $Sp(2)$.

String Universality?

Not clear if ALL 434,509 models are realized in string theory.

E.g. $G = SO(32) \times SO(8)$, $\mathscr{H} = (\mathbf{32}, \mathbf{8}_v) + (\mathbf{32}, \mathbf{8}_s) + (\mathbf{32}, \mathbf{8}_c)$.

Completely consistent at the level of effective field theory.

From string theory perspective,

- Any geometric construction (like F-theory) does not lead to the above spectra.
- I am unaware of a non-geometric construction (such as asymmetric orbifold) realizing this.

New String construction or New Consistency relation?

Summary

- Classification of 6d $T = 1$ supergravity.
- Quantum Gravity Bounds.