Relationship between Grosse-Wulkenhaar model & N-body harmonic oscillator or Calogero-Moser model

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Exact Solvable Models @ ESI
24 th – 26 th July, 2024

Happy 80th Birthday, Harold !!

Quiz). In Japanese, "80 years old" is called "San-ju " meaning "Happy Umbrella age".

Why is it called the age of "Umbrella" ?

Age of

?

Hint: In Japanese (Chinese), 80 is written as

八 eight

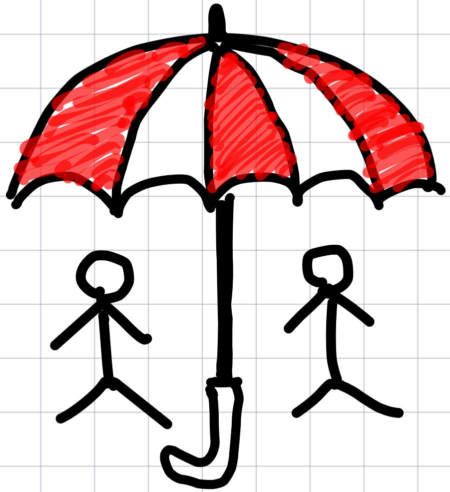
十 ten

Hint: In Japanese (Chinese), "80" is written as

八 8

十 0

Hint 2: "Umbrella" is written as

傘 = 

Talk Plan

1 History and Overview

2 Matrix model from Non-Com. Quantum Field Theory

3. Φ^4 model $\Rightarrow$ Harmonic Oscillator System

Calogelo - Moser Model

4. Viraso (Witt) alg

5. Summary

§1 History

▶ 90s' Matrix model

◎ 2D gravity $\Leftrightarrow$ random matrix

Brezin - Kazakov, Gross - Migdal, etc.

Kontsevich model (Witten conjecture)

Fukuma-Kawai
Nakayama

$$Z[J] = \int \mathcal{D}\Phi \exp\left(-\mathrm{tr}\left(\Lambda \Phi^2 + \Phi^3\right)\right)$$

Φ : Hermite matrix, $\Lambda = \mathrm{diag}(\Lambda_1, \Lambda_2, \cdots)$

Makeenko - Semenoff solve this in $n \to \infty$

2000's QFT on N.C. space

N.C. field Theory $\Rightarrow$ Matrix model.

Grosse-Steinacker ('05, '06)

Φ^3 model (basically Kontsevich model) Renormalizable

Grosse-Wulkenhaar ('04)

Φ^4 models in 2, 4 dim are Renormalizable

Φ^4 model is solvable

(SD-eq is recursivly determined.)

§2. ~The Origin from N.C. field Theory~

$\mathbb{R}^2_\theta$: Moyal plane $\quad [A,B] := AB - BA$

$$[x^1, x^2] = i\underbrace{\theta}_{\text{N.C. parameter}} \Leftrightarrow [z, \bar{z}] = 2\theta$$

- Annihilation $\left(a := \frac{z}{\sqrt{2\theta}} \right)$ Creation $\left(a^\dagger := \frac{\bar{z}}{\sqrt{2\theta}} \right)$ op.

$$\Rightarrow [a, a^\dagger] = 1, \quad [a, a] = [a^\dagger, a^\dagger] = 0$$

- $\frac{\partial}{\partial z} = -\frac{1}{\sqrt{2\theta}} [a^\dagger, \ \ ], \quad \frac{\partial}{\partial \bar{z}} = \frac{1}{\sqrt{2\theta}} [a, \ \ ]$

- Fock sp. $[a, a^\dagger] = 1$, $[a, a] = [a^\dagger, a^\dagger] = 0$

$|0\rangle$: $a|0\rangle = 0$, $|n\rangle := \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$

Number op. $N := a^\dagger a$ $\quad N|n\rangle = n|n\rangle$

$\langle n|$ = dual of $|n\rangle$

$\langle n|m\rangle = \delta_{nm}$

- Scalar field $\Phi = \underline{\sum \phi_{nm}|m\rangle\langle n|}$
  ($C^\infty(\mathbb{R}^d)$)

Hermitian Matrix !!

$\int d^2x \quad \rightarrow \quad \theta^2 \, \mathrm{Tr}$

Action

$$S_1 = \int d^4x \, -\phi \left(\frac{\partial}{\partial z} \frac{\partial}{\partial \bar{z}} \right) \phi$$

$$= \frac{\theta^2}{2\theta} \mathrm{Tr}\, \phi \, [a^\dagger, [a, \phi]] \qquad N = a^\dagger a$$

$$= \theta \; \mathrm{Tr} \left(\phi N \phi \underline{- \theta a^\dagger \phi a \phi} \right)$$

Removing this term by a counter Lagrangian

Renormalizable model is obtained.

$$S_m = \theta \, \mathrm{Tr} \, \frac{\mu^2}{2} \phi^2$$

μ: const. (mass)

ex). Φ^3 model $\quad \Phi$: $N\times N$ Hermitian matrix

$$S = N\,\mathrm{tr}\left(E\Phi^2 - A\Phi + \frac{2}{3}\Phi^3\right) \longleftrightarrow \begin{cases}\text{Kontsevich model}\\ \text{KdV hierarchy}\end{cases}$$

$$E_{ij} = \left(\frac{1}{2}\mu^2 + i\right)\delta_{ij}\,, \quad A : \text{const.}$$

$$Z[J] = \int D\Phi\, e^{-S + N\,\mathrm{tr}(J\Phi)}$$

$$\left(D\Phi = \prod_i d\Phi_{ii} \prod_{i<j} d\Phi^{Re}\, d\Phi^{Im} \right)$$

$$\log\frac{Z[J]}{Z[0]} = \sum_{B=1}^{\infty}\ \sum_{1\le N_1\le\cdots\le N_B}^{\infty}\ \sum_{p_i^j=0}^{N} N^{2-B}\,\frac{G_{|p_1^1\cdots p_{N_1}^1|\cdots|p_1^B\cdots p_{N_B}^B|}}{S(N_1,\cdots,N_B)}$$

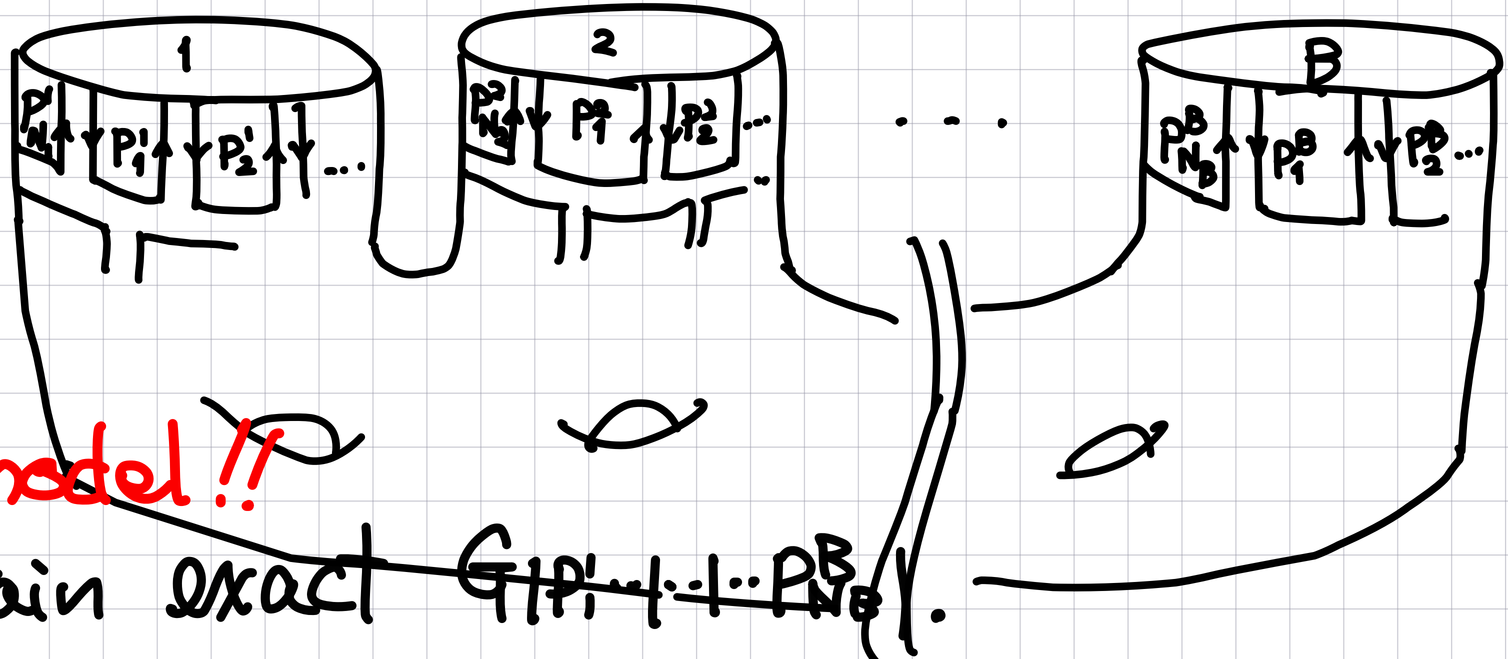
$$\times \prod_{\beta=1}^{B}\frac{\prod_{k=1}^{N_\beta} J_{p_k p_{k+1}}}{N_\beta}$$

statistical factor

$N_1+N_2+\cdots+N_B$ pts connected Green fun.

$G|P^1_1P^1_2\cdots P^1_{N_1}|P^2_1P^2_2\cdots P^2_{N_2}|\cdots|P^B_1P^B_2\cdots P^B_{N_B}|$

$= \sum$



Solvable model!!

We can obtain exact $G|P^1_1\cdots|\cdots|\cdots P^B_{N_B}|$.

H.Grosse-A.S.-R.Wulkenhaar ('16, '17) for $N\to\infty$

N.Kanomata-A.S. ('23) for finite N

Φ^4 model is also solvable

H.Grosse-R.Wulkenhaar-A.Hock ('19, '21)

J.Branahl-A.Hock-R.Wulkenhaar ('22)

$\uparrow$ topological recursion

Hermitian matrix

§3 Φ^4 model $\Rightarrow$ Harmonic Oscillator System

$$S = N\,\mathrm{tr}\left(E\Phi^2 + \frac{\eta}{4}\Phi^4\right) \qquad \eta \in \mathbb{R}_{>0}$$

$$Z = \int d\Phi\, e^{-S} \qquad E = \mathrm{diag}(E_1, E_2, \cdots, E_N)$$

$d\Phi = \prod_i d\Phi_{ii} \prod_{i<j} d\Phi_{ij}^{Re}\, d\Phi_{ij}^{Im}$

$E_i > 0, \quad E_i \neq E_j$

Main Thm. 1

$$\Psi(E,\eta) := e^{-\frac{N}{2\eta}\sum_i E_i^2}\,\Delta(E)\, Z,$$

where $\Delta(E) = \prod_{k<l}(E_l - E_k)$ Vandermonde

$\Rightarrow \mathcal{H}_{HO}\Psi = 0$, Schrödinger eq

$$\mathcal{H}_{HO} = -\frac{\eta}{N}\sum_{i=1}^{N}\left(\frac{\partial}{\partial E_i}\right)^2 + \frac{N}{\eta}\sum_{i=1}^{N}(E_i)^2 \ : \text{Hamiltonian}$$

N-body (N-dim) Harmonic Oscillator System

Proof) $E = \begin{pmatrix} E_1 & & O \\ & E_2 & \\ O & & \ddots \; E_N \end{pmatrix} = U H U^\dagger$ $H = (H_{ij})$

Hermite non-diagonal

$$S = N \,\mathrm{tr}\left(H \Phi^2 + \frac{\eta}{4} \Phi^4 \right), \quad Z = \int d\Phi \, e^{-S}$$

S-D eq $\int d\Phi \, \frac{\partial}{\partial \Phi_{ij}} \left(\Phi_{ij} e^{-S} \right) = 0$

$$\Leftrightarrow Z - N \sum_k \left(H_{ki} \langle \Phi_{ij} \Phi_{jk} \rangle + H_{jk} \langle \Phi_{ki} \Phi_{ij} \rangle \right)$$

$$- N\eta \sum_{k,l} \langle \Phi_{jk} \Phi_{kl} \Phi_{li} \Phi_{ij} \rangle = 0$$

, where $\langle \mathcal{O} \rangle = \int d\Phi \, \mathcal{O} \, e^{-S}$

$$\frac{\partial Z}{\partial H_{ij}} = -N \sum_k \langle \Phi_{jk} \Phi_{ki} \rangle, \quad \frac{\partial^2 Z}{\partial H_{ij} \partial H_{mn}} = N^2 \sum_{k,l} \langle \Phi_{jk} \Phi_{ki} \Phi_{nm} \Phi_{lm} \rangle$$

Diff eq.

$$\underbrace{\left(N^2+2\sum_{i,k}H_{ki}\frac{\partial}{\partial H_{ki}}-\frac{\gamma}{N}\sum_{i,k}\left(\frac{\partial}{\partial H_{ki}}\frac{\partial}{\partial H_{ik}}\right)\right)}\, Z=0$$

@ Change of valiables

$\mathbb{R}^{N^2}\to\mathbb{R}^N\times U(N)$ $\quad H_{ij}\mapsto E_i+$ ~~others~~ $\quad Z$ depends on only E_i.

$$\begin{cases}\bullet\ \text{Laplacian}: \displaystyle\sum_{i,j}\frac{\partial}{\partial H_{ji}}\frac{\partial}{\partial H_{ij}}=\sum_i\left(\frac{\partial}{\partial E_i}\right)^2+\sum_{i\neq j}\frac{1}{E_i-E_j}\left(\frac{\partial}{\partial E_i}-\frac{\partial}{\partial E_j}\right)\\ \bullet\ \displaystyle\sum_{i,j}H_{i,j}\frac{\partial}{\partial H_{i,j}}=\sum_k E_k\frac{\partial}{\partial E_k}\end{cases}$$

$$\underbrace{\left\{\frac{\gamma}{N}\sum_{i=1}^{N}\left(\frac{\partial}{\partial E_i}\right)^2+\frac{\gamma}{N}\sum_{i\neq j}\frac{1}{E_i-E_j}\left(\frac{\partial}{\partial E_i}-\frac{\partial}{\partial E_j}\right)-2\sum_k E_k\frac{\partial}{\partial E_k}-N^2\right\}}_{=:\mathcal{L}_{SD}}Z=0$$

Lem. Diagonalization

N-Body harmonic oscillator Hamiltonian

$$\mathcal{H}_{HO} := -\frac{\eta}{N}\sum_{i=1}^{N}\left(\frac{\partial}{\partial E_i}\right)^2 + \frac{N}{\eta}\sum_{i=1}^{N}(E_i)^2 ,$$

$$e^{-\frac{N}{2\eta}\sum_i E_i^2}\Delta(E)\,\mathcal{L}_{SD}\,\Delta(E)^{-1}\,e^{\frac{N}{2\eta}\sum_i E_i^2} = -\mathcal{H}_{HO}$$

$\Downarrow$

Thm.

$$\Psi(E,\eta) := e^{-\frac{N}{2\eta}\sum_i E_i^2}\Delta(E)\,Z(E,\eta)$$

$$\Rightarrow \qquad \mathcal{H}_{HO}\,\Psi(E,\eta) = 0$$

Ψ is a 0-energy solution of HO system //

▷ Concrete expression $\swarrow$ IZ integral

$$Z(E,\eta)=\frac{c}{\Delta(E)}\int_{\mathbb{R}^N}\left(\prod_{i=1}^{N}dx_i\, e^{-N\left(\frac{\eta}{4}x_i^4+E_ix_i^2\right)}\right)\left(\prod_{k<l}\frac{x_k-x_l}{x_k+x_l}\right)$$

$$\Updownarrow$$

$$\Psi(E,\eta)=c\int_{\mathbb{R}^N}\left(\prod_{i=1}^{N}dx_i\, e^{-N\left(\frac{\eta}{4}x_i^4+E_ix_i^2+\frac{1}{2\eta}E_i^2\right)}\right)\left(\prod_{k<l}\frac{x_k-x_l}{x_k+x_l}\right)$$

$\Downarrow$ Bruijn's formula

$$Z(E,\eta)=\frac{c}{\Delta(E)}\,\mathrm{Pf}_{i,j}\,M_{ij},\quad \Psi=c\,e^{-\frac{N}{2\eta}\sum_i E_i^2}\,\mathrm{Pf}_{i,j}\,M_{ij}$$

where

$$M_{i,j}=\int_{\mathbb{R}^2}dx\,dy\left(\frac{x-y}{x+y}\right)e^{-2N\left(V(x)+E_ix^2+V(y)+E_jy^2\right)}$$

$$V(x)=\frac{\eta}{4}x^4$$

Note $\mathcal{H}_{HO} = \sum_{i=1}^{N} \left\{ -\left(\frac{\partial}{\partial u_i}\right)^2 + u_i^2 \right\} > 0$ possitive

In $L^2(\mathbb{R}^N)$ there is no solution of

$$\mathcal{H}_{HO}\Psi = 0 \quad \text{without} \quad \Psi = 0 .$$

What's happen ?

Rem. $N=1$ case

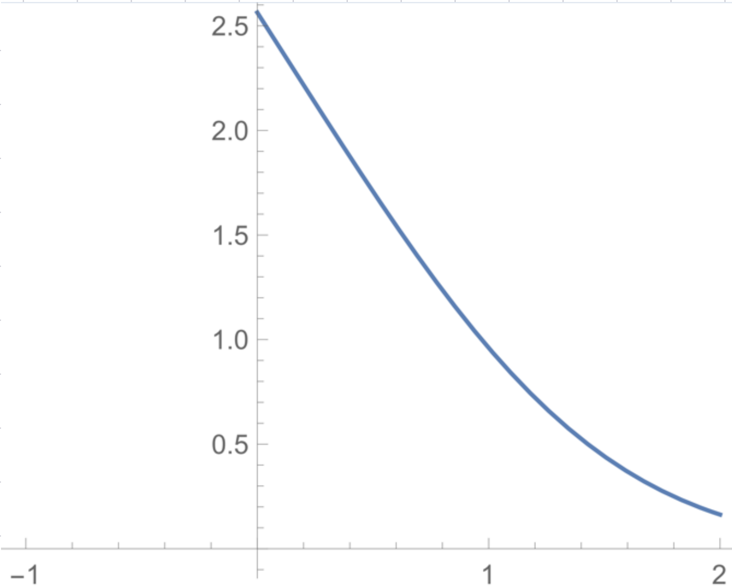
H-O (Weber eq) $y''(u) = u^2 y(u) \quad (u = \sqrt{\tfrac{N}{\eta}}E)$

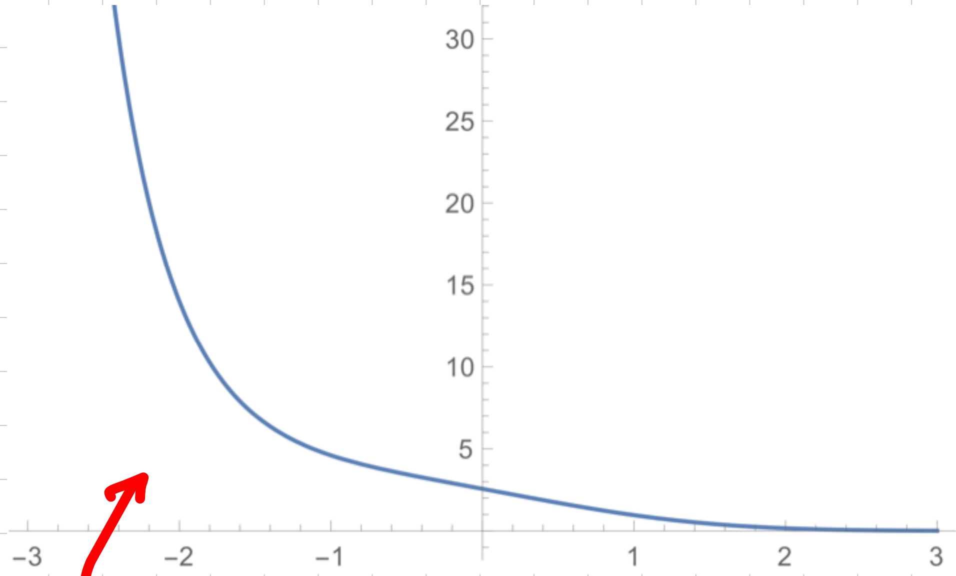
$$\Psi(u) = e^{-\frac{u^2}{2}} \int_{-\infty}^{\infty} dx\, e^{-\sqrt{\eta}\, u x^2 - \frac{\eta}{4} x^4} = \frac{1}{\eta^{\frac{1}{4}}} \sqrt{u}\, K_{\frac{1}{4}}\left(\frac{u^2}{2}\right)$$

modified Bessel fun of 2nd kind

$$\sqrt{u}\,K_{\frac{1}{4}}\left(\frac{u^2}{2}\right)$$

$$e^{-\frac{u^2}{2}}\int_{-\infty}^{\infty}dx\, e^{-ux^2-\frac{1}{4}x^4}$$





Not $L^2(\mathbb{R})$ function

For $N>1$, the nature of

$$\Psi(E,\eta) = C\int_{\mathbb{R}^N}\left(\prod_{i=1}^{N} dx_i\, e^{-N\left(\frac{\eta}{4}x_i^4 + E_i x_i^2 + \frac{1}{2\eta}E_i^2\right)}\right)\left(\prod_{k<l}\frac{x_k-x_l}{x_k+x_l}\right)$$

$\left(= C e^{-\frac{N}{2\eta}\sum_i E_i^2}\,\underset{i,j}{\mathrm{pf}}\, M_{ij}\right)$ is still unknown.

§3 Φ^4 model $\Rightarrow$ Harmonic Oscillator System

← Hermitian matrix

$$S = N\,\mathrm{tr}\left(E\Phi^2 + \frac{\eta}{4}\Phi^4\right) \qquad \eta \subset \mathbb{R}_{>0}$$

$$Z = \int d\Phi \; e^{-S} \qquad E = \mathrm{diag}(E_1, E_2, \cdots, E_N)$$

$d\Phi = \prod_i d\Phi_{ii} \prod_{i<j} d\Phi_{ij}^{Re}\, d\Phi_{ij}^{Im}$ $\qquad E_i > 0, \quad E_i \neq E_j$

Main Thm. 1

$$\Psi(E,\eta) := e^{-\frac{N}{2\eta}\sum_i E_i^2}\,\Delta(E)\,Z,$$

where $\Delta(E) = \prod_{k<l}(E_l - E_k)$ Vandermonde

$$\Rightarrow \mathcal{H}_{HO}\Psi = 0, \qquad \text{Schrödinger eq}$$

$$\mathcal{H}_{HO} = -\frac{\eta}{N}\sum_{i=1}^{N}\left(\frac{\partial}{\partial E_i}\right)^2 + \frac{N}{\eta}\sum_{i=1}^{N}(E_i)^2 \;: \text{Hamiltonian}$$

N-body (N-dim) Harmonic Oscillator System

▶ Hermitian matrix ⇒ Real Sym matrix

Φ^4 model ⇒ Calogero-Moser System

$$S = N\,\mathrm{tr}\left(E\Phi^2 + \frac{\eta}{4}\Phi^4\right) \qquad \eta \in \mathbb{R}_{>0}$$

$$Z = \int d\Phi\, e^{-S} \qquad E = \mathrm{diag}(E_1, E_2, \cdots, E_N)$$

$d\Phi = \prod_i d\Phi_{ii} \prod_{i<j} \Phi_{ij}$ $\qquad E_i > 0,\ E_i \neq E_j$

Main Thm. 2

$$\Phi(E,\eta) := e^{-\frac{N}{\eta}\sum_i E_i^2}\, \Delta^{\frac{1}{2}}(E)\, Z,$$

where $\Delta(E) = \prod_{k<l}(E_l - E_k)$ Vandermonde

⇒ $\mathcal{H}_c \Phi = 0$, Schrödinger eq

$$\mathcal{H}_c = -\frac{\eta}{2N}\left(\sum_{i=1}^{N}\left(\frac{\partial}{\partial E_i}\right)^2 + \frac{1}{4}\sum_{i\neq j}\frac{1}{(E_i - E_j)^2}\right) + \frac{2N}{\eta}\sum_{i=1}^{N}(E_i)^2 : \text{Hamiltonian}$$

Calogero-Moser model

§4 Virasoro (Witt) alg. for Hermitian Φ

$$\mathcal{H}_{HO} = -\frac{\eta}{N}\sum_{i=1}^{N}\left(\frac{\partial}{\partial E_i}\right)^2 + \frac{N}{\eta}\sum_{i=1}^{N}(E_i)^2 \text{ : Hamiltonian}$$

$$\downarrow \quad y_i := \sqrt{\frac{N}{\eta}}\, E_i$$

$$\mathcal{H}_{HO} = \sum_i \left(-\left(\frac{\partial}{\partial y_i}\right)^2 + y_i\right)^2$$

$$= \frac{1}{4}\sum_i \{a_i, a_i^\dagger\}$$

$\{A, B\} := AB + BA$

Creation, anihilation

$$\begin{cases} a_i^\dagger := \frac{1}{\sqrt{2}}(y_i - \partial_i), \\ a_i := \frac{1}{\sqrt{2}}(y_i + \partial_i) \\ [a_i, a_j^\dagger] = \delta_{ij} \end{cases}$$

▷ Virasoro generator

$$L_{-n} := \sum_{i=1}^{N}\{\alpha (a_i^\dagger)^{n+1} a_i + (1-\alpha)\, a_i (a_i^\dagger)^{n+1}\} \qquad \alpha \in \mathbb{R}$$

$$\Downarrow$$

$$[L_n, L_m] = (n-m) L_{n+m}$$

$$L_0 = \frac{1}{2}\mathcal{H}_{HO} + \frac{N}{2} - \alpha N$$

i.e). $[\frac{1}{2}\mathcal{H}_{HO}, L_{-m}] = m L_{-m}$

$$\Downarrow \quad \begin{cases} g := e^{\frac{N}{2n}\sum_i E_i^2} \Delta^{-1}(E) \\ \tilde{L}_n := g L_n g^{-1}, \end{cases} \qquad \mathcal{L}_{SD} := -g\mathcal{H}_{HO}g^{-1}$$

$$[\mathcal{L}_{SD}, \tilde{L}_{-m}] = -2m \tilde{L}_{-m}$$

Thm3.

$$\boxed{\mathcal{L}_{SD}(\tilde{L}_{-m} Z(E,\eta)) = -2m(\tilde{L}_{-m} Z(E,\eta))}$$

↑ The same thm is obtained for Real Sym Φ.

▷ Bergshoeff and Vasiliev ('95)

Calogero-Moser model

$$H_C = \frac{1}{2}\sum_{i=1}^{N}\left(-\frac{\partial^2}{\partial y_i^2} + y_i^2\right) + \sum_{j>k}\frac{\beta(\beta-1)}{(y_j-y_k)^2}$$

$\left(y_i = \sqrt{\frac{2N}{g}}\, E_i \ , \ \beta = \frac{1}{2} \quad \text{for the real } \Phi^4 \text{ model} \right)$

∘ creation anhilation op.

$$a_i = \frac{1}{\sqrt{2}}(y_i + D_i) \ , \quad a_i^\dagger = \frac{1}{\sqrt{2}}(y_i - D_i)$$

where $D_i = \frac{\partial}{\partial y_i} + \beta \sum_{j \neq i} \frac{1-K_{ij}}{y_i - y_j}$, Dunkl op.

K_{ij} : permutation op $\quad K_{ij}\, y_j = y_i$

$$\Rightarrow [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0 \ , \quad [a_i, a_j^\dagger] = \delta_{ij}\left(1 + \beta\sum_{l=1}^{N} K_{il}\right) - \beta K_{ij}$$

$$L_{-n} := \sum_{i=1}^{N} \left(\alpha (a_i^\dagger)^{n+1} a_i + (1-\alpha) a_i (a_i^\dagger)^{n+1} \right)$$

$\Rightarrow$ Virasoro (Witt) alg. $[L_n, L_m] = (n-m) L_{n+m}$

$$H := \frac{1}{2} \sum_{i}^{N} \{ a_i, a_i^\dagger \}$$

$$= L_0 - \left(\frac{1}{2} - \alpha \right) N + \frac{1}{2} \left(\alpha - \frac{1}{2} \right) \sum_{i \neq j} K_{ij}$$

$\mathrm{Rest}(H)$: Restriction to K_{ij} inv sp

$$\Rightarrow \mathrm{Rest}(H) = \prod_{j>k} (y_j - y_k)^{-\frac{1}{2}} H_c \prod_{j>k} (y_j - y_k)^{\frac{1}{2}}$$

$$= -\frac{1}{2} e^{-\sum_i y_i^2} \mathcal{L}_{SD} \, e^{\sum_i y_i^2}$$

S-D op for real Sym Φ^4 matrix model

Note $K_{ij} Z = Z$

$K_{ij}\, e^{\sum_i y_i^2} = e^{\sum_i y_i^2}, \cdots$

$\Rightarrow$ We can ignore "Rest"

$\Rightarrow$ The following can be discussed in the same way as in the case of H-O.

$$\tilde{L}_{-m} := e^{\frac{1}{2}\sum_i y_i^2} L_{-m}\, e^{-\frac{1}{2}\sum_i y_i^2}$$

$$[\mathcal{L}_{SD}, \tilde{L}_{-m}] = -2m\, \tilde{L}_{-m}$$

<u>Thm3.</u>

$$\boxed{\mathcal{L}_{SD}\left(\tilde{L}_{-m} Z(E,\eta)\right) = -2m\left(\tilde{L}_{-m} Z(E,\eta)\right)}$$

§5 Summary

Typical quantum integrable

@ Schrödinger eq for 0-energy $\mathcal{H}\Psi = 0$

$$\mathcal{H} = -\frac{\hbar}{2N}\left(\sum_{i=1}^{N}\left(\frac{\partial}{\partial E_i}\right)^2 + \frac{1}{4}\sum_{i\neq j}\frac{1}{(E_i - E_j)^2}\right) + \frac{2N}{\hbar}\sum_{i=1}^{N}(E_i)^2 \quad : \text{Hamiltonian}$$

Calogero - Moser model or Harmonic Oscillators

$$\Psi = e^{-\frac{2N}{2\hbar}\sum_i E_i^2}\,\Delta(E)^{\frac{1}{2}}\,Z$$

$$\mathcal{H} = \frac{1}{2}\sum_i \{a_i, a_i^\dagger\}$$

@ G-W type Φ^4 matrix model (Hermitian / Real Sym)

$$Z = \int d\Phi\, e^{-S},$$

$$S = N\,\mathrm{tr}\left\{E\Phi^2 + \frac{\eta}{4}\Phi^4\right\}$$

S-Deq $\mathcal{L}_{SD} Z = 0$

gauge trans

@ Virasoro alg.

$$[\tilde{L}_n, \tilde{L}_m] = (n-m)\tilde{L}_{n+m}$$

$$\mathcal{L}_{SD}(\tilde{L}_{-m} Z) = -2m\,\tilde{L}_{-m} Z$$

Thank you very much for your kind attention!

Happy 80th Birthday, Harald!

By the way, in Japan, the next grand celebration after 80 is at the age of 88.
The age of 88 is called "Beijyu" in Japan, which means "Age of Rice".

Quiz: Why "Rice"?

I'll give you the answer in 8 years.