Discussion of Combinatorial Dyson-Schwinger equations in tensorial field theory

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What is known in ordinary QFTs

- Well-known Hopf algebra in QFT renormalization (Connes, Kreimer)
- Combinatorial Dyson-Schwinger equation in QFT (Broadhurst, Kreimer, Yeates,...)
- Perturbative and non-perturbative results via resurgence (Borinsky, Dunne,...)

What is new by Johannes Thürigen

Definition

2-Graphs: More combinatorial structure as colors, strands, vertex graphs, boundary graphs,...

Examples

- Matrix Models: Kontsevich model, Grosse-Wulkenhaar model, 2D quantum gravity
- all kind of Tensor models

Hopf Algebra of 2-Graphs

Theorem

2-graphs have an underlying (renormalization) Hopf algebra

Different construction of JT in comparison to previous results (by Kreimer and Tanasa on GW-model ϕ^4 matrix model):

- Canonical consideration of the boundary structure (boundary graphs)
- Insertion operator B_+ takes care of these structures $\rightarrow$ natural extension to tensorial models

An Application: BPHZ Theorem

The antipode S of the Hopf algebra gives naturally the forest formula for 2-graphs. Perturbative calculations were explicitly carried out in the

Example

- matrix ϕ^3 model
- matrix ϕ^4 model
- tensor $\phi^4_{1,3}$ model

and compared with the expansion of exact results.

Exact results are available

For the matrix ϕ^3 and ϕ^4 model exact results are known and given by Topological Recursion!

Tensor models $\rightarrow$ Topological Recursion via Hopf algebra?

Perturbative and Non-Perturbative

Our work in progress

Two ways to get results:

1. Hopf algebra $\rightarrow$ combinatorial DSE $\rightarrow$ resurgence
2. Large N limit and genus expansion $\rightarrow$ ordinary DSE $\rightarrow$ exact results for each genus

Relation between 1. and 2.

How are both approaches related? Can we learn something for ordinary QFTs from this?

Question to Johannes

- Are exact results in tensor models derived, and compared with the perturbative expansion renormalized by BPHZ Theorem?