

## References and comment for "Lectures on Poisson sigma model and integrable systems".

These lectures are based on the paper:

A. Cattaneo, P. Mnev, N. Reshetikhin, Poisson sigma model and semiclassical quantization of integrable systems, *Reviews in Mathematical Physics* 30, 93-118 (2018), arXiv:1803.07723

and on unpublished results which will soon appear (with A. Cattaneo and P. Mnev).

### 1. LECTURES 1A

- There are plenty of books on classical integrable systems. See for example, see two surveys and a book below and references therein.
  - V. I. Arnold, A. B. Givental, Symplectic geometry, In: *Dynamical Systems IV*, Volume 4, the series *Encyclopaedia of Mathematical Sciences* pp 1-138, Springer, 2001.
  - Semenov-Tian-Shansky M.A. (1997) *Quantum and classical integrable systems*. In: Kosmann-Schwarzbach Y., Grammaticos B., Tamizhmani K.M. (eds) *Integrability of Nonlinear Systems*. *Lecture Notes in Physics*, vol 495. Springer, Berlin, Heidelberg. <https://doi.org/10.1007/BFb0113700>
  - Olivier Babelon, Denis Bernard, Michel Talon, *Introduction to Classical Integrable Systems*, Cambridge University Press, 2003
- The amount of reviews and books written on quantization is enormous. Here are two of my favorite references which are nice introductions at an undergraduate and graduate level (there are many wonderful other references, I am just a bit biased) to the subject:
  - L. D. Faddeev, O. A. Yakubovskii, *Lectures on Quantum Mechanics for Mathematics Students*, American Mathematical Soc., 2009
  - L. Takhtajan, *Quantum Mechanics for Mathematicians Graduate Studies in Mathematics Volume: 95*; American Mathematical Soc., 2008; 387 pp.
- The paper by M. Kontsevich on star products and formality for Poisson manifolds:
  - M. Kontsevich, *Deformation quantization of Poisson manifolds, I*, *Lett. Math. Phys.*, 66:157-216,2003, arXiv:q-alg/9709040
- Quantum integrable systems and their semiclassical behavior. There are many aspects of this topic and there are plenty of references. Here is my paper on this subject with most relevant references:
  - N. Reshetikhin, *Semiclassical geometry of integrable systems*, arXiv:1802.00416.

## 2. LECTURE 1B

There are many references on formal integrals and Feynman diagrams. Most relevant ones for these lectures can be found in

N. Reshetikhin, Lectures on quantization of gauge systems, Based on lectures given at the Summer School "New paths towards quantum gravity", Holbaek, Denmark, 2008; arXiv:1008.1411

Microlocal treatment of what is called formal integral operator here and many other interesting aspects of formal semiclassical geometry can be found in

B. Tsygan, A microlocal category associated to a symplectic manifold, arXiv:1512.02747.

## 3. LECTURE 1C

The relation between the semiclassical asymptotic of joint eigenfunctions for two quantum integrable systems and the topological quantum mechanics is explained in

A. Cattaneo, P. Mnev, N. Reshetikhin, Poisson sigma model and semiclassical quantization of integrable systems, *Reviews in Mathematical Physics* 30, 93-118 (2018), arXiv:1803.07723

## 4. LECTURE 2A

The Poisson sigma model was originated in

P. Schaller and T. Strobl, Poisson structure induced (topological) field theories, *Modern Phys. Lett. A* 9 (1994), no. 33, 31293136

Formal path integral for the Poisson sigma model was defined in

A. Cattaneo, G. Felder, Poisson sigma models and deformation quantization *Mod.Phys.Lett. A* 16 (2001) 179-190, arXiv:hep-th/0102208

where it was also shown that Feynman diagrams there are exactly coefficients of Kontsevich's star-product.

Boundary structures in classical field theories is a subject that has been discussed a lot in the literature. See the paper below for the relevant discussion and relevant references.

A. Cattaneo, P. Mnev, N. Reshetikhin, Classical BV theories on manifolds with boundary, *Commun. Math. Phys.* 332, 535-603 (2014), arXiv:1201.0290.

## 5. LECTURE 2B

The BV extension of the Poisson sigma is described in  
A. Cattaneo, G. Felder, Poisson sigma models and deformation quantization  
Mod.Phys.Lett. A16 (2001) 179-190, arXiv:hep-th/0102208

## 6. LECTURE 3A

For a systematic exposition of BV framework see

P. Mnev, Lectures on Batalin-Vilkovisky formalism and its applications in topological quantum field theory, arXiv:1707.08096

See

A. Cattaneo, G. Felder, Poisson sigma models and deformation quantization  
Mod.Phys.Lett. A16 (2001) 179-190, arXiv:hep-th/0102208

A. Cattaneo, G. Felder, Coisotropic submanifolds in Poisson geometry and branes  
in the Poisson sigma model, Lett. Math. Phys. 69 (2004) 157-175, arXiv:math/0309180

for propagators with various boundary conditions. See also

A. Ferrario, Poisson Sigma Model with branes and hyperelliptic Riemann surfaces,  
J.Math.Phys.49:092301,2008, arXiv:0709.0635

where propagators were computed for boundary conditions with more than two  
components.

These lectures are based on the forthcoming paper:

A. Cattaneo, P. Mnev, N. Reshetikhin (in progress).