ESI Erwin Schrödinger International Institute for Mathematical Physics

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ABSTRACTS

ESI workshop on “Several complex variables and CR geometry" from November 3rd to 13th, 2015

Soledad Benguria (University of Wisconsin-Madison)

An application of John ellipsoids to the Szego kernel on unbounded convex domains

ABSTRACT: I use classical convexity tools, in particular John ellipsoids, to obtain a size estimate for the Szego kernel on the boundary of a class of unbounded convex domains in $\mathbb{C}^n$. Given a polynomial $b : \mathbb{R}^n \to \mathbb{R}$ satisfying a certain growth condition, I consider domains of the type $\Omega_b = \{z \in \mathbb{C}^{n+1} : \mathrm{Im}[z_{n+1}] > b(\mathrm{Re}[z_1], \ldots, \mathrm{Re}[z_n])\}$.

Franz Berger (University of Vienna)

The essential spectrum of the complex Laplacian on product manifolds

ABSTRACT: We compute the essential spectrum of the complex Laplacian on the product of two Hermitian manifolds in terms of the spectral data of the factors. Applications to (non-)compactness of the minimal solution operators to the $\overline{\partial}$-equation are given, and this generalizes some previously known results.

Shiferaw Berhanu (Temple University)

Some remarks on the weak Hopf lemma

ABSTRACT: We discuss some results on unique continuation for holomorphic maps of one complex variable at the boundary. If time permits, we will present extensions to solutions of planar, complex vector fields.

Mehmet Celik (Texas A&M University Commerce)

Analysis on the intersection of pseudoconvex domains

ABSTRACT: In this talk, we discuss the preservation of certain analytic properties of the $\overline{\partial}$-Neumann operator, Bergman projection and Hankel operators on the intersection of pseudoconvex domains. (Joint work with Yunus Zeytuncu.)

Bo-Yong Chen (Fudan University)

Parameter dependence of the Bergman kernels: Hölder continuity

ABSTRACT: Let $\Omega_t = \{\rho_t < 0\}$, $t \in [0,1]$, be a family of bounded hyperconvex domains such that ρ_t is Hölder continuous of order α in t. Let K_t denote the Bergman kernel of Ω_t. We show that $K_t(z,w)$ is Hölder continuous of order β in t for any $\beta < \alpha$.

Paulo Cordaro (University of São Paulo)

Semi-global solvability with loss of one derivative for linear partial differential operators in dimension 2

ABSTRACT: In this talk I shall present recent results obtained in collaboration with J. Hounie on the improvement of the celebrated Hörmander's theorem on semi-global solvability for LPDO satisfying condition (P). In particular we show that in dimension 2 solutions can be found with the sharp loss of one derivative.

Gian Maria Dall'Ara (Scuola Normale Superiore, Pisa)

The weighted $\overline{\partial}$-problem in $\mathbb{C}^n$: solvability, compactness, and pointwise bounds for the Bergman kernel

ABSTRACT: Let $\varphi : \mathbb{C}^n \to [0, +\infty)$ be a smooth plurisubharmonic weight function, and let us denote by $L^2(\mathbb{C}^n, \varphi)$ the L^2 space with respect to $e^{-2\varphi}$-times Lebesgue measure. The associated weighted $\overline{\partial}$-problem consists in studying existence and properties of the solutions in $L^2(\mathbb{C}^n, \varphi)$ of the equation: $\overline{\partial} f = \omega$, where ω is a $\overline{\partial}$-closed $(0,1)$-form with coefficients in $L^2(\mathbb{C}^n, \varphi)$. A related question is that of studying the Bergman projection operator that maps orthogonally $L^2(\mathbb{C}^n, \varphi)$ onto its subspace consisting of holomorphic functions. These questions have been studied in depth in one complex variable (see in particular [1]), and it appears that their several variables counterparts present serious difficulties and require new ideas.

In the talk we present some results we obtained (see [2], [3], [4]) about the weighted $\overline{\partial}$-problem in $\mathbb{C}^n$, in particular:

1. new pointwise estimates for the integral kernel of the Bergman projection operator that extend those obtained by Christ,
2. sufficient and/or necessary conditions for solvability and compactness of the problem (i.e., compactness of the canonical solution operator) for certain classes of model weights previously studied by Nagel and Pramanik.

A key tool in our analysis is a new "holomorphic uncertainty principle" inspired by the close relationship between the weighted Kohn Laplacian, an elliptic operator naturally arising in this context, and certain generalized Schrödinger operators.

[1] Christ, M., *On the $\overline{\partial}$ equation in weighted L^2 norms in $\mathbb{C}^1$*, Journal of Geometric Analysis 1 (3) (1991) 193-230.

[2] Dall'Ara, G., *Pointwise estimates of weighted Bergman kernels in several complex variables*, Advances in Mathematics 285 (2015) 1706-1740.

[3] Dall'Ara, G., *Coercivity of weighted Kohn Laplacians: the case of model monomial weights in $\mathbb{C}^2$*, arXiv:1502.02598, to appear in Transactions of the American Mathematical Society.

[4] Dall'Ara, G., *Matrix Schrödinger Operators and Weighted Bergman Kernels*, Ph.D. thesis, arXiv:1501.06311

Giuseppe Della Sala (American University of Beirut)

Stationary discs and singular Riemann-Hilbert problems

ABSTRACT: Stationary discs are holomorphic discs that are attached to a real submanifold M of $\mathbb{C}^n$ and satisfy additional lifting properties, ensuring that their family is both finite dimensional and biholomorphically invariant. Because of that, such discs are suitable for studying mapping problems between smooth CR submanifolds. The usual notion of stationarity, however, is only well adapted for Levi non-degenerate submanifolds. Via the treatment of certain linear Riemann-Hilbert problems with singularities, we show how to construct stationary discs (in a generalized sense) for a class of smooth hypersurfaces of finite type, and then obtain finite jet determination results for CR maps between hypersurfaces of this class. This is joint work with Florian Bertrand.

Barbara Drinovec Drnovšek (University of Ljubljana)

Complete proper holomorphic embeddings of strictly pseudoconvex domains into balls

ABSTRACT: We present a construction of a complete proper holomorphic embedding from any strictly pseudoconvex domain with smooth boundary in $\mathbb{C}^n$ into the unit ball of $\mathbb{C}^N$, for N large enough.

Son Ngoc Duong (Texas A&M University at Qatar)

On Schwarzian derivative and CR mappings

ABSTRACT: I would like to report on a study on the notion of Schwarzian derivative for CR mappings. This notion is a CR-analogue of Schwarizan for conformal mappings introduced by Osgood and Stowe in 1992. I will use it to define the notion of Mobius transformations on pseudohermitian manifolds and study geometric properties of manifolds admitting nontrivial Mobius transformation. I will also prove a rigidity result for such transformations on compact manifolds of nonpositive Webster Ricci curvature.

Peter Ebenfelt (University of California, San Diego)

Umbilical points on compact, three dimensional CR manifolds

ABSTRACT: We shall discuss existence of umbilical points on compact, strictly pseudoconvex CR manifolds of dimension three. We consider such CR manifolds M with a free, transverse CR $U(1)$-action and prove a Poincare-Hopf type index formula relating the sum of indices of isolated umbilical $U(1)$-orbits to the Euler characteristic of the Riemann surface $M/U(1)$. As a consequence, we find, e.g., that every compact, circular hypersurface in $\mathbb{C}^2$ has at least one circle of umbilical points.

Franc Forstnerič (University of Ljubljana)

Non-orientable minimal surfaces in $\mathbb{R}^n$

ABSTRACT: In this lecture, I will show how complex analytic methods can be used for effective constructions of non-orientable minimal surfaces in $\mathbb{R}^n$ for any $n > 2$. In particular, we prove the Runge-Mergelyan approximation theorem, construct proper non-orientable minimal surfaces in certain classes of domains in $\mathbb{R}^n$, and construct complete non-orientable minimal surfaces bounded by Jordan curves. (Joint work with A. Alarcon and F.J. Lopez, University of Granada.)

Stefan Fürdös (University of Vienna)

Regularity of infinitesimal CR Automorphisms

ABSTRACT: In this talk we consider infinitesimal CR automorphisms on abstract CR manifolds and their regularity properties. Under rather modest a-priori regularity conditions we prove that every infinitesimal CR automorphism of an abstract CR manifold (satisfying a certain weak nondegeneracy condition) is actually smooth. The proof is based on a regularity result for solutions of a certain system of linear PDEs, where the assumptions on the solution can be reduced to some boundedness condition and a certain microlocal smoothness condition. This microlocal condition corresponds in the embedded case to the holomorphic extendability in a wedge, although our condition is naturally considerably weaker. This is joint work with Bernhard Lamel.

Martin Kolar (Masaryk University, Brno)

Automorphism groups of Levi degenerate hypersurfaces in $\mathbb{C}^3$

ABSTRACT: By the classical Chern-Moser theory, there are no nonlinear automorphisms on any strongly pseudoconvex nonspherical hypersurface. Several recent results indicate that the case of Levi degenerate hypersurfaces is much more interesting. In this talk we will consider hypersurfaces of finite Catlin multitype in $\mathbb{C}^3$ and give a classification of such hypersurfaces which admit nonlinear infinitesimal automorphisms. We will also describe a common source of such vector fields in terms of existence of suitable mappings into hyperquadrics in $\mathbb{C}^K$, $K \geq 3$. This is joint work with Francine Meylan.

Loredana Lanzani (Syracuse University)

Singular Integral techniques in Several complex variables

ABSTRACT: I will give a survey of recent joint work with E. M. Stein concerning the L^p-regularity of the Szegő and Bergman projections, and other singular integral operators, in the non-classical context of domains with non-smooth boundary.

Ingo Lieb (Universität Bonn)

Formulae and estimates for the Cauchy-Riemann equations

ABSTRACT: Integral formulae yield good estimates for solutions of the $\overline{\partial}$ equation. The structure of their kernels gives, moreover, some insight into the nature of the problem. This has been worked out, over the years, in the case of strictly pseudoconvex domains with smooth boundary. More recently, there has been some progress in the case of non-smooth boundaries. I will report on some aspects of this theory.

Jeffery McNeal (Ohio State University)

The Bergman projection on generalized Hartogs triangles

ABSTRACT: We introduce a class of domains in C^2, that interpolate between the classical Hartogs triangle and the product domain $D \times D^*$, and discuss their Bergman theory. The main result is that the Bergman projection, B, of these domains is only bounded on L^p for a restricted range of p that shrinks to 2 as the domains fill out $D \times D^*$. This is somewhat surprising as B on $D \times D^*$ is known to map L^p to itself boundedly for all $1 < p < \infty$. The results are joint work with Luke Edholm.

Francine Meylan (University of Fribourg)

Higher order symmetries of real hypersurfaces in $\mathbb{C}^3$

ABSTRACT: We study nonlinear automorphisms of Levi degenerate hypersurfaces of finite multitype. By recent results, the Lie algebra of infinitesimal CR automorphisms may contain a graded component consisting of nonlinear vector fields of arbitrarily high degree, which has no analog in the classical Levi nondegenerate case, or in the case of finite type hypersurfaces in $\mathbb{C}^2$. We analyze this phenomenon for hypersurfaces of finite Catlin multitype in complex dimension three. The results provide a complete classification of such manifolds. As a consequence, we show on which hypersurfaces 2-jets are not sufficient to determine an automorphism. (Joint work with Martin Kolar.)

Nordine Mir (Texas A&M University at Qatar)

Artin approximation and CR geometry

ABSTRACT: In 1968, Artin proved his famous approximation theorem: given any system of real-analytic equations, if there exists a formal solution to such a system at a given point, then there exists a real-analytic solution that is as close as we want in the Krull topology to the formal solution. One question that naturally thereafter arises is whether the conclusion of Artin's approximation theorem is still preserved if the system of equations is coupled with a specific PDE. In 1978, Milman investigated such a question when the PDE consists of the standard Cauchy-Riemann operator in $\mathbb{R}^{2n} \simeq \mathbb{C}^n$: he showed that any formal solution of a system of real-analytic equations and of the standard CR equations in $\mathbb{C}^n$ can be approximated (in the Krull topology) by a sequence of convergent solutions of the system of analytic and CR equations. In this talk, we will discuss recent results generalizing Milman's theorem when the standard Cauchy-Riemann operator in $\mathbb{C}^n$ is replaced by the tangential Cauchy-Riemann operator associated to a real-analytic CR manifold.

Takeo Ohsawa (Nagoya University)

Backgrounds and updates of L^2 extension theorems

ABSTRACT: After the foundation of several complex variables by K. Oka and H. Cartan, the L^2 method was developed to obtain effective results in complex analysis and geometry. An extension theorem with L^2 growth condition was obtained in this context. It turned out to be useful in complex analysis and geometry. Recently, based on an argument of Q. Guan and X.-Y. Zhou, J. Cao proved in a general context that an L^2 extension theorem implies the semipositivity of the direct image of relative canonical bundles. Vice versa, B. Berndtsson and L. Lempert showed that such a curvature property implies an L^2 extension theorem in an optimal form. Since this progress is closely related to the solution of a long-standing conjecture of N. Suita by Z. Błocki, Guan and Zhou, it might be a good occasion to give an overview of this development starting from the background of the questions. Several new questions will be raised, too.

Andrew Seth Raich (University of Arkansas)

Global L^q-Gevrey Function Spaces

ABSTRACT: In this talk, I will discuss ongoing joint work with Gustavo Hoepfner and Ziad Adwan. I will introduce a class of functions that arose in the work of Boggess and myself when estimating the $\Box_b$-heat kernel on polynomial models. This class of functions captures the quantitative estimates on the Fourier transform needed to characterize exponential decay. I will discuss properties of these functions, give examples, and explore other applications.

Jean Ruppenthal (Bergische Universität Wuppertal)

$\overline{\partial}$-homotopy formulas on singular varieties

ABSTRACT: We will discuss homotopy integral formulas for the $\overline{\partial}$-operator on singular complex varieties, particularly the integral operators of Andersson-Samuelsson Kalm. It will be pointed out that we can expect useful formulas in the situation of mild singularities (so-called canonical singularities). Useful means here for example that the operators can be used to solve $\overline{\partial}$-equations in the L^2-sense. This will be elaborated for affine cones over projective manfifolds. The talk is based on a joined work with Richard Laerkaeng.

Nikolay Shcherbina (Bergische Universität Wuppertal)

Some remarks on analyticity of q-pseudoconcave graphs

ABSTRACT: We make an overview of known results on analyticity of pseudoconvex graphs and discuss their possible generalizations to the q-pseudoconvex case.

Laurent Stolovitch (Université Nice Sophia Antipolis)

Real submanifolds of maximum complex tangent space at a CR singular point

ABSTRACT: We study a germ of real analytic n-dimensional submanifold of $\mathbf{C}^n$ that has a complex tangent space of maximal dimension at a CR singularity. Under some assumptions, we show its equivalence to a normal form under a local biholomorphism at the singularity. We also show that if a real submanifold is formally equivalent to a quadric, it is actually holomorphically equivalent to it, if a small divisors condition is satisfied. Finally, we investigate the existence of a complex submanifold of positive dimension in $\mathbf{C}^n$ that intersects a real submanifold along two totally and real analytic submanifolds that intersect transversally at a possibly non-isolated CR singularity. This is a joined work with X. Gong.

Emil Straube (Texas A&M University)

L^2-Sobolev and compactness estimates for $\overline{\partial}_M$: a (very) brief survey

ABSTRACT: We will survey a few semi-recent (last ten years) results concerning compactness or Sobolev estimates for $\overline{\partial}_M$. Here M is a compact pseudoconvex orientable CR submanifold of $\mathbb{C}^n$ without boundary, of hypersurface type.

Wlodzimierz Zwonek (Jagiellonian University in Krakow)

Multidimensional Suita conjecture

ABSTRACT: In 2013 Z. Blocki proved that the Suita conjecture, i.e. the inequality $c_\Omega^2(z) \leq \pi K_\Omega(z,z)$, $z \in \Omega$ holds for any planar domain Ω. Here c_Ω denotes the logarithmic capacity of the complement of Ω whereas K_Ω denotes the Bergman kernel. The method used in the proof of the above inequality is of higher dimensional character and relies upon the estimates between the volume of the sublevel sets of the pluricomplex Green function and the Bergman kernel which holds in higher dimensional domains. This allows us to formulate and prove the higher dimensional version of the above inequality which is the following $F_\Omega(z) := \lambda(I_\Omega^A(z))K_\Omega(z,z) \geq 1$, where Ω is a pseudoconvex domain in $\mathbb{C}^n$ and $I_\Omega^A(z)$ is the Azukawa indicatrix at z. It is interesting that in the case of convex domain Ω an upper estimate for F_Ω also holds. Analysis of (non-trivial) examples of convex domains Ω with $F_\Omega \not\equiv 1$ will also be presented. The results presented in the lecture come from the joint work with Z. Blocki.