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Holography of new conformal higher spin gravities in three dimensions

Geometry for Higher Spin Gravity: Conformal Structures, PDEs, and Q-manifolds

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Outline

- Introduction
- Conformal gravity and Higher spin algebras
- Conformal higher spin gravity
- New conformal higher spin theories
- Holography
- Boundary conditions
- Conclusion

Introduction

- Chern-Simons theory in three dimensions is very special
- Its action for $so(2,2) \sim sl_2 \oplus sl_2$ is equivalent to Einstein-Hilbert action, and this construction was first step to get higher spin extension by replacing sl_2 algebra
- Conformal gravity in three dimensions can be interpreted as Chern-Simons theory for the conformal algebra so(3,2)
- Conformal higher spin algebra in 3D is AdS algebra in 4D
- Chern Simons action for this algebra gives a consistent theory for CHS
- They first appeared in study of Pope and Towsend, and Fradkin and Linetsky.
- Conformal higher spin fields, Fradkin-Tseytlin fields, are symmetric traceless tensors with linearised gauge symmetries $\delta\phi_{a_1\ldots a_s} = \partial_{a_1}\ldots\partial_{a_t}\xi_{a_t+1\ldots a_s} + \text{permutations} - \text{traces}$

Introduction

- In the literature so far, there have been CHS theories that were studied using higher spin algebras known from the symmetry studies of higher derivative free CFT's and partially massless fields in AdS_4, however this is not our case
- The difference to earlier studied CHS theories, is that here the theories have finitely many fields, and the construction permits for a large class of new higher spin algebras

Introduction

Holography

- HS theory is from several aspects important in string theory. One of them is to better understand AdS/CFT correspondence.
- In 2+1 D the theory is simple with rich asymptotic symmetry
- The pioneering work by Campoleoni, Fredenhagen and Theisen, was a generalisation of the Brown and Henneaux study of the 3D gravity. It showed that massless higher spins are dual to two copies of W algebra with central charge as the one of pure gravity

Introduction

Holography

- The works so far have generalised the spin two analyses of 3D Chern-Simons gravity, we proceed along this direction
- The Brown-Henneaux bcs are not the only ones that incorporate stationary black hole solutions in General Relativity with negative cosmological constant in 2+1D. There are also black flower configurations, that have infinite number of u(1) charges [Afshar, Detourney, Grumiller, Merbis, Perez, Tempo]
- That modified bcs accommodate respective black hole solutions with higher spin fields [Grumiller, Perez, Prohazka, Tempo, Troncoso]
- The most general boundary conditions (bcs) for the Chern-Simons gravity with so(2,2) gauge group have six charges and six chemical potentials, and define ASA which consists of two copies of affine $sl(2)_k$ algebra [Grumiller, Riegler]

Conformal gravity and conformal higher spin theories

Conformal gravity

$$S(\tilde{\omega}) = \int Tr\left(\tilde{\omega} \wedge d\tilde{\omega} + \frac{2}{3}\tilde{\omega} \wedge \tilde{\omega} \wedge \tilde{\omega}\right)$$

spin connection

generators of the Lorentz algebra

$$\tilde{\omega} \equiv \frac{1}{2}\tilde{\omega}^{a,b} L_{ab} \qquad \tilde{\omega}^{a,b} = \tilde{\omega}^{a,b}_{\mu} dx^{\mu}$$

$$e^a \equiv e^a_\mu dx^\mu$$

dreibein

Torsion constraint

$$\nabla e^a = de^a + \tilde{\omega}^{a,}{}_b \wedge e^b = 0$$

- CG is gauge theory of conformal algebra in 3D [Horne,Witten]

$$S(\omega) = \int Tr\left(\omega \wedge d\omega + \frac{2}{3}\omega \wedge \omega \wedge \omega\right)$$

Generators of so(3,2) algebra: Lorentz rotations, translations, SCTs and dilatations

$$\omega = \frac{1}{2}\tilde{\omega}^{a,b} L_{ab} + e^a P_a + f^a K_a + bD$$

Conformal gravity and conformal higher spin theories

Conformal gravity

$$F = d\omega + \frac{1}{2}[\omega,\omega]$$

Equations of motion set curvature to zero

gauge parameter

$$\Xi = \frac{1}{2}\eta^{a,b}L_{ab} + \xi^a P_a + \zeta^a K_a + \rho D$$

Solving equations of motion and taking into account gauge transformation:

➡

$$C_{\mu\nu} = 0$$

Cotton tensor

$$\nabla = d + \tilde{\omega}$$

Torsion constraint

Sets $f_{\mu\nu}$ to be symmetric

$$F_P^a = \nabla e^a - b \wedge e^a\,,$$

$$F_D = \nabla b + e_m \wedge f^m\,,$$

$$F_L^{a,b} = R^{a,b} - e^a \wedge f^b + e^b \wedge f^a\,,$$

$$F_K^a = \nabla f^a + b \wedge f^a$$

Riemann 2-form

Schouten tensor

Conformal gravity and conformal higher spin theories

Higher spin algebras

- We are going to use the action which is the same Chern-Simons action for a higher spin algebras for partially massles fields from AdS4
- Earlier considered algebras are usually symmetries of $\Box^k \Phi(x) = 0$ which is a conformally invariant higher derivative equation
- However we are going to consider new algebras
- That will lead to finite number of Fradkin-Tseyltin fields with higher derivative gauge transformations
- To get higher spin extension of so(3,2) algebra, we need algebras that have so(3,2) as an subalgebra.

Conformal gravity and conformal higher spin theories

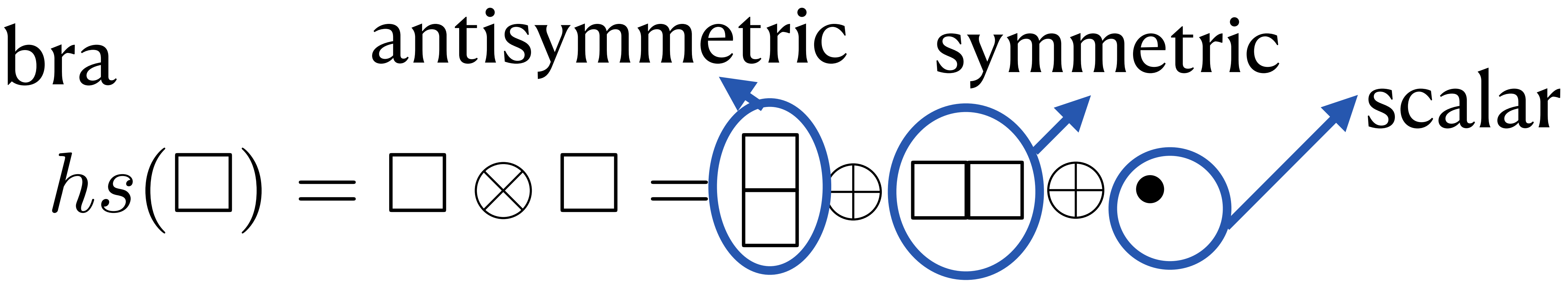
Higher spin algebras

- To construct a finite spectrum of Fradkin-Tseytlin fields one needs to take non-trivial finite-dimensional irreducible representation V of so(3,2). Which is irreducible tensor or a spin-tensor
- Then, one needs to evaluate U(so(d,2)) in V, which means multiply the generators of so(3,2) in this representation to find the algebra "hs(V)" that they generate. The algebra is the algebra of all the the matrices of size dimV, $hs(V) = End(V) = V \otimes V^*$
- We want to decompose the hs(V) into irreducible so(3,2) modules which determines the spectrum of Fradkin-Tseytlin fields
- $hs(V) = gl(V) = sl(V) \oplus u(1)$

Conformal gravity and conformal higher spin theories

Higher spin algebras

- To better show this we consider an example of the Young diagram and a corresponding algebra



$$hs(\square) = \square \otimes \square = \begin{smallmatrix}\square\\ \square\end{smallmatrix} \oplus \square\!\square \oplus \bullet$$

- In general, the algebra is matrix algebra of $(3+2)^2$ generators $t_A{}^B$
- they are decomposed with respect to so(3,2)
- The commutation relations are those of gl_{3+2} $[t_A{}^B, t_C{}^D] = -\delta_A{}^D t_C{}^B + \delta_C{}^B t_A{}^D$
- in the so(3,2) base we have

$$T_{AB} = t_{A|B} - t_{B|A}, \qquad S_{AB} = t_{A|B} + t_{B|A} - \frac{2}{d+2} t_C{}^C, \qquad R = t_C{}^C$$

antisymmetric

symmetric

scalar

Conformal gravity and conformal higher spin theories

Higher spin algebras

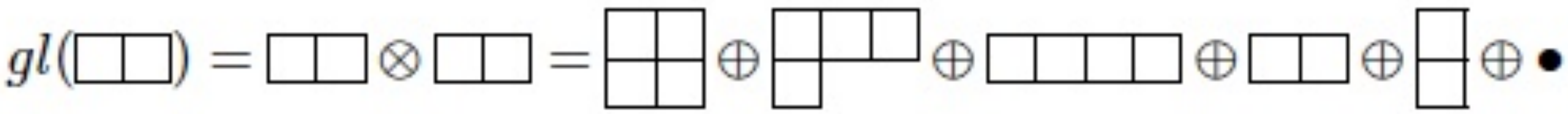
- The commutation relations

$$[T_{AB}, T_{CD}] = \eta_{BC} T_{AD} - \eta_{AC} T_{BD} - \eta_{BD} T_{AC} + \eta_{AD} T_{BC} \tag{1}$$

$$[T_{AB}, S_{CD}] = \eta_{BC} S_{AD} - \eta_{AC} S_{BD} + \eta_{BD} S_{AC} - \eta_{AD} S_{BC} \tag{2}$$

$$[S_{AB}, S_{CD}] = \eta_{BC} T_{AD} + \eta_{AC} T_{BD} + \eta_{BD} T_{AC} + \eta_{AD} T_{BC} \tag{3}$$

- example: symmetric rank two representation with $t^{AB}{}_{CD}$ generators and decomposition in so(3,2)



Conformal higher spin gravity

Free Fradkin-Tseytlin Fields

- To identify the higher spin content for one of the higher spin algebras above we have to linearise the theory over the Minkowski vacuum.
- In Cartesian coordinates the vacuum is chosen as $\omega_0 = h^a P_a$

$$h^a \equiv h^a{}_\mu dx^\mu \qquad h^a{}_\mu = \delta^a{}_\mu \qquad P_a \in so(3,2) \subset hs$$

Lie algebra valued one form

$$\omega \equiv \omega^\Lambda t_\Lambda$$

- free equations and the linearised gauge symmetries

$$d\omega + \omega_0 \wedge \omega + \omega \wedge \omega_0 = 0 \qquad \delta\omega = d\xi + [\omega_0, \xi]$$

$$\xi \equiv \xi^\Lambda t_\Lambda$$

gauge parameter

Lie algebra valued zero form

Conformal higher spin gravity

Free Fradkin-Tseytlin Fields

- When we introduce the definition of the vacuum we can write the main equations that we are considering

$$d\omega^{\Lambda} t_{\Lambda} + h^{a} \wedge \omega^{\Lambda}[P_a, t_{\Lambda}] = 0 \ \ (\star) \qquad\qquad \delta\omega^{\Lambda} t_{\Lambda} = d\xi^{\Lambda} t_{\Lambda} + h^{a} \wedge \xi^{\Lambda}[P_a, t_{\Lambda}]$$

- we need a dictionary between irreducible modules of so(3,2) that appear in hs and Fradkin-Tseytlin fields

Conformal higher spin gravity

Conformal gravity

- The dictionary between conformal algebra and T_{AB} is

$$P_a = T_{a+}, K_a = T_{a-}, D = -T_{+-}, L_{ab} = T_{ab}$$

- In the light cone coordinates one decomposes the equations and gauges to get the equations and fields for each of the higher spin theories

$$\eta_{+-} = \eta_{-+} = 1 \qquad\qquad A = a, +, -$$

- For Conformal Gravity we have adjoint module $\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}$ and $t_{AB} = -t_{BA}$

$$\omega = \omega^{a+} t_{a+} + \frac{1}{2} \omega^{a,b} t_{ab} + \omega^{+-} t_{+-} + \omega^{a-} t_{a-}$$

- insert that in equation $(\star)$

Conformal higher spin gravity

Conformal gravity

sets antisymmetric part of $\omega^{a-|m}$ to zero

gauges away ω^{+-}

- the linearised eom for CG and their gauge symmetries

$$
\begin{aligned}
t_{+-}: &\quad d\omega^{+-} - h_m \wedge \omega^{m-} = 0\,, &\quad \delta\omega^{+-} &= d\xi^{+-} - h_m \xi^{m-}\,,\\
t_{a+}: &\quad d\omega^{a+} + h_m \wedge \omega^{m,a} - h^a \wedge \omega^{+-} = 0\,, &\quad \delta\omega^{a+} &= d\xi^{a+} + h_m \xi^{m,a} - h^a \xi^{+-}\,.\\
t_{ab}: &\quad d\omega^{a,b} - h^a \wedge \omega^{b-} + h^b \wedge \omega^{a-} = 0\,, &\quad \delta\omega^{a,b} &= d\xi^{a,b} - h^a \xi^{b-} + h^b \xi^{a-}\\
t_{a-}: &\quad d\omega^{a-} = 0\,, &\quad \delta\omega^{a-} &= d\xi^{a-}\,.
\end{aligned}
$$

gives $\omega^{ab|m}$

eliminates antisymmetric part of $\omega^{a+|m}$

removes the trace

$$\omega_{a-|m} = S_{ab} = \frac{1}{2}(-\Box\phi_{ab} + \partial_a\partial^m\phi_{mb} + \partial_b\partial^m\phi_{ma} - \frac{1}{2}\eta_{ab}\partial^m\partial^n\phi_{mn})$$

Linearized Schouten tensor

Conformal higher spin gravity

Conformal gravity and Spin-2

- The last equation is dynamical one and sets Cotton tensor to zero

$$C_{ab} = \epsilon_a{}^{mn}\partial_m S_{bn} + \epsilon_b{}^{mn}\partial_m S_{an} = 0 \qquad \delta\phi_{ab} = \partial_a\xi_b + \partial_b\xi_a - \frac{2}{3}\eta_{ab}\partial^m\xi_m$$

- The next new example is the vector representation of so(3,2) denoted with $\square$

$$\omega = \omega^a t_a + \omega^+ t_+ + \omega^- t_-$$

- similarly ➡ $\delta\phi_{ab} = \partial_a\partial_b\xi = -\frac{1}{3}\eta_{ab}\Box\xi$

Depth-two Fradkin-Tseytlin spin-two conformal field

$$C_{ab} = \epsilon^{mn}{}_a\partial_m\phi_{bn} + \epsilon^{mn}{}_b\partial_m\phi_{an} = 0$$

Cotton tensor

New conformal higher spin theories

Spin-3

- The simplest higher spin example is the irreducible rank-two representation of so(3,2) □□ $t_{AB} = t_{BA}$

$$\omega = \omega^{a+} t_{a+} + \frac{1}{2}\omega^{ab} t_{ab} + \frac{1}{2}\omega^{++} t_{++} + \frac{1}{2}\omega^{--} t_{--} + \omega^{a-} t_{a-}$$

- using the same procedure we obtain depth-three Fradkin Tseytlin field

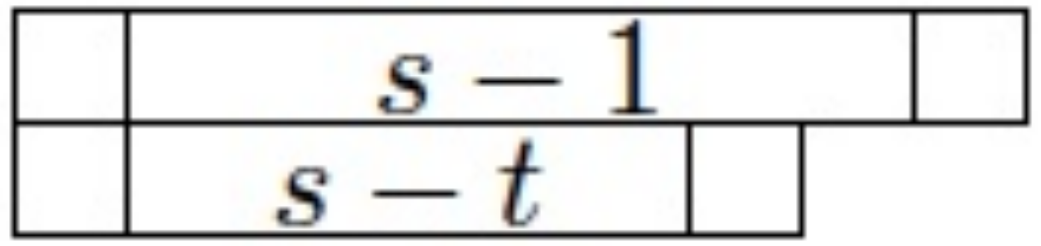
$$\delta\phi^{abc} = \partial^a \partial^b \partial^c \xi - \frac{1}{5}(\eta^{ab}\partial^c \Box\xi + \eta^{ac}\partial^b \Box\xi + \eta^{ac}\partial^b \Box\xi)$$

New conformal higher spin theories

Spin-s

- In general, for the higher spin algebra with an embedding of so(3,2) we need to decompose hs into so(3,2)-modules.
- The spectrum of the Fradkin-Tseytlin fields is determined by the free field limit

$$\delta\phi_{a_1\ldots a_s} = \partial_{a_1}\ldots\partial_{a_t}\xi_{a_t+1\ldots a_s} + \ldots$$



Gauge field: $\omega^{A(s-1),B(s-t)}$

Gauge parameter: $\zeta^{A(s-1),B(s-t)}$

- gauge parameters except $\zeta^{a(s-t)+(t-1),+(s-t)}$ can be used to gauge away certain components of $\omega^{A(s-1),B(s-t)}$

Denotes +…+ (s-t) times

New conformal higher spin theories

Spin-s

- All components of ω are gauged away except of

$$\phi^{a_1 \ldots a_s} = \omega_\mu^{a_1 \ldots a_{s-1}, +(s-t)} h^{\mu a_s} + symmetrization - traces$$

- Dynamical equations of motion can be written in terms of the Cotton tensor, symmetric and traceless operator of the order 2s-2t+1 $C_s = \epsilon^{\cdots} \partial^{2s-2t+1} \phi_s$
- Higher order equations are obtained by solving auxiliary fields in terms of $\phi_{a_1 \ldots a_s}$ one by one

Conformal higher spin gravity

Interacting Fradkin-Tseytlin fields

- If we want to express our formulation of conformal higher spin gravities in terms of the Fradkin-Tseytlin fields we obtain more complicated description, from which one explicitly sees interactions

$$\delta\phi_{a_1\dots a_s} = \partial_{a_1}\dots\partial_{a_t}\xi_{a_{t+1}\dots a_s} + \text{permutations} - \text{traces}$$

- linearised gauge symmetries
- fields of weight t+1-s

$$\partial^{b_1}\dots\partial^{b_t}J_{b_1\dots b_t a_{t+1}\dots a_s} = 0$$

(partially)-conserved tensors

$$S = \int d^3x\, \epsilon^{\cdots}\left(\sum_{s,t}\phi_{s,t}\,\partial^{2s-2t+1}\phi_{s,t} + \sum_{s_i,t_i}\partial^{N_{s_i,t_i}}\phi_{s_1,t_1}\phi_{s_2,t_2}\phi_{s_3,t_3} + \dots\right)$$

Action in terms of Fradkin-Tseytlin fields

number of derivatives

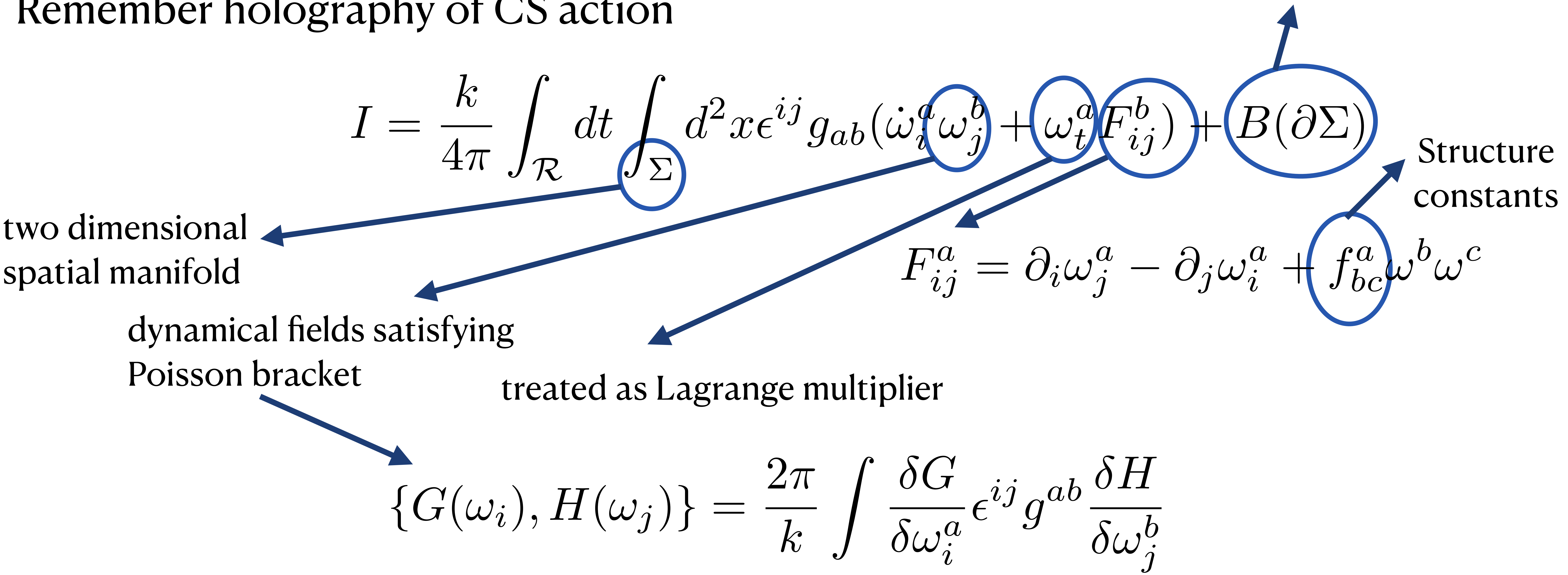
Conformal higher spin gravity

Interacting Fradkin-Tseytlin fields

- The important feature of conformal higher spin theories is that the number of derivatives in a vertex is fixed for any given s_i, t_i.
- The equations of motion should give the higher spin Cotton tensor (certain multiplet of Cotton tensors, as required by a given higher spin algebra).

Holography

- Remember holography of CS action



Boundary term. Depends on the boundary conditions and needs to be included to ensure that the action is gauge invariant

$$I = \frac{k}{4\pi}\int_{\mathcal{R}} dt \int_{\Sigma} d^2x\, \epsilon^{ij} g_{ab}(\dot{\omega}_i^a \omega_j^b + \omega_t^a F_{ij}^b) + B(\partial\Sigma)$$

Structure constants

two dimensional spatial manifold

$$F_{ij}^a = \partial_i \omega_j^a - \partial_j \omega_i^a + f_{bc}^a \omega^b \omega^c$$

dynamical fields satisfying Poisson bracket

treated as Lagrange multiplier

$$\{G(\omega_i), H(\omega_j)\} = \frac{2\pi}{k}\int \frac{\delta G}{\delta \omega_i^a}\epsilon^{ij} g^{ab}\frac{\delta H}{\delta \omega_j^b}$$

- variation w.r.t. ω_t^a gives a constraint G_a which satisfies Poisson bracket algebra
- Therefore G_a are the generators of gauge transformations acting on phase space

Holography

- One can write the generators as a smeared generators with added boundary term J which makes it differentiable

$$G(\eta^a) = \int_\Sigma \eta^a G_a + J(\eta)$$

- The parameters of the gauge transformation that we will observe depend on the field $\eta^a = -\xi^i \omega_i^a$

- Variation of the generator gives bulk and boundary term. To have well defined generator one needs to demand charge to be given by

$$\delta J = \frac{k}{2\pi} \int \xi^i \omega_i \delta \omega_j dx^j \qquad (\star\star\star)$$

- The algebra of global charges that we obtain, (after fixing the boundary conditions in standard way $\delta \omega_r^a = 0$)

$$\{J(\xi^i), J(\zeta^j)\}^* = \boxed{J([\xi, \zeta]^i)} + \frac{k\alpha^2}{2\pi} \int \xi^r \partial_\phi \zeta^r d\phi$$

Charge J evaluated on the parameter equal to the Lie bracket of the deformation vectors ξ^i and ζ^i

Boundary conditions

partially fix to radial gauge

$\omega = b^{-1}[3 + \Omega_{CG}(t,\phi)]b$

General boundary conditions

Fields which we call charges, allowed to vary

- Conformal gravity

diffeo charges LT charges SCT charges

$$\Omega_{\rm CG} = e^a(t,\varphi)P_a + \omega^a_b(t,\varphi)L^b_a + \sigma^a(t,\varphi)K_a + \phi(t,\varphi)D$$

Weyl charge

$$\delta\Omega_{\rm CG} = \delta e^a(t,\varphi)P_a + \delta\omega^a_b(t,\varphi)L^b_a + \delta\sigma^a(t,\varphi)K_a + \delta\phi(t,\varphi)D$$

$$\xi_{\rm CG} = b^{-1}[\xi^a_1(t,\varphi)P_a + \xi^a_{2b}(t,\varphi)L^b_a + \xi^a_3(t,\varphi)K_a + \xi_4(t,\varphi)D]b$$

- any transformation $\delta_\xi\omega = d\omega + [\omega,\xi] = \mathcal{O}(\delta\omega)$ with $\xi = \xi_{CG}$ conserves the bc's
- ➡ we can determine the infinitesimal changes of the state dependent functions under transformations that preserve the bc's, which we need to evaluate the charge

Boundary conditions

General boundary conditions

- Using the expression $(\star\star\star)$ we can write the canonical boundary charge

$$Q_{\mathrm{CG}} = \frac{k}{2\pi}\oint d\varphi\left(2\xi_1^a\sigma_a - 2\xi_3^a e_a + 4\xi_2^{ab}\omega_{ab} + \xi_4\phi\right)$$

Invariant bilinear form of the representation

- expand fields and parameters in terms of the Fourier modes: ⇒

$$i\{T^n_{AB}, T^m_{CD}\} = \eta_{BC}T^{n+m}_{AD} - \eta_{AC}T^{n+m}_{BD} - \eta_{BD}T^{n+m}_{AC} + \eta_{AD}T^{n+m}_{BC} - knTr(T_{AB}T_{CD})\delta_{n+m,0}$$

- ⇒ asymptotic symmetry algebra is a loop algebra of underlying gauge algebra

Boundary conditions

General boundary conditions

- Analogously to conformal gravity example we consider conformal gravity with a vector field

$$\Omega = \Omega_{\text{CG}} + \Omega_s \qquad \Omega_s = \omega^+(t,\varphi)t_+ + \omega^-(t,\varphi)t_- + \omega^c(t,\varphi)t_c$$

$$\xi = \xi_{\text{CG}} + \xi_s \qquad \xi_s = b^{-1}[\xi^+(t,\varphi)t_+ + \xi^-(t,\varphi)t_- + \xi_5^c(t,\varphi)t_c]b$$

$$Q_s = \frac{k}{2\pi}\oint d\varphi(\xi^+\omega^- + \xi^-\omega^+ + \xi_5^a\omega^b)$$

→ Expand in the Fourier modes

→

$$i\{T^m_{AB}, U^n_C\} = \eta_{BC}U^{m+n}_A - \eta_{AC}U^{m+n}_B$$

$$i\{U^m_A, U_C\}^n = T^{m+n}_{AC} - knTr[U_A U_C]\delta_{m+n,0}$$

← no central extension

- asymptotic symmetry algebra is a loop algebra of underlying gauge algebra

Boundary conditions

General boundary conditions

Decomposition in terms of the generators irreducible rank-two representation of so(3,2 as we have seen above

- Conformal gravity with a spin three field

$$\Omega = \Omega_{\text{CG}} + \Omega_w$$

$$\Omega_w = w^{a+}(t,\varphi)t_{a+} + \frac{1}{2}w^{ab}(t,\varphi)t_{ab} + \frac{1}{2}w^{++}(t,\varphi)t_{++} + \frac{1}{2}w^{--}(t,\varphi)t_{--} + w^{a-}(t,\varphi)t_{a-}$$

$$\xi_w = b^{-1}[\xi^{a+}(t,\varphi)t_{a+} + \frac{1}{2}\xi^{ab}(t,\varphi)t_{ab} + + \frac{1}{2}\xi^{++}(t,\varphi)t_{++} + \frac{1}{2}\xi^{--}(t,\varphi)t_{--} + \xi^{a-}(t,\varphi)t_{a-}]b$$

- analogous procedure as above:

$$i\{T^m_{AB}, S^n_{CD}\} = \eta_{BC}S^{m+n}_{AD} - \eta_{AC}S^{m+n}_{BD} + \eta_{BD}S^{m+n}_{AC} - \eta_{AD}S^{m+n}_{BC} - knTr(T_{AB}S_{CD})\delta_{m+n,0}$$

$$i\{S^m_{AB}, S^n_{CD}\} = \eta_{BC}T^{m+n}_{AD} + \eta_{AC}T^{m+n}_{BD} + \eta_{BD}T^{m+n}_{AC} + \eta_{AD}T^{m+n}_{BC} - knTr(S_{AB}S_{CD})\delta_{m+n,0}$$

- asymptotic symmetry algebra is a loop algebra of underlying gauge algebra

Boundary conditions

Near horizon inspired boundary conditions

- First we have write the matrices that satisfy the algebra of the examples we consider: → PKLD basis
- Find a map that transforms these matrices to the W-L basis.
- For the W-L basis we know how to impose near horizon boundary conditions
- Technically the boundary conditions require diagonal matrices so we could have imposed similar bcs for matrices that were diagonal in PKLD basis
- In our consideration we didn’t focus on a particular embedding

Boundary conditions

Near horizon inspired boundary conditions

Diagonal generators of sl4 in WL representation

fixed at the boundary

- Conformal gravity with vector field $\Longleftrightarrow$ sl_4

$$\Omega = (\mathcal{J} d\varphi + \zeta dt) L_0 + (\mathcal{J}_{(3)} d\varphi + \zeta_{(3)} dt) V_0^{(3)} + (\mathcal{J}_{(4)} d\varphi + \zeta_{(4)} dt) V_0^{(4)} \qquad (\star\star)$$

dynamical fields

$$\epsilon = \eta L_0 + \eta_{(3)} V_0^{(3)} + \eta_{(4)} V_0^{(4)}$$

Lagrange multipliers, chemical potentials

$$\mathcal{Q}[\eta, \eta_{(3)}, \eta_{(4)}] = -\frac{k}{4\pi} \int d\varphi \left(\eta(\varphi) \mathcal{J}(\varphi) + \alpha_3 \eta_{(3)}(\varphi) \mathcal{J}_{(3)}(\varphi) + \alpha_4 \eta_{(4)}(\varphi) \mathcal{J}_{(4)}(\varphi) \right)$$

$$i\{J_n, J_m\} = \frac{k}{2} n \delta_{m+n,0} \qquad i\{J_n^{(3)}, J_m^{(3)}\} = \frac{\alpha_3 k}{2} n \delta_{m+n,0} \delta_{3,3}$$

$$i\{J_n^{(4)}, J_m^{(4)}\} = \frac{\alpha_4 k}{2} n \delta_{m+n,0} \delta_{4,4} \qquad i\{J_n^{(3)}, J_m^{(4)}\} = 0$$

- asymptotic symmetry algebra corresponds to u(1) currents

Boundary conditions

- one can translate this boundary conditions to the metric formulation, since solutions there are usually more intuitive
- We have to calculate ω using $\omega = b^{-1}[d + \Omega]b$, so we need to choose b

defines the metric

$$b = e^{Y} \qquad Y = a(\rho)(P_y + P_t - D) \quad \text{and} \quad (\star\star) \Rightarrow e_{\mu a} \Rightarrow g_{\mu\nu}$$

$$ds^2 = \frac{7}{256r^2}\left(r^2 + \frac{1}{r^2} - 2\right)dr^2 + \frac{(5\mathcal{K} - 3\mathcal{K}_4)\,(r-1)^2(r+1)}{20r^3}drd\phi + \frac{(5\xi - 3\xi_4)\,(r-1)^2(r+1)}{20r^3}drdt$$
$$+ 16\mathcal{K}_3^2\left(\frac{1}{r} - 1\right)^2 d\phi^2 + 32\mathcal{K}_3\xi_3\left(\frac{1}{r} - 1\right)^2 d\phi dt + 16\xi_3^2\left(\frac{1}{r} - 1\right)^2 dt^2$$

- conformally flat metric which satisfies Cotton tensor, it has curvature invariants that are zero

Boundary conditions

Near horizon inspired boundary conditions

- Conformal gravity

$$\Omega = \boxed{(\mathcal{K}(\varphi, t)} d\varphi + \xi(\varphi, t)dt)D + (\boxed{\mathcal{K}_d(\varphi, t)} + \xi_d(\varphi, t)dt)L_d$$

dynamical fields

$$\epsilon = \mu D + \mu_d L_d$$

$$Q[\mu, \mu_d] = \frac{k}{2\pi} \oint d\varphi (2\mu(\varphi)\mathcal{K}(\varphi) + \frac{3}{2}\mu_d(\varphi)\mathcal{K}_d(\varphi))$$

$$i\{K_n, K_m\} = kn\delta_{m+n,0} \qquad i\{K_n^{(d)}, K_m^{(d)}\} = \frac{3k}{4}n\delta_{m+n,0}$$

- asymptotic symmetry algebra corresponds to u(1) currents of k and $\frac{3k}{4}$

Boundary conditions

- To go to metric formulation, we use Y as above

$$Y = \rho(P_y + P_t - D)$$

- leads to a metric which after multiplication with right conformal factor can lead to the flat metric, as well as to metric with Ricci scalar equal to -6.
- Possible form to which metric can be brought to is

$$ds^2 = -\frac{dtdy\left(K\xi + 3K_3\xi_3\right)}{2\xi y^2} + \frac{K_3 dy^2\left(K_3\xi^2 - 2K\xi_3\xi - 3K_3\xi_3^2\right)}{\xi^2 y^2} - \frac{\xi dtd\phi}{2\xi_3 y^2} - \frac{3dt^2}{16y^2} + \frac{d\phi^2}{y^2}$$

- The metric can be further transformed to familiar form of the extremal black hole

Boundary conditions

Near horizon inspired boundary conditions

- Conformal gravity with spin-3 field ⟷ sl_5

$$\Omega = (\mathcal{J} d\varphi + \zeta dt)L_0 + (\mathcal{J}_{(3)} d\varphi + \zeta_{(3)} dt)V_0^{(3)} + (\mathcal{J}_{(4)} d\varphi + \zeta_{(4)} dt)V_0^{(4)}$$
$$+ (\mathcal{J}_{(5)} d\varphi + \zeta_{(5)} dt)V_0^{(5)}$$

dynamical fields

$$\epsilon = \eta L_0 + \eta_{(3)} V_0^{(3)} + \eta_{(4)} V_0^{(4)} + \eta_{(5)} V_0^{(5)}$$

$$\mathcal{Q}[\eta, \eta_{(3)}, \eta_{(4)}, \eta_{(5)}] = -\frac{k}{4\pi}\int d\varphi(\eta(\varphi)\mathcal{J}(\varphi) + \beta_3\eta_{(3)}(\varphi)\mathcal{J}_{(3)}(\varphi) + \beta_4\eta_{(4)}(\varphi)\mathcal{J}_{(4)}(\varphi)$$
$$+ \beta_5\eta_{(5)}(\varphi)\mathcal{J}_{(5)}(\varphi))$$

- asymptotic symmetry algebra corresponds to u(1) currents of $\frac{1}{2}k$ and $\frac{1}{2}k\beta_i$, i=3,4,5

Boundary conditions

- In the metric formulation, the diagonal generators are those of time translations and s_{yy}, $\quad s_{yy} \equiv SL_{yy}$ (remember the symmetric field from the beginning)

$$Y = a(\rho)(P_t - SL_{yy})$$

$$ds^2 = d_1(\rho)dr^2 + d_2(\rho, \mathcal{K}, \mathcal{K}_3, \mathcal{K}_5)d\phi^2 + 2d_3(\rho, \mathcal{K}, \mathcal{K}_3, \mathcal{K}_5, \xi, \xi_3, \xi_5)d\phi dt + d_2(\rho, \xi, \xi_3, \xi_5)dt^2$$

- $\rho \to \infty$: Ricci scalar of the metric and first several curvature invariants vanish, as well as Einstein tensor
- One could analogously apply same boundary conditions to other theories from new conformal higher spin gravities and consider asymptotic solutions

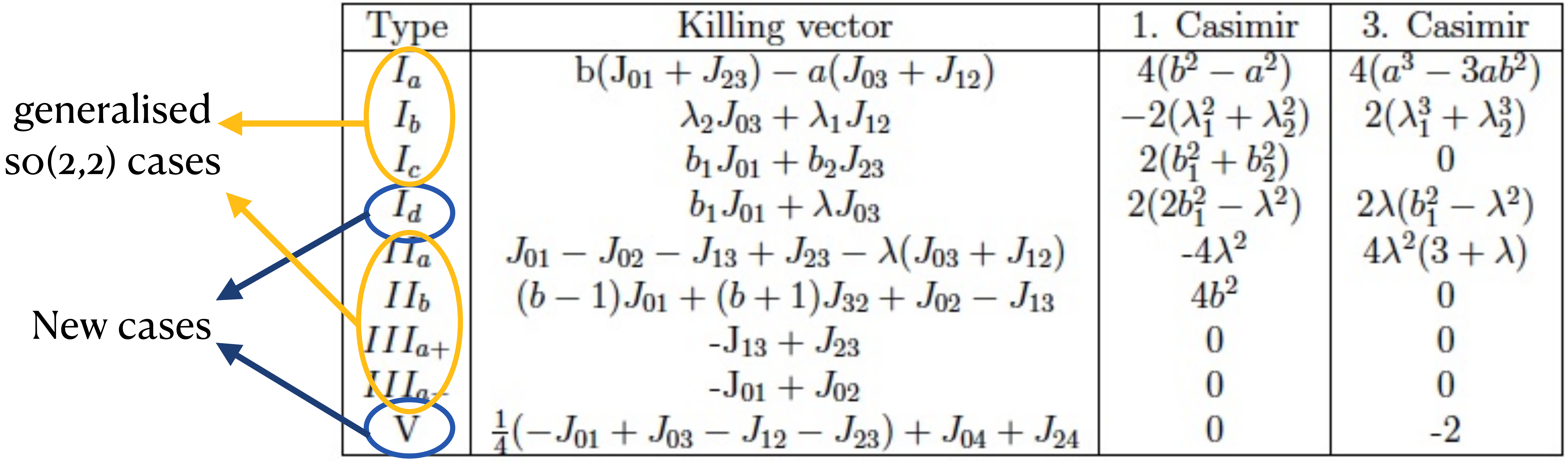
Conclusion

- We have presented an infinite class of models for conformal higher spin gravities that extends that of Pope and Townsend
- There are infinitely many examples of the new theories with a finite number of higher spin fields
- The new theories contain finite number of Fradkin-Tseytlin fields with higher derivatives in the gauge transformations
- The holography of these theories admits near horizon inspired boundary conditions, which can be generalised to all the models of new theories
- Imposing most general boundary conditions leads to loop algebra of the underlying gauge algebra as expected, that is as well true for all the examples of the new theories

Thank you!

One parameter subgroups of so(3,2)

- Rotating Einstein gravity solutions in 3d, of the group so(2,2) have been classified by Banados, Henneaux, Teitelboim, Zanelli
- Classification of the solutions of conformal gravity in 3d



generalised so(2,2) cases

New cases

Type	Killing vector	1. Casimir	3. Casimir
I_a	$b(J_{01} + J_{23}) - a(J_{03} + J_{12})$	$4(b^2 - a^2)$	$4(a^3 - 3ab^2)$
I_b	$\lambda_2 J_{03} + \lambda_1 J_{12}$	$-2(\lambda_1^2 + \lambda_2^2)$	$2(\lambda_1^3 + \lambda_2^3)$
I_c	$b_1 J_{01} + b_2 J_{23}$	$2(b_1^2 + b_2^2)$	0
I_d	$b_1 J_{01} + \lambda J_{03}$	$2(2b_1^2 - \lambda^2)$	$2\lambda(b_1^2 - \lambda^2)$
II_a	$J_{01} - J_{02} - J_{13} + J_{23} - \lambda(J_{03} + J_{12})$	$-4\lambda^2$	$4\lambda^2(3 + \lambda)$
II_b	$(b-1)J_{01} + (b+1)J_{32} + J_{02} - J_{13}$	$4b^2$	0
III_{a+}	$-J_{13} + J_{23}$	0	0
III_{a-}	$-J_{01} + J_{02}$	0	0
V	$\frac{1}{4}(-J_{01} + J_{03} - J_{12} - J_{23}) + J_{04} + J_{24}$	0	-2

Classification of holographic solutions

General rotating solution of conformal gravity

$$ds^2 = \frac{4r^2 dr^2}{a^2 - 2ar(er - 2c) + 4r^2\left(br^2 + c^2 - cer\right)} + \frac{2c_1(a + 2r(-br + c + r))dT d\phi}{a} + r^2 d\phi^2$$
$$+ \frac{2c_1^2\left(a(-2b + e + 2) + 2r\left(\left(b^2 - 3b + 1\right)r + c(-2b + e + 2)\right)\right)dT^2}{a^2}$$

- Setting asymptotically AdS bcs in the metric formulation [Afshar,Grumiller,...] fixes coefficients and determines mass and angular momentum
- There is third coefficient that stays unfixed under this condition
- The solutions we obtained can be related to this one by fixing coefficients, coordinate transformations or Weyl transformations

Conformal gravity and conformal higher spin theories

Higher spin algebras

- The spectrum is given by

$$\mathfrak{hs}\Big|_{so(3,2)} = \bigoplus_{s=0}^{\infty} \bigoplus_{k=0}^{k=q-1} \begin{array}{|c|}\hline \qquad s-1 \qquad \\ \hline s-2k-1 \\ \hline \end{array}$$

- The invariant trace is also known from the literature, so one can write down the Chern-Simons action [Mkrtchyan, Joung]