Quantum Phase Transition and Naturalness in Generalized Effective Action

ESI Workshop on Large-N Matrix Models and Emergent Geometry

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Extensive and intensive parameters
and
(Multi-)critical point

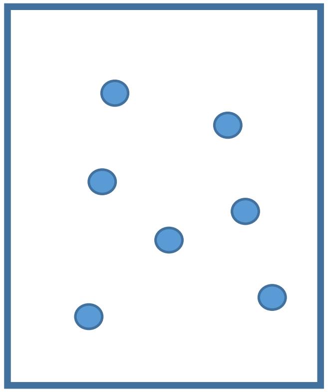
extensive and intensive variables

extensive variable (parameters) $\mathcal{O} \propto V^1$

intensive variable $\mathcal{O} \propto V^0$

V: volume

Example molecules in a box



extensive var.

V: volume

N: # of molecules

E: total energy

S: total entropy

intensive var.

p: pressure

μ: chemical pot.

T: temperature

$n = \frac{N}{V}$: number density

$\varepsilon = \frac{E}{V}$: energygy density

extensive parameters as fundamental parameters

Some of the variables are averaged quantities that appear only in a large system limit $V \to \infty$.

(ex.) $S,\ p,\ \mu,\ T$

From microscopic point of view, extensive parameters are more fundamental:

microcanonical ensemble $\rho \propto \delta(\hat{H} - E)\,\delta(\hat{N} - N) \cdots$

On the other hand, intensive parameters are easy to handle although not fundamental:

canonical ensemble $\rho \propto \mathrm{e}^{-\frac{\hat{H}}{T}}\,\mathrm{e}^{\frac{\mu}{T}\hat{N}} \cdots$

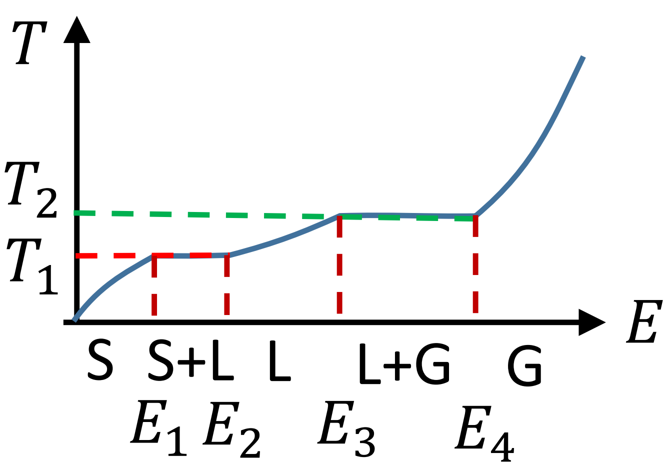
Tuning of intensive parameters without fine tuning

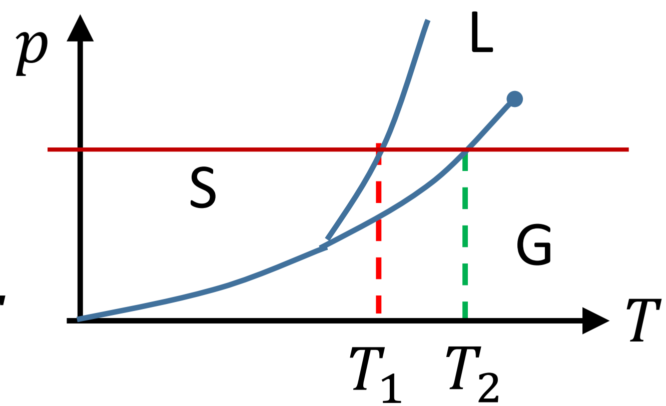
Suppose: Extensive parameters are controlled.

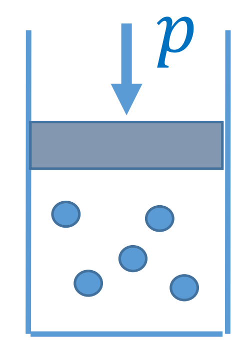
$$\Downarrow$$

Corresponding intensive parameters are on a (multi-) critical point of first order phase transition with a non-zero probability.

Example molecules in a box with constant pressure







$p, \ N$: fixed

The mapping $E \mapsto T$ is not $1:1$.

In order to reach one of the critical points,

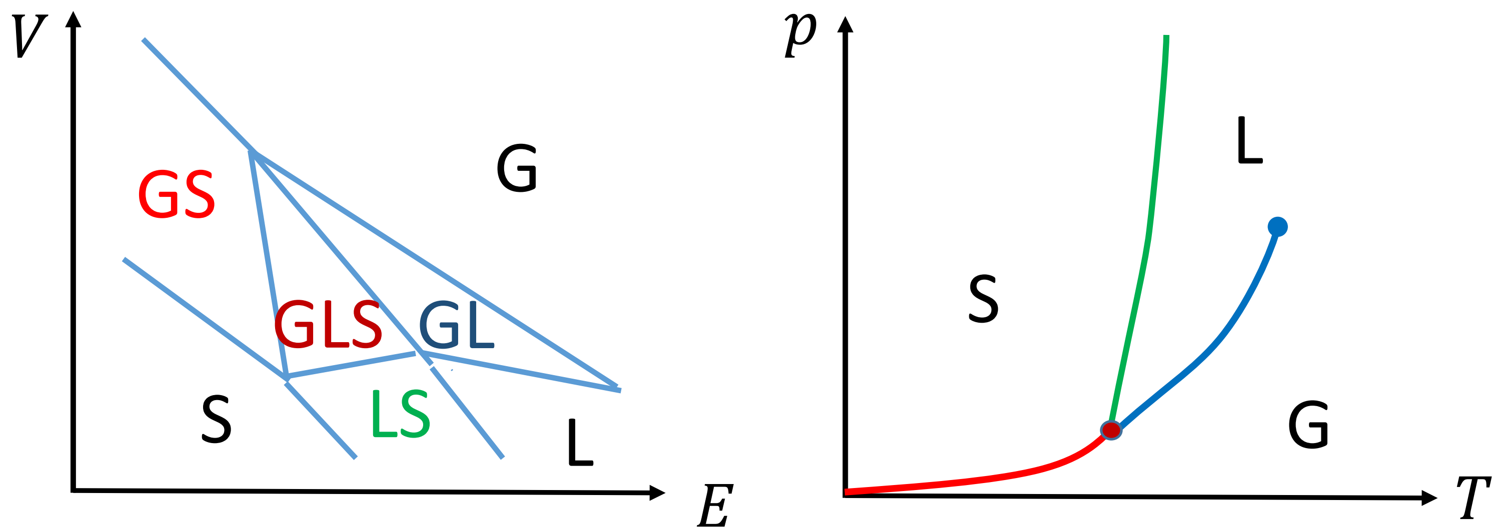
if we control T, we need one parameter fine tuning such that $T = T_1 \text{ or } T_2$.

If we control E, we do not need fine tuning but tuning to finite domain such that $E_1 < E < E_2$ or $E_3 < E < E_4$.

To reach one of the first-order phase transition points, fine tuning is not necessary if the extensive parameters are controlled.

Example 2 Two control parameters V, E

molecules in a box with fixed N



In order to obtain the coexisting phases,
need fine tunings in terms of intensive parameters,
do not need them in terms of extensive parameters.

Even the triple point can be realized with a finite probability
if the extensive parameters are controlled.

In statistical mechanics, there are two ways to control the system for each parameter.

extensive parameter ↔ micro-canonical ensemble

intensive parameter ↔ canonical ensemble

If the intensive parameters are controlled, we need to tune the parameters to realize a (multi-)critical point of first-order phase transition.

In other words, if the intensive parameters are probabilistically distributed uniformly, the probability of a critical point being realized is zero.

On the other hand, if the extensive parameters are probabilistically distributed uniformly, the probability of a critical point being realized is finite.

Multicritical Point Principle (MPP)

"Multi-critical point is natural."

Generalized QFT
and
Multicritical Point Principle

Analogous situation in QFT (or QM)? Nambu, ...

Ordinary QFT (canonical QFT)

$$\langle t_2, q_2 | t_1, q_1 \rangle = \int_{\substack{q(t_2)=q_2 \\ q(t_1)=q_1}} \mathcal{D}q \; e^{\frac{i}{\hbar}S[q]} \sim \sum_n e^{-\frac{E_n}{T}}$$

↓ tempting

microcanonical QFT $\displaystyle \int \mathcal{D}q \, \delta(S[q]-A) \sim \sum_n \delta(E_n - E)$

Generalized QFT $\displaystyle \int \mathcal{D}q \, f(S[q]-A)$

We will see that under some circumstances they are equivalent in the large volume limit.

Furthermore, the naturalness problem is absent in the microcanonical or generalized QFT.

In fact, we can show

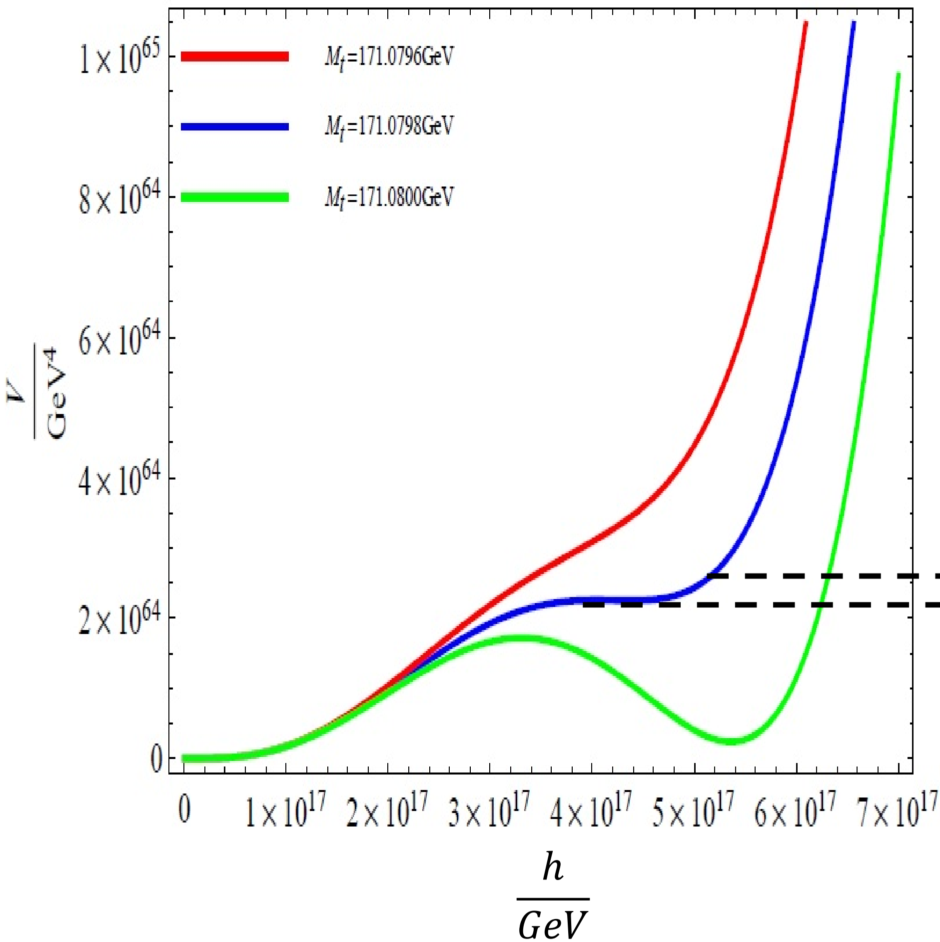
MPP of Bennett Froggatt and Nielsen:
“ Coupling constants are fixed such that the vacuum is at a (multi-) critical point.”

$\Downarrow$ Consider the time evolution of universe.

Generalized MPP
“ Coupling constants are fixed to such values that significantly change the history of universe when they are changed.”

Hamada, Kawana, HK ‘14, ‘15

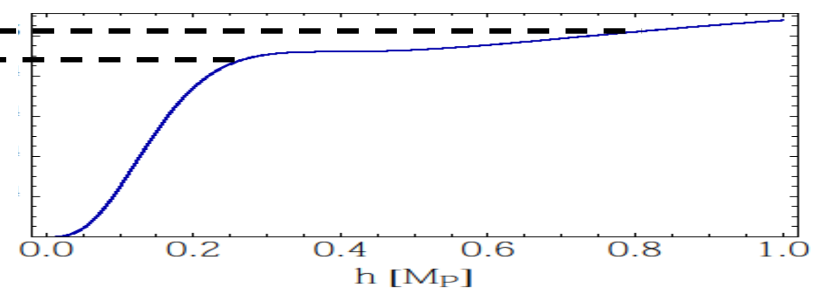
SM Higgs is close to MPP.



non-renormalizable coupling $\xi R h^2$ with $\xi \sim 10$.

In the Einstein frame the effective potential becomes

$$V(\varphi_h), \quad \varphi_h = \frac{h}{\sqrt{1+\xi h^2/M_P{}^2}}.$$



Hamada, Oda, Park and HK '14
Bezrukov,Shaposhnikov

Motivation
of
Generalized QFT

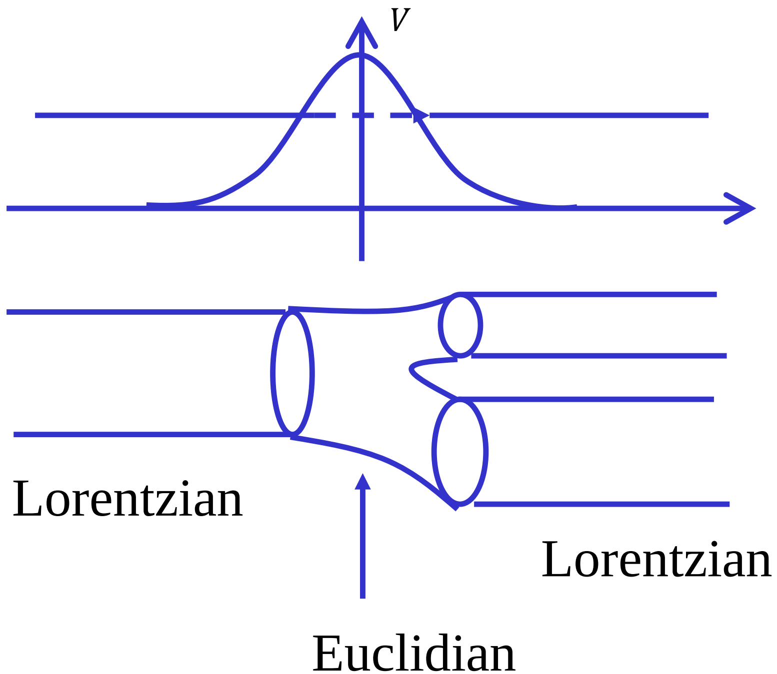
multiverse and baby universes in MM

Universe arises from matrices. Steinacker, Nishimura, and all.
By considering block diagonal configuration, multiverse appears naturally.

In terms of the WKB approximation of GR, topology change can be described by connecting Lorentzian and Euclidian signature.

QM

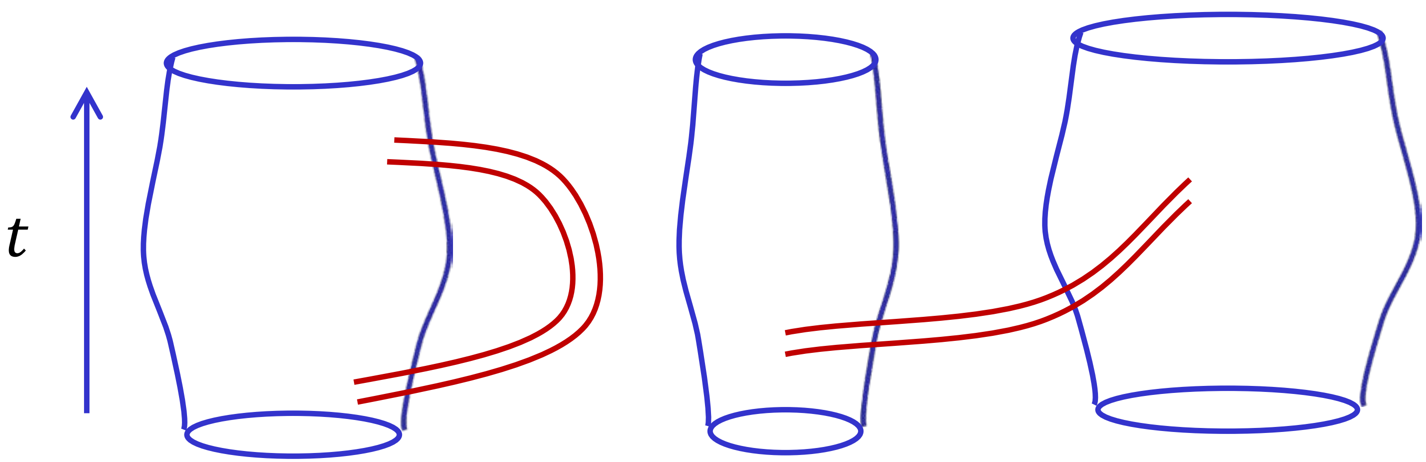
Gravity



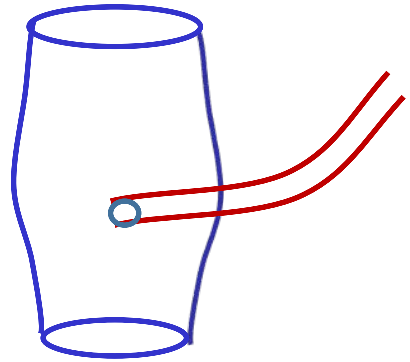
For macroscopic universes topology change is highly suppressed, because the transition probability is proportional to exp(−classical Euclidean action).

On the other hand, if one of the universes is small (baby universe), topology change becomes important.

What we should consider is emission and absorption of BU's by multiverse.

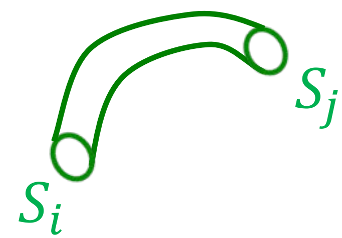


For the large universe, emission or absorption of BU looks like an insertion of a local operator.



Therefore, the emission and subsequent absorption of a BU modify the effective action as

$$S \to S + \sum_{i,j} c_{ij} S_i S_j \ ,$$



where S_i is a space-time integral

$$S_i = \int d^4x \sqrt{-g(x)}\, O_i(x)$$

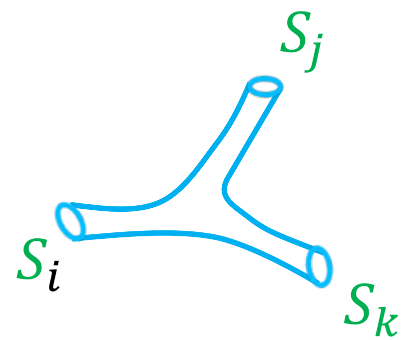
of a scalar operator O_i such as

$$1\ , R\ , R_{\mu\nu}R^{\mu\nu}\ , F_{\mu\nu}\ F^{\mu\nu}, \overline{\psi}\gamma^{\mu} D_{\mu}\psi\ , \cdots .$$

Each S_i has a form of local action.

Furthermore, bifurcated BU's contribute to the effective action as

$$S \to S + \sum_{i,j,k} c_{ijk} S_i S_j\, S_k \ .$$



Thus the low energy effective theory of QG / MM is not a simple local action but a generalized QFT:

$$S_{\mathrm{eff}} = f\,(S_1, S_2, \cdots).$$

Coleman '89
Tsuchiya-Asano-K

Other possibilities of generalized QFT

(1) microcanonical QFT

ordinary matrix model Ishibashi-K-Kitazawa-Tsuchiya

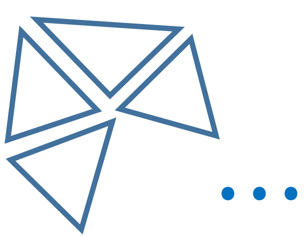
$$Z = \int dA d\psi \ e^{\, iS_{IIB}[A,\psi]}$$

microcanonical matrix model Jevicki-Yoneya

$$Z = \int dA d\psi \ \delta(S_{IIB}[A,\psi] - 1)$$

A priori, we don't know which is more fundamental.

(2) M.C. simulation of dynamical triangulation of QG



# of D-simplexes $= N_D$: fixed $\leftrightarrow \delta(\int d^D x \sqrt{g} - N_D)$

discretized version of Microcanonical C.C.

Equivalence of canonical and microcanonical ensembles

Equivalence of canonical and microcanonical ensembles

Two ensembles are equivalent in the $V \to \infty$ limit.

$e^{S(E,V)} := \sum_n \delta(E_n - E)$ S: entropy from micr. ens.

basic assumption $S(E,V) \sim V\, s\left(\frac{E}{V}\right)$ for large V

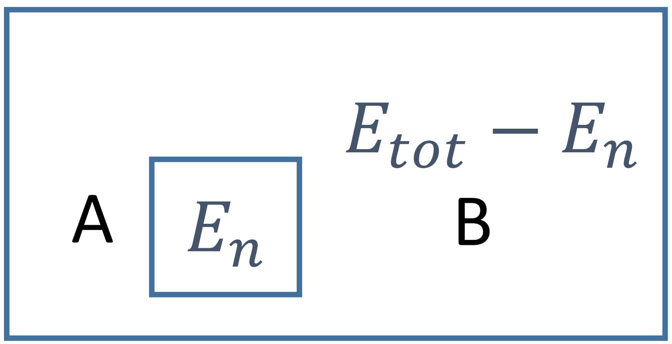
$e^{-\frac{F}{T}} := \sum_n e^{-\frac{E_n}{T}}$ F:free energy from can. ens.

$$
\begin{aligned}
&= \sum_n \int dE\, \delta\,(E_n - E)\, e^{-\frac{E}{T}} \\
&= \int dE\;\; e^{-\frac{E}{T}+S(E,V)} \\
&= V \int d\varepsilon\;\; e^{V\left(-\frac{\varepsilon}{T}+s(\varepsilon)\right)} \quad \leftarrow\ \varepsilon = \frac{E}{V} \\
&\backsimeq\ e^{-\frac{E}{T}+S(E,V)} \quad \leftarrow \frac{\partial S}{\partial E} = \frac{1}{T}\ \text{ for } V \to \infty
\end{aligned}
$$

Saddle point dominates.

Another view of equivalence

"A small subsystem of a large system is described by canonical ensemble."



"total system" very large DOF

E_{tot}: total energy, fixed

micricanonical ensemble

$$\rho \propto \delta(H - E_{tot})$$

A : subsystem, small compared with the total system

B:= (total system) – A >> A (DOF)

basic assumption $E_{tot} = E_A + E_B$ ← energy conservation

P_n: probability that A takes a microstate n with energy E_n

$P_n \propto$ # of the microstates of B with energy $E_{tot} - E_n$

$\propto e^{S_B(E_{tot}-E_n)}$

$$S_B(E_{tot} - E_n) = S_B(E_{tot}) - \frac{\partial S_B}{\partial E}(E_{tot})E_n + \frac{1}{2}\frac{\partial^2 S_B}{\partial E^2}(E_{tot})E_n^2 + \cdots$$

$\uparrow$ $O(V^1)$ $\uparrow$ $O(V^0)$ $\uparrow$ $O(V^{-1})$

As a function of E_n , we have

$$\frac{1}{T} = \frac{\partial S_E}{\partial E}(E_{tot})$$

$$S_B(E_{tot} - E_n) = const. - \frac{E_n}{T}\left(1 + O\left(\frac{E_n}{V}\right)\right).$$

Thus we have $P_n = \mathrm{const.}\, e^{-\frac{E_n}{T}}$ in the large V limit.

"The canonical ensemble is an effective theory of small subsystems of a large system."

The FT counterpart:

ordinary path integral (Schrödinger equation) $\Leftarrow$ underlying large system (Boltzmann entropy)

Are generalized QFT equivalent to the ordinary QFT?

Ordinary QFT (canonical QFT)

$$\langle t_2, q_2 | t_1, q_1 \rangle = \int_{\substack{q(t_2) = q_2 \\ q(t_1) = q_1}} \mathcal{D}q \;\; e^{\frac{i}{\hbar} S[q]} \quad \sim \sum_n e^{-\frac{E_n}{T}}$$

microcanonical QFT $\displaystyle \int \mathcal{D}q \, \delta(S[q] - A) \sim \sum_n \delta(E_n - E)$

Generalized QFT $\displaystyle \int \mathcal{D}q \, f(S_i[q] - A_i)$

$$S_i = \int d^4x \sqrt{-g(x)} \, O_i(x)$$

$$O_i = 1\,, R\,, R_{\mu\nu} R^{\mu\nu}\,, F_{\mu\nu} F^{\mu\nu}, \overline{\psi} \gamma^\mu D_\mu \psi\,, \cdots .$$

Under what circumstances are they equivalent?

microcanonical QFT

$$\int \mathcal{D}q \, \delta(S[q] - A)$$

$$= \int \mathcal{D}q \; \int d\alpha \, e^{i\alpha \, (S[q] - A\,)}$$

$$= \int d\alpha \, e^{-i\alpha A} \; \int \mathcal{D}q \, e^{i\alpha \, S[q]}$$

Generalized QFT

$$\int \mathcal{D}q \, f(S_i[q] - A_i)$$

$$= \int \mathcal{D}q \; \int \prod_i d\alpha_i \, \tilde{f}(\alpha) \, e^{i \sum_i \, \alpha_i \, (S_i[q] - A_i\,)}$$

$$= \; \int \prod_i d\alpha_i \, w(\alpha) \boxed{\int \mathcal{D}q \, e^{i \sum_i \, \alpha_i \, S_i[q]}}$$

$$w(\alpha) = \tilde{f}(\alpha) \, e^{-\, i \, \sum_i \alpha_i A_i}$$

Ordinary FT with coupling constants α_i .

Generalized QFT = superposition of ordinary QFT

$$Z = \int \prod_i d\alpha_i \, w(\alpha) Z(\alpha)$$

$$Z(\alpha) = \int \mathcal{D}q \, e^{i \sum_i \alpha_i S_i[q]} \qquad \leftarrow S_i = \int d^4x \sqrt{-g(x)} \, O_i(x)$$

$$w(\alpha) = \tilde{f}(\alpha) \, e^{-i \sum_i \alpha_i A_i}$$

(1) Does one point in the α space, $\alpha_i = \alpha_i^{(0)}$, dominate the integral? Then the theory is equivalent to the ordinary QFT with coupling constants $\alpha_i^{(0)}$.

(2) If it is the case, are $\alpha_i^{(0)}$ good values so as to solve the naturalness problem?

A model – Microcanonical mass term

Scalar field with microcanonical mass term

Instead of the ordinary path integral for massive free scalar field

$$Z(m^2) = \int d\phi\, e^{i\int d^4x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \underline{\frac{m^2}{2}\phi^2}\right)},$$

canonical mass term

m^2: intensive

we consider

$$\Omega(A) = \int d\phi\, e^{i\int d^4x\, \frac{1}{2}\partial_\mu\phi\partial^\mu\phi}\ \underline{\delta\left(\int d^4x\, \frac{1}{2}\phi^2 - A\right)}.$$

microcanonical mass term

A: extensive

We examine

- whether they are equivalent in the large volume limit,
- if so, the mass obtained is natural or not.

Evaluation of $\Omega(A)$

$$\Omega(A) = \int d\phi \, e^{i\int d^4x \frac{1}{2}\partial_\mu\phi\partial^\mu\phi} \, \delta\left(\int d^4x \, \frac{1}{2}\phi^2 - A\right)$$

$$= \int d\phi \, \int_{-\infty}^{\infty} dm^2 \, e^{i\int d^4x \frac{1}{2}\partial_\mu\phi\partial^\mu\phi} \, e^{-im^2\left(\int d^4x \frac{1}{2}\phi^2 - A\right)}$$

$$= \int_{-\infty}^{\infty} dm^2 \, e^{im^2 A} \, Z(m^2)$$

$$\log Z(m^2) = -\frac{1}{2} \operatorname{Tr}\log\left(-\partial_\mu\partial^\mu - m^2 + i\,0\right)$$

$$= -\frac{1}{2} V \int \frac{d^4p}{(2\pi)^4} \log(p^2 - m^2 + i\,0)$$

V: space-time volume

$p^0 = ip^4$

$$= -\frac{i}{2} V \int \frac{d^4p_E}{(2\pi)^4} \log(p_E^2 + m^2 - i\,0) \; + \text{const.}$$

$$\Omega(A) = \int_{-\infty}^{\infty} dm^2 \, e^{iVf(m^2)}$$

$$f(m^2) = m^2 a \; - \frac{1}{2}\int \frac{d^4p_E}{(2\pi)^4} \log(p_E^2 + m^2 - i\,0) \,, \; a = \frac{A}{V} \,.$$

large V ⇒ highly oscillating ⇒ stationary phase?

more complicated!

$$\Omega(A) = \int_{-\infty}^{\infty} dm^2\, e^{iVf(m^2)}$$

$$f(m^2) = m^2 a \; - \frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4} \log(p_E^2 + m^2 - i0)$$

$$= m^2 a \; - \frac{1}{16\pi^2}\int_0^{\Lambda} dp\, p^3 \log(p^2 + m^2 - i0)$$

$$= m^2 a \; - \frac{\Lambda^4}{32\pi^2} \int_0^1 dx\, x \left\{ \log\left(x + \frac{m^2}{\Lambda^2} - i0 \right) + \log \Lambda^2 \right\}$$

$$= \xi\, \Lambda^2 a - \frac{\Lambda^4}{32\pi^2} g(\xi) + (\text{const. depending only on } \Lambda)$$

$a = \frac{A}{V}$

Λ: UV cutoff

$\leftarrow x = \frac{p^2}{\Lambda^2}$

$$g(\xi) = \int_0^1 dx\, x \log(x + \xi - i0)\,, \;\; \xi = \frac{m^2}{\Lambda^2}.$$

$$\int_{-\infty}^{\infty} dm^2 \;\Rightarrow\; \int_{-\infty}^{\infty} d\xi$$

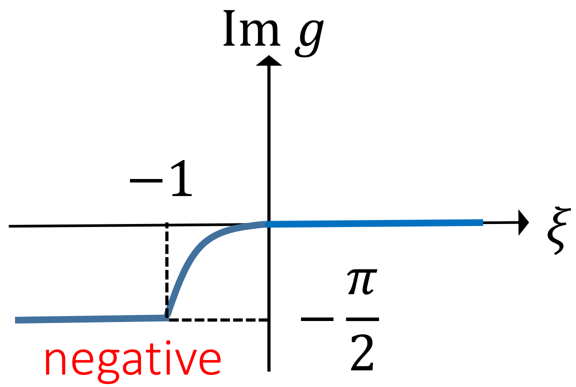
As a function of ξ, f is a linear combination of ξ and g.

We next examine the ξ dependence of g.

$$g(\xi) = \int_0^1 dx\, x \log(x + \xi - i0)$$

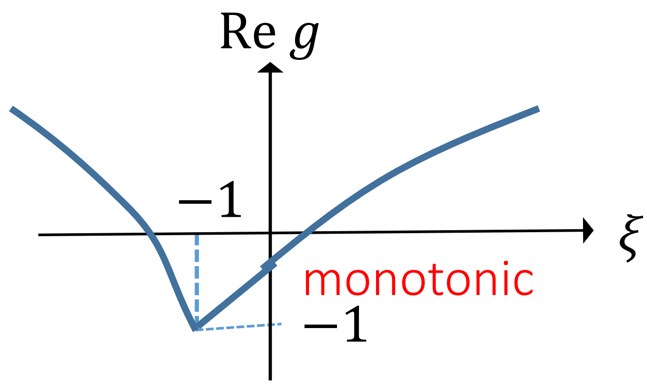
$g(\xi)$ has negative imaginary part for $\xi < 0$:

$$\mathrm{Im}\, g(\xi) = \begin{cases} 0 & (0 < \xi) \\ -\dfrac{\pi}{2}\xi^2 & (-1 < \xi < 0) \\ -\dfrac{\pi}{2} & (\xi < -1) \end{cases}$$



$g(\xi)$ is monotonic for $\xi > 0$:

$$\mathrm{Re}\, g(\xi) = \int_0^1 dx\, x \log|x + \xi|$$



Contribution from $m^2 < 0$ to $\Omega(A) = \int_{-\infty}^{\infty} dm^2\, e^{iVf(m^2)}$ is exponentially small in $V \to \infty$ limit:

$$\left|e^{iVf(m^2)}\right| = e^{\frac{V\Lambda^4}{32\pi^2} Im\, g(\xi)} = \begin{cases} 1 & (m^2 > 0) \\ e^{-\frac{Vm^4}{64\pi}} & (-\Lambda^2 < m^2 < 0) \\ e^{-\frac{V\Lambda^4}{64\pi}} & (m^2 < -\Lambda^2) \end{cases}$$

$$f = \xi\, \Lambda^2 a - \frac{\Lambda^4}{32\pi^2} g(\xi)$$

On the other hand, as we will see, contribution from $m^2 > 0$ is $O(V^{-\frac{1}{2}})$ or $O(V^{-1})$.

Therefore, it is enough to consider

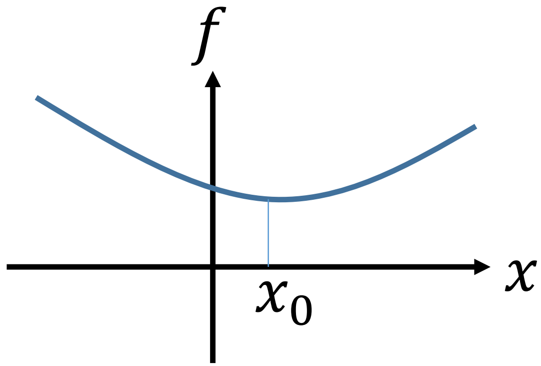
$$\Omega(A) = \int_0^{\infty} dm^2\, e^{iVf(m^2)}.$$

Formulas on $e^{i\,Vf(x)}$ for large V

$e^{iVf(x)}$ for large V

$f(x)$: real function

(1)

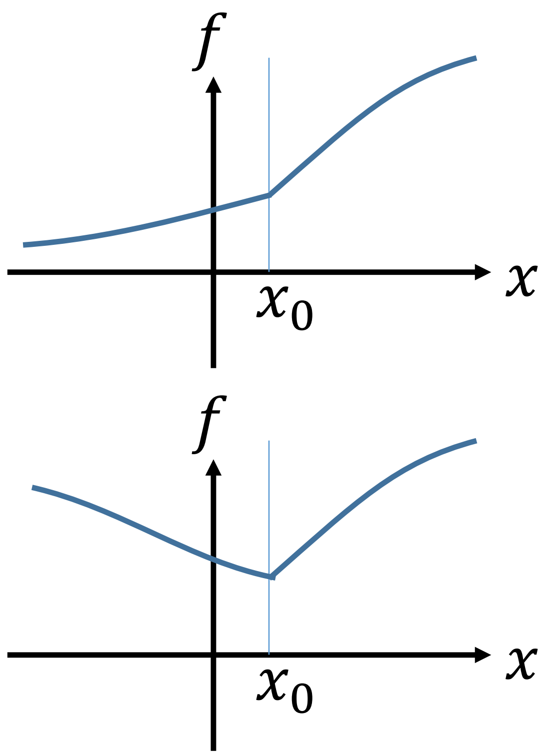


smooth and one extremum

$$e^{iVf(x)}$$

$$\sim \frac{1}{\sqrt{V}}\, e^{i\,Vf(x_0)}\delta(x-x_0)$$

(2)



f is continuous

f' is discontinuous at x_0

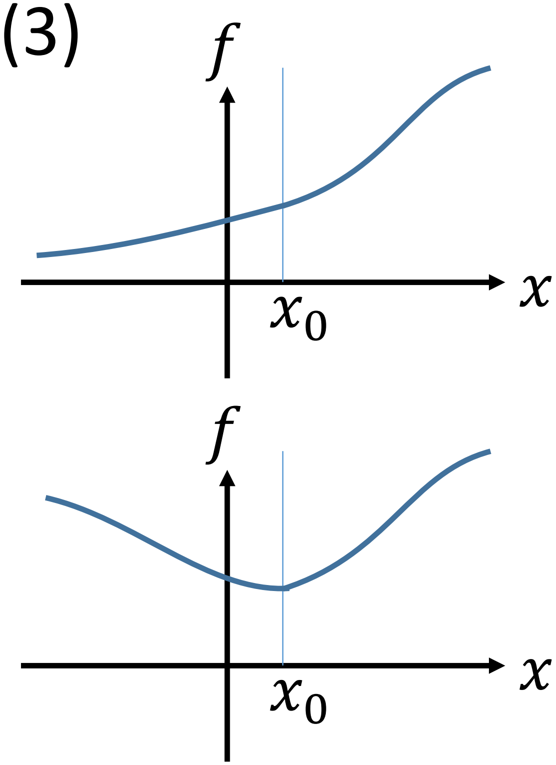
x_0 need not be extremum

monotonic on each side $x \lesseqgtr x_0$

$$e^{iVf(x)}$$

$$\sim \frac{1}{V}\left(\frac{1}{f'(x_0+0)} - \frac{1}{f'(x_0-0)}\right) \cdot e^{i\,Vf(x_0)}\,\delta(x-x_0)$$

(3)



$f,\ f'$ are continuous

f'' is discontinuous at x_0

x_0 need not be extremum

monotonic on each side $x \lesseqgtr x_0$

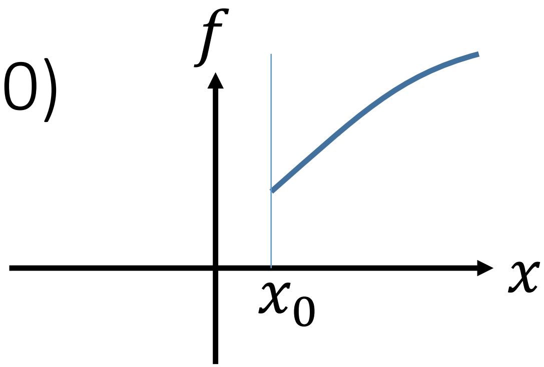
$$e^{iVf(x)}$$

$$\sim \frac{1}{V^2}\frac{1}{f'(x_0)^3}\ \left(f''(x_0+0) - f''(x_0-0)\right)$$

$$\cdot\, e^{i\,Vf(x_0)}\ \delta(x - x_0)$$

proof of (2), (3)

(0)



$f:[\mathrm{x}_0,\infty]\to\ \mathbb{R}$
smooth
monotonically increasing

For any test function,
$\varphi\in\mathcal{C}^{\infty}$, finite support,

$$\int_{x_0}^{\infty}dx\ \ e^{iVf(x)}\ \varphi(x)$$

$\leftarrow y=f(x),y_0=f(x_0),y_\infty=f(\infty)$

$$=\int_{y_0}^{y_\infty}dy\frac{dx}{dy}\ e^{iVy}\varphi\big(f^{-1}(y)\big)$$

$\leftarrow g(y)=\frac{dx}{dy}\varphi(f^{-1}(y))$

$$=\int_{y_0}^{y_\infty}dy\,e^{iVy}g(y)$$

$$=\left[\frac{1}{iV}e^{iVy}g(y)\right]_{y_0}^{y_\infty}-\int_{y_0}^{\infty}dy\frac{1}{iV}e^{iVy}g'(y)$$

$$=\frac{i}{V}e^{iVy_0}g(y_0)+O\left(\frac{1}{V^2}\right)$$

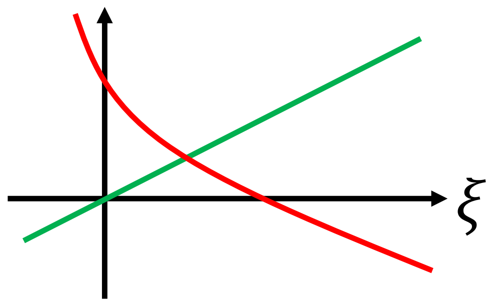
$\leftarrow\varphi$ has finite support $\Rightarrow g(y_\infty)=0$

$$=\frac{i}{V}e^{iVf(x_0)}\frac{1}{f'(x_0)}\ \varphi(x_0)+O(\frac{1}{V^2})$$

Behavior of $\Omega(A)$

$$\Omega(A) = \int_0^\infty dm^2\, e^{iVf(m^2)},$$

$$f = \xi\, \Lambda^2 a - \frac{\Lambda^4}{32\pi^2}\, g(\xi), \quad g(\xi) = \int_0^1 dx\, x \log(x+\xi)\,.$$



Λ: UV cutoff

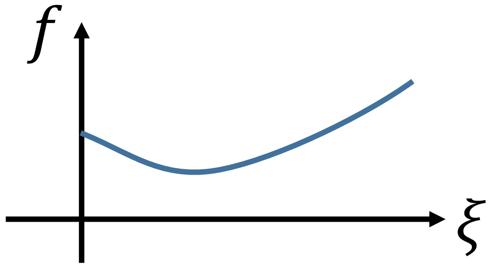
$a = \frac{A}{V} \quad \leftarrow \delta\left(\int d^4x\, \frac{1}{2}\phi^2 - A\right)$

$-g(\xi)$ is convex

$-g'(0) = -1$

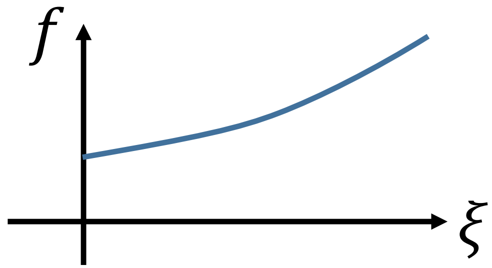
$\Rightarrow$ two cases

case 1. $\Lambda^2 a < \frac{\Lambda^4}{32\pi^2}$



$\exists$ minimum

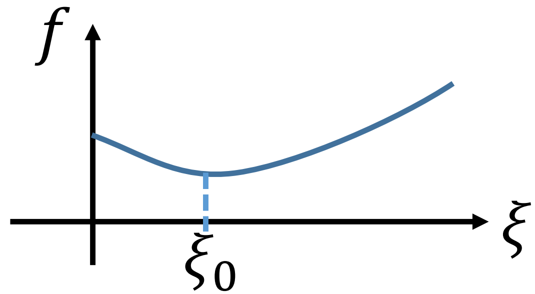
case 2. $\Lambda^2 a > \frac{\Lambda^4}{32\pi^2}$



monotonic

$$\Omega(A) = \int_0^\infty dm^2\, e^{iVf(m^2)}.$$

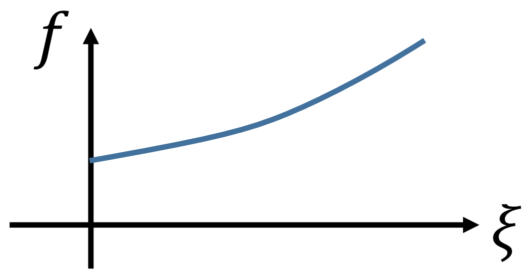
case 1. $\Lambda^2 a < \frac{\Lambda^4}{32\pi^2}$



$$e^{iVf(m^2)} \sim \frac{1}{\sqrt{V}}\delta(\xi - \xi_0) \;\Rightarrow\; \Omega(A) \simeq e^{iVf(\xi_0\Lambda^2)}$$

$\xi = \xi_0 \;\Leftrightarrow\; \frac{m^2}{\Lambda^2} = \xi_0 \sim O(1)$ quadratic divergence

case 2. $\Lambda^2 a > \frac{\Lambda^4}{32\pi^2}$



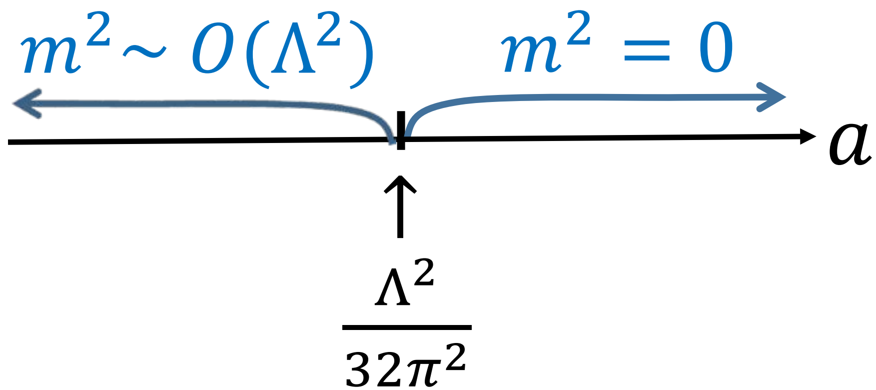
$$e^{iVf(m^2)} \sim \frac{1}{V}\frac{1}{f'(0)}\delta(\xi) \Rightarrow \;\Omega(A) \simeq e^{iVf(0)}$$

$\xi = 0 \;\Leftrightarrow\; \frac{m^2}{\Lambda^2} = 0$ mass is automatically tuned to 0

phase diagram

$$\delta\left(\int d^4x\, \frac{1}{2}\phi^2 - A\right),\ a = \frac{A}{V}$$

space of A



In a finite region of the parameter space, the physical mass is automatically tuned to 0.

$m^2 = 0$ is as natural as quadratic divergence $m^2 \sim O(\Lambda^2)$.

More general cases

Generalized mass term

So far, we have considered microcanonical mass term

$$\Omega(A) = \int d\phi\, e^{i\int d^4x\,\frac{1}{2}\partial_\mu\phi\partial^\mu\phi}\ \delta\left(\int d^4x\ \frac{1}{2}\phi^2 - A\right).$$

Here we discuss more general form

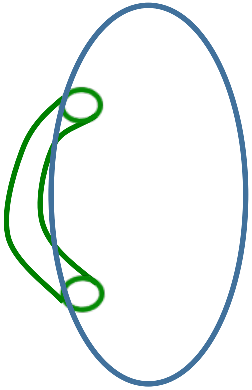
$$\Omega = \int d\phi\, e^{i\int d^4x\,\frac{1}{2}\partial_\mu\phi\partial^\mu\phi}\ f\left(\int d^4x\ \frac{1}{2}\phi^2 - A\right).$$

To be concrete we consider

$$f(x) = e^{i\frac{\alpha}{2}x^2}.$$

This can be thought of as the result of baby universe:

$$S_{\mathrm{eff}} = \frac{\alpha}{2}\left(\int d^4x\,\sqrt{-g}\,\frac{1}{2}\phi^2\right)^2 + \frac{\gamma}{2}\left(\int d^4x\,\sqrt{-g}\right)^2 + \beta\left(\int d^4x\ \sqrt{-g}\,\frac{1}{2}\phi^2\right)\left(\int d^4x\ \sqrt{-g}\right) + \cdots$$



$$\Omega = \int d\phi\, e^{i\int d^4x \frac{1}{2}\partial_\mu\phi\partial^\mu\phi}\, \mathrm{e}^{i\frac{\alpha}{2}\left(\int d^4x \frac{1}{2}\phi^2 - c\,V\right)^2}$$

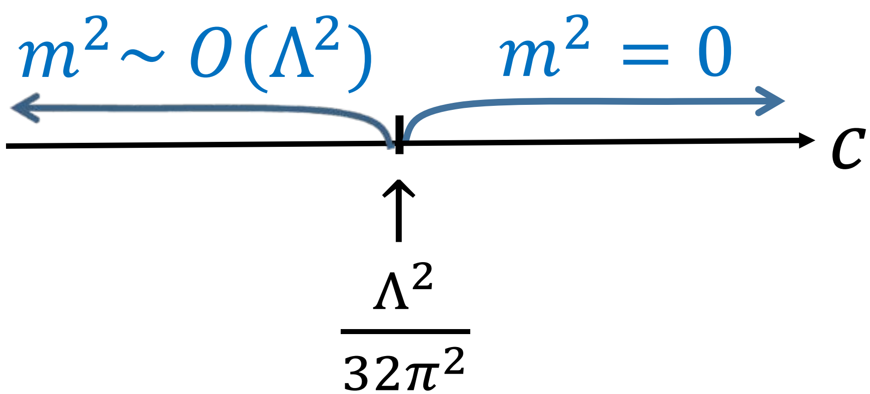
$$= \int dm^2\; e^{-i\frac{1}{2\alpha}\left(m^2\right)^2} \int d\phi\, e^{i\int d^4x \frac{1}{2}\partial_\mu\phi\partial^\mu\phi}\, e^{-im^2\left(\int d^4x \frac{1}{2}\phi^2 - cV\right)}$$

$$= \int dm^2\; e^{-i\frac{1}{2\alpha}(m^2)^2}\; \mathrm{e}^{iV\left(\xi\,\Lambda^2 c - \frac{\Lambda^4}{32\pi^2}g(\xi)\right)} \qquad \leftarrow \xi = \frac{m^2}{\Lambda^2}$$

↑

The only difference from microcanonical mass term.
This factor does not depend on V.

The large-V behavior is the same as before.



Comment on $m^2 < 0$

In the ordinary Euclidian FT, free scalar field with $m^2 < 0$ is not well defined:

$$Z(m^2 < 0) = \infty.$$

On the other hand, in the Lorentzian case we found

$$|Z(m^2)| = \begin{cases} 1 & (m^2 > 0) \\ e^{-\frac{Vm^4}{64\pi}} & (-\Lambda^2 < m^2 < 0) \\ e^{-\frac{V\Lambda^4}{64\pi}} & (m^2 < -\Lambda^2)\,, \end{cases}$$

which may represent the decay of the vacuum.

In this way we can formally define $Z(m^2)$ for all m^2, but the physical meaning is not clear.

It is better to consider models without such problem.

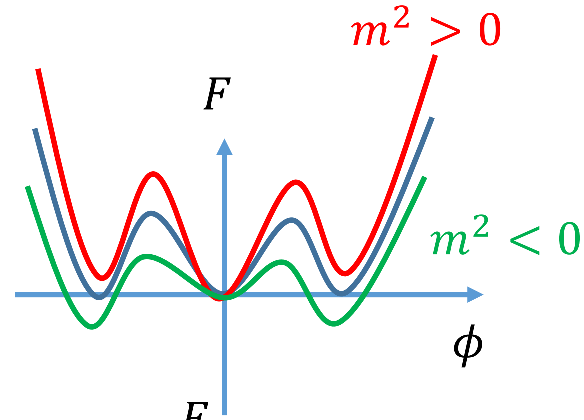
Models with well-defined ground state.

Order of phase transition does not matter.

1^{st} order phase transition

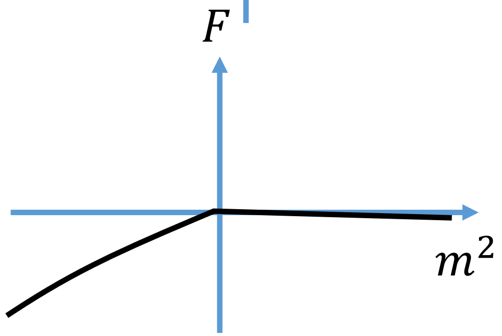
GL theory for 1^{st} order phase transition

$$F = m^2\phi^2 + (\phi^2 - 1)^2\phi^2$$



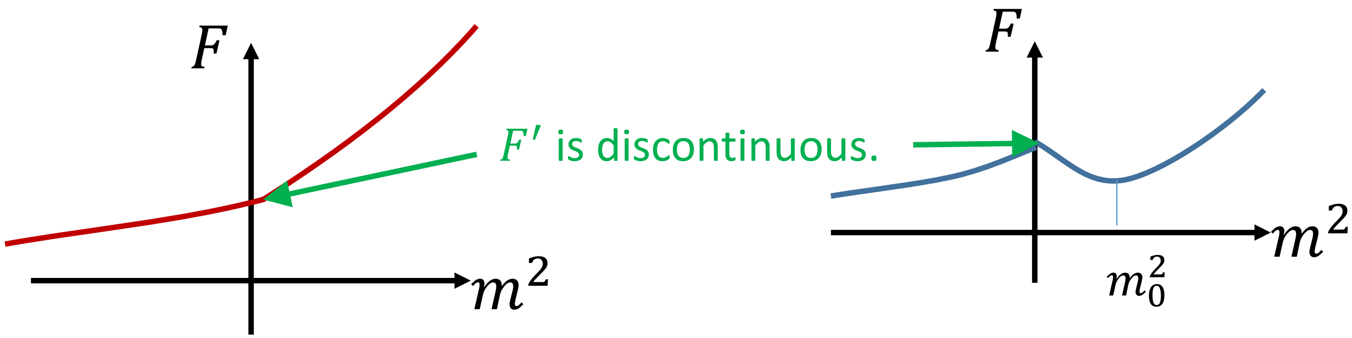
As a function of m^2, the minimum of F behaves as

$$F = \begin{cases} 0 & (m^2 > 0) \\ \sim m^2 & (m^2 < 0) \end{cases}$$



F' is discontinuous at the phase transition point.

In general if $m^2 = 0$ is the 1st order phase transition point, we have two cases depending on the parameters:



F is monotonic on each side $m^2 \lessgtr 0$.

$$e^{iVF} \sim \frac{1}{V}\delta(m^2)$$

F has an extremum at $m^2 = m_0^2$.

$$e^{iVF} \sim \frac{1}{\sqrt{V}}\delta(m^2 - m_0^2)$$

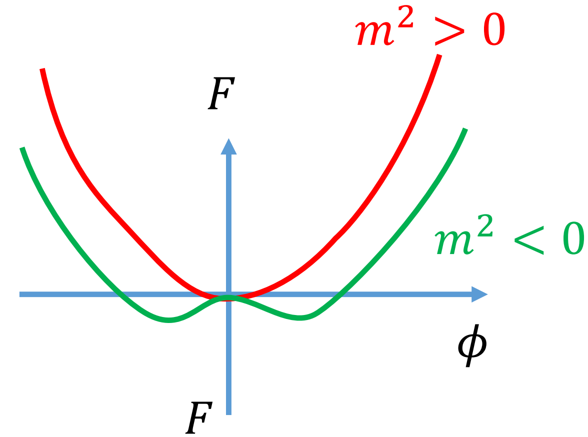
The same situation as the free field.

In a finite region of the parameter space of generalized QFT, automatic fine tuning to 1st order phase transition point occurs. The same as classical MPP.

2nd order phase transition

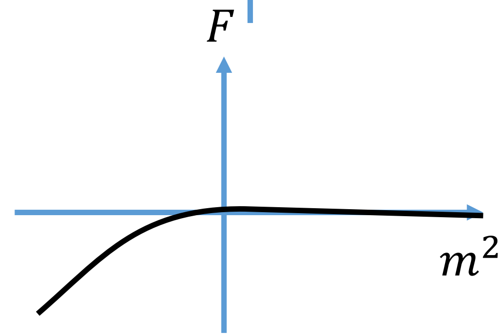
GL theory for 2nd order phase transition

$$F = m^2\phi^2 + \phi^4$$



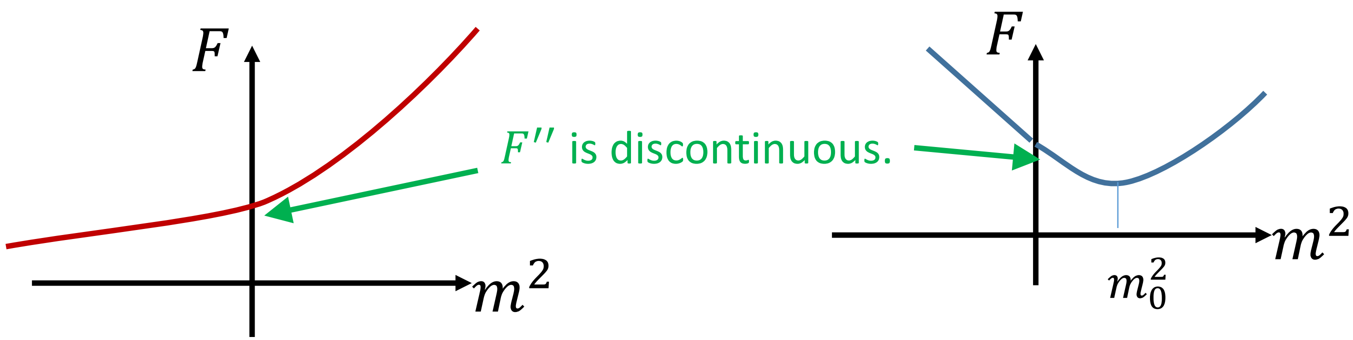
As a function of m^2, the minimum of F behaves as

$$F = \begin{cases} 0 & (m^2 > 0) \\ \sim(m^2)^2 & (m^2 < 0) \end{cases}$$



F'' is discontinuous.

In general, if $m^2 = 0$ is the 2nd order phase transition point, we have two cases depending on the parameters:



F is monotonic on each side $m^2 \lessgtr 0$.

$$e^{iVF} \sim \frac{1}{V^2}\delta(m^2)$$

F has an extremum at $m^2 = m_0^2$.

$$e^{iVF} \sim \frac{1}{\sqrt{V}}\delta(m^2 - m_0^2)$$

The coupling constants are automatically tuned to either a phase transition point or the minimum of F.

The same is true for any higher-order phase transition.

Quantum version of MPP:

In generalized QFT, the coupling constants of the equivalent canonical QFT are automatically adjusted either to minimize the vacuum energy density or to one of the critical points of the quantum phase transition.

Time evolution of universe

Time evolution of universe

So far we have considered static vacuum to evaluate the path integral:

$$Z = \int \prod_i d\alpha_i \, w(\alpha) Z(\alpha)$$

In this case, we can regard

$$Z(\alpha) = e^{i\,V\varepsilon(\alpha)}$$

where ε is the vacuum energy density.

If we consider the time evolution of universe, the notion of critical point should be generalized to critical point of the history of universe, which means coupling constants that significantly change the time evolution of universe when they are changed.

Generalize MPP:

The coupling constants of the low energy effective canonical FT of MM or quantum gravity are automatically adjusted either to minimize the vacuum energy density or to one of the critical points of the history of universe.

Examples

1. QCD θ-parameter

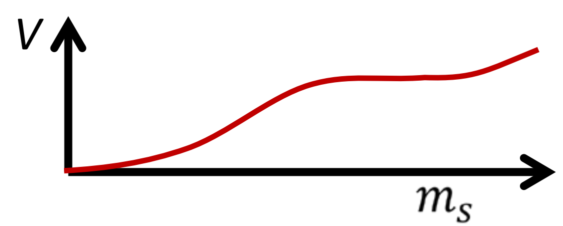
   $\theta = 0$ minimizes the vacuum energy.

2. Cosmological constant

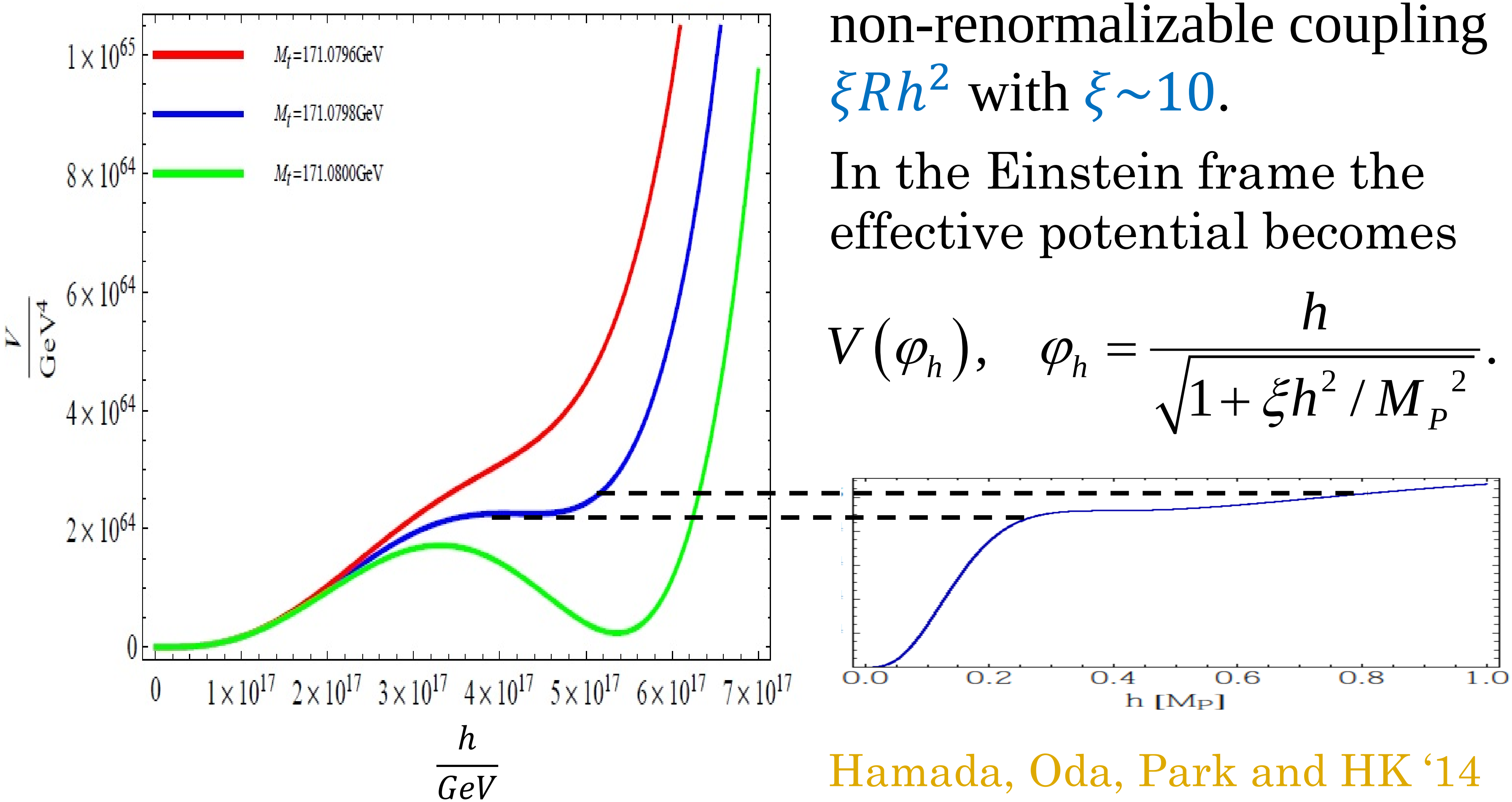
   $\lambda = 0$ is the critical point.

3. Higgs inflation at criticality

   Flat potential is the critical point of the history of universe.



SM Higgs is close to MPP.



non-renormalizable coupling $\xi R h^2$ with $\xi \sim 10$.

In the Einstein frame the effective potential becomes

$$V(\varphi_h), \quad \varphi_h = \frac{h}{\sqrt{1+\xi h^2 / M_P{}^2}}.$$

Hamada, Oda, Park and HK '14
Bezrukov,Shaposhnikov

Summary

We have discussed the following possibilities:

The low energy effective theory of the matrix model or quantum gravity is a generalized QFT.

Generalized MPP is realized.

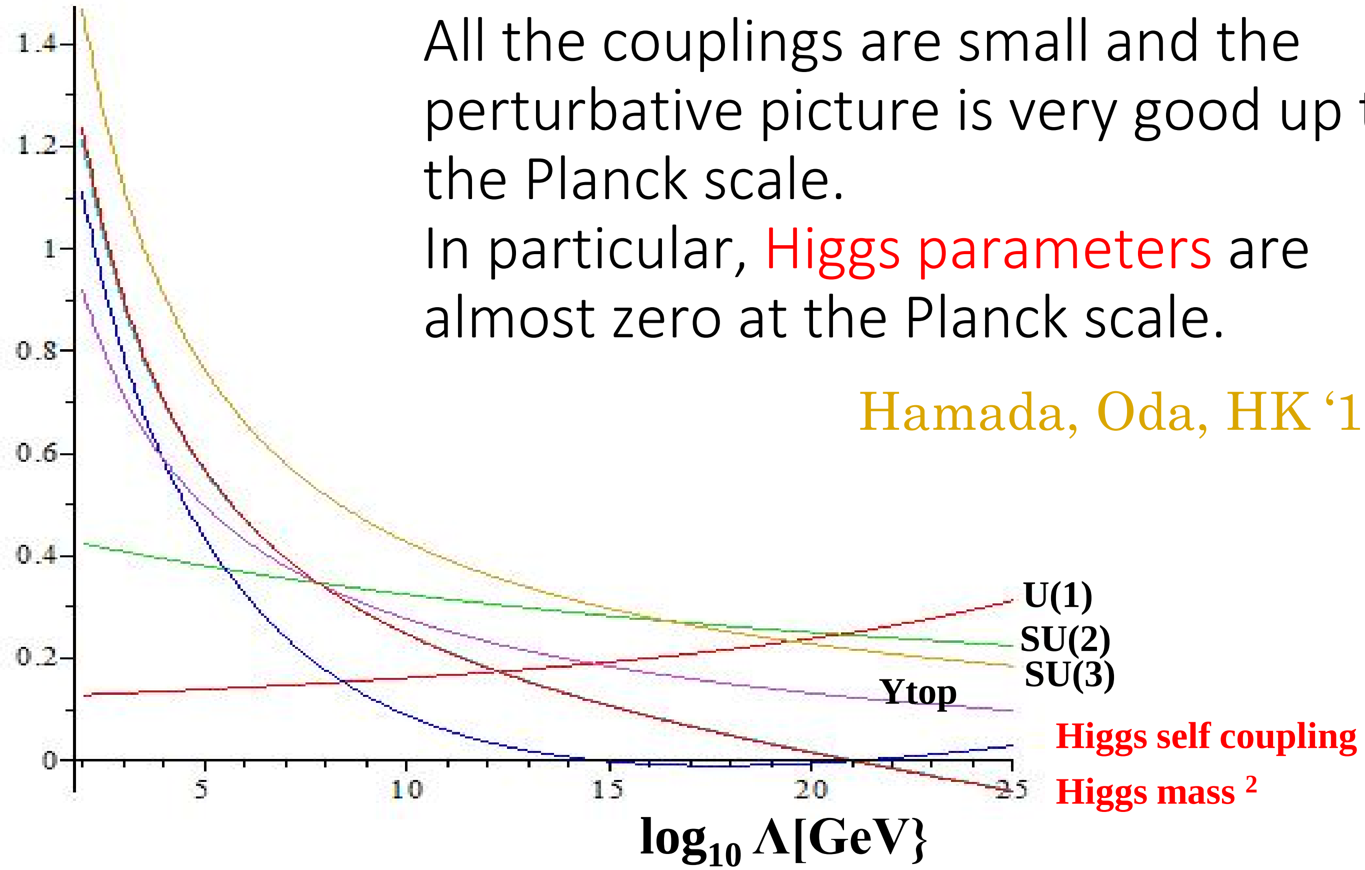
> The coupling constants of the low energy effective canonical FT of MM or quantum gravity are automatically adjusted either to minimize the vacuum energy density or to one of the critical points of the history of universe.

Some of the parameters of the (modified) standard model may be fixed by GMPP, independent of the detailed dynamics.

Thank you.

Appendix

RG analysis of SM



All the couplings are small and the perturbative picture is very good up to the Planck scale.
In particular, Higgs parameters are almost zero at the Planck scale.

Hamada, Oda, HK '13

It is natural to imagine that SM is directly connected to the string theory at the Planck scale without large modification.

subsystem in IIB MM

$A_\mu, \Psi : N \times N,$

can. or microcan. ens.

$n \times n$ submatrices

$$A_\mu = \begin{pmatrix} \overbrace{a_\mu}^{n} & \cdot \\ \cdot & \cdot \end{pmatrix}, \Psi = \begin{pmatrix} \psi & \cdot \\ \cdot & \cdot \end{pmatrix} \quad n \ll N$$

effective action for the submatrices

chemical pot.

$$S_{\text{eff}} \simeq -\frac{\mathrm{z}}{4} \operatorname{tr}\left([a_\mu, a_\nu]^2\right) - \frac{\mathrm{z}'}{2} \operatorname{tr}(\bar{\psi} \cdots) + \mu \underbrace{\operatorname{tr}(1)}_{n}$$

corresponding continuum action

$$S_{\text{Schild}} = \frac{z}{4} \int \omega \, \{X^\mu, X^\nu\}^2 + \cdots + \mu \int \omega$$

$\omega = \rho \, d^2\xi$: symplectic (volume) form

$\{f, g\} = \frac{1}{\rho} \epsilon^{ab} \partial_a f \, \partial_b g$: Poisson bracket

eliminate $\rho \Rightarrow S = \kappa \int \sqrt{(\sigma^{\mu\nu})^2}, \; \kappa = \sqrt{z\mu}.$ $\sigma^{\mu\nu} = \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu$

String appears as a subsystem of IIB MM.

comment on Green's functions

$$\int \mathcal{D}q\, f(S_i[q] - A_i)\, O[q]$$

$\leftarrow O[q]$: product of local operators

$$= \int \mathcal{D}q \int \prod_i d\alpha_i\, \tilde{f}(\alpha)\, e^{i \sum_i \alpha_i (S_i[q] - A_i)}\, O[q]$$

$$= \int \prod_i d\alpha_i\, w(\alpha) \int \mathcal{D}q\, e^{i \sum_i \alpha_i S_i[q]} O[q]$$

$$= \int \prod_i d\alpha_i\, w(\alpha) \frac{\int \mathcal{D}q e^{i \sum_i \alpha_i S_i[q]} O[q]}{\int \mathcal{D}q e^{i \sum_i \alpha_i S_i[q]}} \int \mathcal{D}q\, e^{i \sum_i \alpha_i S_i[q]}$$

$$= \int \prod_i d\alpha_i\, w(\alpha)\, \langle O[q] \rangle_\alpha \int \mathcal{D}q\, e^{i \sum_i \alpha_i S_i[q]}$$

$\langle O[q] \rangle_\alpha$: Ordinary FT with coupling constants α_i .

$$w(\alpha) = \tilde{f}(\alpha)\, e^{-i \sum_i \alpha_i A_i}$$

$\langle O[q] \rangle_\alpha$ is intensive. $\Rightarrow$ Does not affect $\alpha_i^{(0)}$.