A Philosopher Looks at Epstein-Glaser Renormalization

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> The aim of philosophy, abstractly formulated, is to understand how things in the broadest possible sense of the term hang together in the broadest possible sense of the term. (Sellars 1962)

If one is interested in how the world hangs together at what philosophers call the fundamental level, then they are naturally drawn to quantum field theory.

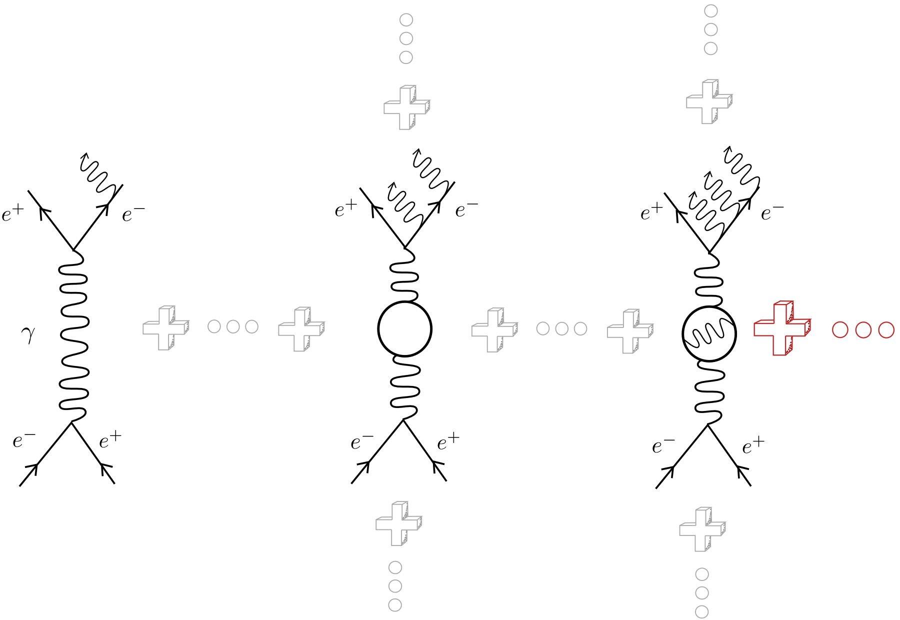
$$a_e \text{ (theory)} = 0.00115965218178(77)$$

$$a_e \text{ (experiment)} = 0.00115965218073(28)$$

(Theory) T. Aoyama et. al. Prog. Th. Ex. Phys. **A01** 107 (2012).
(Experiment) D. Hanneke et. al. Phys. Rev. Lett. **100** 120801 (2008).

This kind of empirical success suggests that the theory is getting some aspects of the stucture of the world correct.

So the structure lying behind the theoretical prediction seems like the best available guide to how things hang together at the fundamental level. But what is that structure?



e^+
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$$
\begin{aligned}
\text{s-matrix} &= \sum_{n=0}^{\infty} a_n e^n \\
&= \text{finite} + \infty + \infty + \ldots
\end{aligned}
$$

ultraviolet/infrared divergences

regularization/renormalization

$$
\begin{aligned}
&= \text{finite} + \text{finite} + \text{finite} + \ldots \\
&= \infty
\end{aligned}
$$

large-order divergence

Two views have dominated philosophical discussions of the nature of scientific theories:

- Syntactic View: theories are collections of sentences
- Semantic View: theories are collections of models

> ...philosophers of physics have taken their object of study to be theories, where theories correspond to mathematical objects (perhaps sets of models). But it is not so clear where "quantum field theory" can be located in the mathematical universe. In the absence of some sort of mathematically intelligible description of QFT, the philosopher of physics has two options: either find a new way to understand the task of interpretation, or remain silent about the interpretation of quantum field theory. (Halvorson 2006, pp. 731-2)

“

Given a theory T, . . . we confront the exemplary interpretive question of how exactly to establish a correspondence between T's models and worlds possible according to T. That is, we confront that question *if* T is the sort of thing that has models. ‘A collection of partially heuristic technical developments’ isn’t readily attributed a set of models about whose underlying ontology or principles of individuation philosophical questions immediately arise. This isn’t to say that ‘a collection of partially heuristic technical developments’ is unworthy of philosophical attention. It is in itself a philosophically provocative circumstance that such a collection can enjoy stunning empirical success. (Ruetsche 2012, pp. 102-3)

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How can such a broken theory be so profoundly empirically successful? Can we separate out what we are getting right from what seems to be going wrong?

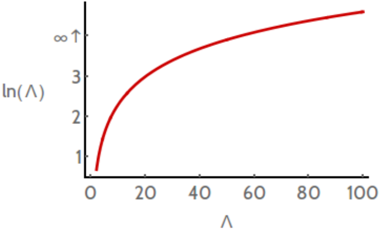
Epstein-Glaser renormalization provides us with the resources to provide a partial answer to this question.

The standard story: the infrared and large-order divergences are conceptually unproblematic.

The standard story: ultraviolet divergences result from the theory treating arbitrarily short distances.

The standard story:

$$I \propto \int^{\infty} \frac{k^3 dk}{k^4} \Longrightarrow \int^{\Lambda} \frac{dk}{k} = \ln(\Lambda)$$



The integral is finite for finite Λ. By studying the behavior of the theory as $\Lambda \to \infty$, we can determine redefinitions of parameters in the theory to generate a collection of infinite counterterms to add to the Lagrangian.

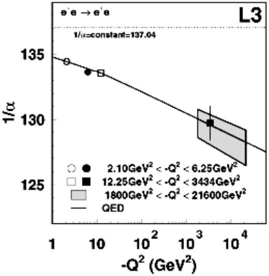
The standard story: power counting methods show us how many counterterms are required to render the theory ultraviolet finite.

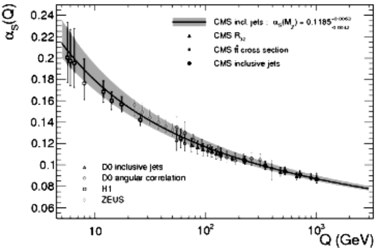
In perturbatively renormalizable theories, only a finite number of redefinitions are required, and these can be fixed with experimental data.

Assuming infrared divergences are also handled during this process, the output of this procedure is a well-defined formal power series.

The standard story: renormalization was once rightly regarded with suspicion, but now we know how to do better.

The standard story: the renormalization group provides a physically well-motivated account of why renormalization is required, and it has led to novel empirical predictions.





There are different ways that one might go about adjusting the language in which a theory is articulated in face of difficulties.

Recast: One might completely recast the theory and search for a model to show that the principles constitutive of the theory are consistent.

Repair: But one might also attempt to directly identify the source of the difficulties in the language of the theory as it was originally articulated and repair it.

This approach has played an important role in understanding the status of perturbative ultraviolet divergences in quantum field theory.

Question: If renormalization is about describing a real physical process, and not about cancelling divergences, why are the divergences present in the theory at all?

Can't we just write down a completely finite theory? In order to answer this question, we first need to understand the origin of the divergences.

An alternative story: renormalization is required to resolve ambiguities stemming from the distributional character of field theoretic expressions.

The original architects of field theory did not realize the expressions they were writing contained distributions (a systematic theory of such objects was not yet available).

Products and divisions of objects which are distributional in character can lead to ambiguities. Just because an expression is ambiguous doesn't entail that it is meaningless!

La normalisation des constantes dans la theorie des quanta*)

par **E. C. G. Stueckelberg** et **A. Petermann.**

(Lausanne et Genève.)

(28. III. 53.)**)

Summary. This article proposes a mathematical foundation to the method previously employed (Stueckelberg and Rivier[1])), (Stueckelberg and Green[2])) to give a definit meaning to the products of invariant distributions such as $\left(\Delta^{(1)}_{x-y} D^{(s)}_{x-y} + {}^{(s)(1)}\right)$, $\left(\Delta^{(1)}_{x-y} \Delta^{(s)}_{y-z} D^{(s)}_{x-z} + \ldots\right)$, etc. in terms of arbitrary constants $c_1, c_2 \ldots c_i \ldots c_{r(n)}$. The n'th approximation $\boldsymbol{S}^{(n)}$ of the $\boldsymbol{S}[V]$ *matrix* (defined for a given space-time region V) depends on these $r(n)$ arbitrary constants in addition to the arbitrary physical parameters (masses $\varkappa$, μ, and coupling constants $e, g \ldots$).

In the introduction (§ 1), we see that a definit physical meaning can be given to the masses $\varkappa$, μ. A coupling parameter, however, can only be specified in terms of a chosen development of a *function* $S\,(xy\ldots, \varkappa\ldots, c_1\ldots)$ of physical significance. However, the terms of the actual correspondence development (in terms of e^2) $S = S_2 + S_4 + \ldots$ have no physical meaning. Therefore the coefficient e^2 in S_2 has only a mathematical significance. It requires that the functions of $xy\ldots\ S_4,\ S_6,\ \ldots S_n$ have all been specified. As this specification involves the c_i's, we must expect that a group of infinitesimal operations $\boldsymbol{P}_i = (\partial/\partial c_i)_{c=0}$ exists, satisfying

$$\boldsymbol{P}_i\, S = h_{i\,e}\,(\varkappa, \mu, e)\ \partial\, S(\varkappa\ \mu\ e,\ 00\ldots)/\partial\, e,$$

admitting thus a *renormalization* of e.

§ 2 contains an outline of the general problem without referring to correspondence.

However the only way of attack being the correspondance principle, we discuss (§ 3) the invariance properties of a classical theory, linear in the Dirac field. In addition to the Weyl group of Gauge invariance, a group exists whose consequence is *the equivalence theorem between pseudoscalar and pseudovector coupling* with the pseudoscalar meson field. In § 4, we show that the definition of distributions in terms of the c_i's is a generalization of the method of M. L. Schwartz[6]). This permits to discuss the *group of normalization* given by the $\boldsymbol{P}_i$'s. § 5 imposes certain restrictions on this group, if we require invariance of $\boldsymbol{S}$ with respect to the corresponding classical groups (Weyl and equivalence). The limiting case of photons with zero rest mass then can be defined.

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*) Recherche subventionnée par la Commission Suisse d'énergie Atomique (C.S.A.).

**) Le présent travail constitue, à des détails près, une thèse présentée par l'un de nous (P) à l'Université de Lausanne, le 9 mai 1952, pour l'obtention du grade de Docteur ès Sciences.

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Realization that ambiguities stem from products and divisions of distributions.

Ann. Inst. Henri Poincaré, Section A :
Vol. XIX, n° 3, 1973, p. 211-295. *Physique théorique.*

The role of locality
in perturbation theory

by

H. EPSTEIN and V. GLASER
C. E. R. N., Geneva

ABSTRACT. — It is shown how an inductive construction of the renormalized perturbation series of quantum field theory automatically yields, at each order, finite terms satisfying the requirements of locality. This method whose result is equivalent to the Bogoliubov-Parasiuk-Hepp prescriptions, also establishes the usual classification between renormalizable and non-renormalizable theories.

RÉSUMÉ. — On montre qu'une construction récurrente de la série des perturbations renormalisée, en théorie quantique des champs, fournit automatiquement, à chaque ordre, des termes finis satisfaisant aux conditions de localité. Cette méthode, dont le résultat équivaut aux prescriptions de Bogoliubov-Parasiuk-Hepp, établit également la classification habituelle des théories renormalisables et non renormalisables.

INTRODUCTION

The theory of renormalization in perturbative Lagrangian quantum field theory [*1*]-[*6*] (1) has been brought by recent investigations ([*7*]-[*11*]) to a high degree of elegance and mathematical rigour. However, it does not seem to have been proved, so far, that the renormalized series, as a formal series, satisfies the two requirements of microcausality (or local

(1) It is impossible to quote all the original papers about the theory of renormalization. Many of them are reprinted in [*1*] while [*2*]-[*6*] give a sample of works not contained in [1].

Connection between the Bogoliubov-Parasiuk causality condition and distribution theoretic ambiguities.

Standard perturbative QFT:

$$\text{s-matrix} = \sum_{n=0}^{\infty} \frac{-i^n}{n!} \int \mathcal{T}(\phi(x_1)\cdots\phi(x_n))d^4x_1 \ldots d^4x_n$$

$$\downarrow$$

Causal perturbation theory:

$$\text{s-matrix} = \sum_{n=0}^{\infty} \frac{-i^n}{n!} \int \mathcal{T}(T(x_1)\cdots T(x_n))g(x_1)\ldots g(x_n)d^4x_1 \ldots d^4x_n$$

The aim of CPT is to produce an order-by-order construction of the S-matrix where each term, S_n, is a well-defined operator valued distribution.

What you find when you produce this construction is that

$$S_n \in \mathcal{D}'(\mathbb{R}^n \setminus \{0\}) = \{T : \mathcal{D}(\mathbb{R} \setminus \{0\}) \to \mathbb{C}\}$$

where,

$$\mathcal{D}(\mathbb{R} \setminus \{0\}) = \{f \in \mathcal{D}(\mathbb{R}^n) | 0 \notin \operatorname{supp}(f)\}$$

So we almost get elements of $\mathcal{D}'(\mathbb{R}^n)$ but not quite.

Can we uniquely extend a $T^0 \in \mathcal{D}'(\mathbb{R}^n \setminus \{0\})$ to a $T \in \mathcal{D}'(\mathbb{R}^n)$?

To answer this question, we need a measure of the singularity of a distribution at the origin.

The scaling degree of $T \in \mathcal{D}'(\mathbb{R}^n)$ at $x = 0$ is given by:

$$\mathrm{sd}(T) = \inf\{\omega \in \mathbb{R} | \lambda^\omega T(\lambda) \xrightarrow{\lambda \to 0} 0\}$$

The power counting arguments of standard perturbative field theory can be thought of as estimating this scaling degree.

If $T^0 \in \mathcal{D}'(\mathbb{R}^n \setminus \{0\})$ is a distribution with $\mathrm{sd}(T^0) < n$, there is a unique distribution $T \in \mathcal{D}'(\mathbb{R}^n)$ with $\mathrm{sd}(T) = \mathrm{sd}(T^0)$ extending T^0.

What happens when $\mathrm{sd}(T^0) \geq n$? Then there is not a unique extension. However, there is a unique extension of:

$$T^0 + \sum_{\alpha \leq \mathrm{sd}(T^0)} C_\alpha \partial^\alpha \delta(x)$$

To produce a unique extension we need to fix a finite set of numbers, the C_α.

These are analogs of the counterterms in the standard story.

Moreover, one can even recover the scaling behaviour in this formalism.

Causal perturbation theory shows us how to write down a perturbatively ultraviolet finite theory from the outset.

This demonstrates that the perturbative ultraviolet divergences that occur in the standard story are artifacts of the wrong choice of mathematical objects.

There is a sense in which this incorrect choice was forced on the original architects of the theory because a systematic theory of the correct objects was not yet available.

There are different ways that one might go about adjusting the language in which a theory is articulated in face of difficulties.

Recast: One might completely recast the theory and search for a model to show that the principles constitutive of the theory are consistent.

Repair: But one might also attempt to directly identify the source of the difficulties in the language of the theory as it was originally articulated and repair it.

Causal perturbation theory thus provides us with the resources to address Ruetsche's question.

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Additional slides

> The quantum theory of fields never reached a stage where one could say with confidence that it was free from internal contradictions – nor the converse. In fact, the Main Problem of quantum field theory turned out to be to kill it or cure it: either to show that the idealizations involved in the fundamental notions of the theory (relativistic invariance, quantum mechanics, local fields, etc.) are incompatible in some physical sense, or to recast the theory in such a form that it provides a practical language for the description of elementary particle dynamics.
> (Streater and Wightman, 1964)

The Dirac delta function

$$\delta(x) = \begin{cases} 0 & x \neq 0 \\ \infty & x = 0 \end{cases}$$

subject to the constraint

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

was in wide use in physics and was known not to be well-defined.

Beginning in 1943, Schwartz, a mathematician in the Bourbaki group developed a systematic theory of this type of object. He published a number of articles on his theory throughout the 1940's (Schwartz 1945; Schwartz 1947a; Schwartz 1947b; Schwartz 1948).

By 1951, Schwartz had developed a reasonably general theory of distributions and published a two-volume textbook on the subject (Schwartz 1951a; Schwartz 1951b).

Schwartz considered the space of infinitely differentiable functions with compact support, $\mathcal{D}(\mathbb{R}^n)$.

A distribution T is then defined to be a functional which maps each element $\varphi(x) \in \mathcal{D}$ to $\mathbb{R}$.

The Schwartz space of distributions, $\mathcal{D}'$, is the set of such functionals which are linear and continuous in an appropriate topology:

$$\mathcal{D}' =_{\mathsf{def}} \{T : \mathcal{D}(\mathbb{R}^n) \to \mathbb{R} \mid T \text{ is linear and continuous}\}.$$

The mapping is generated by integrating the distribution against the test function.

It is possible to uniquely associate a locally integrable function, $f(x)$, with a functional that takes test functions, $\varphi(x) \in \mathcal{D}$, to the numbers:

$$T_f : \varphi(x) \to \int_{-\infty}^{\infty} f(x)\varphi(x)dx.$$

$\mathcal{D}'$ thus includes all of the functionals associated with locally integrable functions, the so-called regular distributions.

But $\mathcal{D}'$ also contains additional objects, the singular distributions.

The first example that Schwartz provides of such an object is the Dirac delta distribution, defined above, which can be identified with the distribution:

$$T_\delta : \varphi(x) \to \int_{-\infty}^{\infty} \delta(x)\varphi(x)dx = \varphi(0), \tag{1}$$

This is a well-defined distribution which is not well-defined as a function.

Schwartz showed that it is possible to extend many standard mathematical operations on functions, such as differentiation, integration and the Fourier transform, to the broader space of objects, $\mathcal{D}'$.

However, there are two very basic operations on functions which do not straightforwardly generalize to distributions; multiplication and division.

The singular support of a distribution T is the closed set of $\mathbb{R}^n$ in which it is not possible to find a locally integrable function, $f(x)$, such that $T = T_f$.

The singular support of the Dirac distribution, for instance, is $\{0\}$ since at all other points it can be identified with the regular distribution associated with $f(x) = 0$.

If two distributions have disjoint singular support we can give a natural meaning to their product as it is always possible to define the product of a singular distribution with a locally integrable function.

The product of two distributions with overlapping singular support, such as δ^2 however, is not a well-defined distribution in general.

A closely connected problem arises for the inverse problem of the division of distributions. Schwartz observes that for a given distribution S, and some $\varphi(x) \in \mathcal{D}$ which has no zeros, there is a unique distribution T which satisfies, $S = \varphi T$.

This ensures that S can be divided by φ and yields a unique, well-defined distribution. However, if φ has zeros, there is no guarantee that there is a unique well-defined T.

Schwartz investigates this problem but does not resolve it in complete generality.

He considers the restricted case where S is a distribution on $\mathbb{R}$ and he shows that there are an infinity of distributions T satisfying $S = xT$ which differ from one another by terms like $C\delta$ for C a constant and δ is the Dirac distribution.

As a corollary he is able to show that for S a distribution on $\mathbb{R}$, there are infinity of distributions T satisfying $S = x^l T$.

In this case the T differ between each other by linear combinations of arbitrary constants multiplied by derivatives of order $\leq l - 1$ of the Dirac distribution.

These results show that division of a well-defined distribution by a function does not always yield a unique distribution and such expressions can be ambiguous.

Further progress on the problem of division in Hormander (1958) and Lojasiewicz (1961). A complete solution was developed in a series of papers by Malgrange (1959-1963).

In the process of solving the problem of division, Malgrange also solved two closely related problems: the problem of splitting and the problem of extension.

Each case involves ambiguities of the form:

$$T(X) = T'(X) + \sum_{i \leq \omega} c_i \partial^i \delta(X).$$

It is well known the [products of $D(xy)$, $\Delta(xy)$, etc.], considered as products of functions, lead to summation difficulties (divergences) as well as inconsistencies (ambiguities such as the loss of gauge invariance in Fourier space, etc.). Furthermore, the D, $\Delta \ldots$, are distributions, tempered solutions of hyperbolic equations. Their analysis relies strictly on the theory of distributions established in detail by M. L. Schwartz. Unlike the other recent formalisms (Dyson and others), in which the divergences are accepted as such and "renormalized" by means of an algebra of infinite quantities, the constants of the problem, we consider that the multiplicative products T of distributions $A, B, \ldots,$ that is to say $T = AB \ldots$, are not in general well defined. The series expansion that involves such products thus doesn't have a precise meaning. (Stueckelberg and Petermann 1953, p. 509)

It is however possible to define the products T each time that they appear in the series by using the following detour: One first searches for a distribution $Q = \theta T$ defined uniquely for any combination of factors θ and T. Then, by division of Q by θ, one obtains the definition of T, up to the indeterminacy of the problem of division. This definition can thus be implemented in two distinct steps:

1) Find distributions Q which, by division by θ give the desired product T.
2) Discussion of the indeterminacy induced by the division. (Stueckelberg and Petermann 1953, p. 509, original emphasis)

The definition by division makes available an infinity of different distributions which differ by terms of the type [linear combinations of the Dirac distribution and its derivatives]. The problem of determining the S-matrix can thus at first sight seem completely indeterminate. However, it is possible to show that changes in the (real) parameters of the group δc_i, don't do anything other than to change the values of the constants $\chi, \mu_0 \ldots, g$. (Stueckelberg and Petermann 1953, pp. 513-4)