Microscopic Entanglement Wedges from Bilocal Holography

Large-N Matrix Models and Emergent Geometry
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Message in a nutshell

The duality between vector models and higher spin gravity is a simple but instructive example of holography. For this example the dual gravity theory can be constructed starting from CFT.

This is achieved by giving a holographic map constructed by:

1. Reducing gravity to physical and independent degrees of freedom.
2. Reducing CFT to its independent degrees of freedom.
3. Identifying the complete set of degrees of freedom of CFT with those of gravity.

We work at large N.

The mapping reproduces general expectations of holography including bulk reconstruction, subregion duality and the holography of information.

Outline

Vector model/higher spin duality.

Gauge fixing higher spin gravity.

Reduction of the vector model.

Bilocal holography for the vector model.

Testing the holographic map.

Conclusions.

Vector model / Higher spin duality

Holographic duality between (Vasiliev's) higher spin gravity theory in AdS_4 and $O(N)$ vector models (Klebanov, Polyakov, arXiv:hep-th/0210114, Sezgin, Sundell, arXiv:hep-th/0305040).

Perhaps simplest, nontrivial example of AdS/CFT:

1. Spectrum of operators in CFT not renormalized at infinite N (Giombi, Minwalla, Prakash, Trivedi, Wadia, et al. arXiv:1110.4386), and
2. Spectrum of fields in the bulk theory is simple. (Vasiliev, arXiv:hep-th/0106149 [hep-th])

Nontrivial content of duality (at least in perturbation theory) is in agreement between correlation functions in the bulk higher spin gauge theory and boundary vector model CFT. Giombi, Yin, arXiv:0912.3462 [hep-th] demonstrated match for free and critical $O(N)$ vector models.

Vector model / Higher spin duality

Free $O(N)$ vector model in $2+1$ dimensions

$$S = \int d^3x \sum_{a=1}^{N} \left(\frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a \right)$$

Single trace primaries: $O_{\Delta=1}(t,\vec{x}) = \sum_{a=1}^{N} \phi^a(t,\vec{x})\phi^a(t,\vec{x})$

$$J_{\mu_1\mu_2\cdots\mu_{2s}}(t,\vec{x})\alpha^{\mu_1}\alpha^{\mu_2}\cdots\alpha^{\mu_{2s}} = \sum_{a=1}^{N}\sum_{k=0}^{2s} \frac{(-1)^k \ :(\alpha\cdot\partial)^{2s-k}\phi^a \ (\alpha\cdot\partial)^k\phi^a:}{k!(2s-k)!\Gamma(k+\frac{1}{2})\Gamma(2s-k+\frac{1}{2})}$$

The gravity dual is higher spin gravity in AdS_4, with a bulk scalar and a gauge field $A_{\mu_1\cdots\mu_{2s}}$ for each conserved current.

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Large N Higher spin equations of motion

Following Metsaev, arXiv:hep-th/9906217 work in lightcone gauge, with lightcone coordinates, in Poincare patch of AdS_4

$$
\begin{aligned}
X^+ &\equiv X^2 + X^0, \quad X^- \equiv X^2 - X^0, \quad X \equiv X^1, \quad Z \\
ds^2 &= \frac{dX^+ dX^- + dX^2 + dZ^2}{Z^2}
\end{aligned}
$$

In lightcone gauge

$$
A^{+\mu_2 \cdots \mu_{2s}} = 0
$$

all components $A^{-\mu_2 \cdots \mu_{2s}}$ are determined by constraints. Dynamical fields are X, Z polarizations: $A^{XZXZ \cdots ZZ}$. Since gauge field is symmetric and traceless, **there are two independent physical degrees of freedom at each spin.**

Free equation of motion is (obtained after gauge fixing and solving the constraint)

$$
\left(\frac{\partial}{\partial X^+} \frac{\partial}{\partial X^-} + \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Z^2} \right) \frac{A^{XXZ \cdots ZX}}{Z} = 0
$$

Conformal symmetry

To work out the so(2,3) AdS isometry (= conformal) generators after reducing to physical degrees of freedom we need to:

• Fix a gauge and solve the associated gauge constraint. Isometries are generated using the Killing vectors as usual.

• Since conformal transformations move out of lightcone gauge, each conformal transformation must be supplemented with a compensating gauge transformation, that restores the gauge.

• Reduce to independent degrees of freedom by solving the symmetric and traceless constraints.

Result is a set of transformation defined on $A^{XX\cdots X}$ and $A^{ZX\cdots X}$ fields.

Repackaging Higher Spin Gravity

Replace an infinite number of spinning fields in AdS_4 with a single field on $\mathrm{AdS}_4 \times \mathrm{S}^1$

Co-ordinates: $X^+ \equiv X^2 + X^0 \qquad X^- \equiv X^2 - X^0, \qquad X \equiv X^1 \qquad Z$

Metric: $ds^2 = \frac{dX^+ dX^- + dX^2 + dZ^2}{Z^2}$

Fields: $A^{XX\cdots X}(X^+, X^-, X, Z)$, $A^{ZX\cdots X}(X^+, X^-, X, Z)$, $\Phi(X^+, X^-, X, Z)$

Co-ordinates: $X^+ \equiv X^2 + X^0 \qquad X^- \equiv X^2 - X^0, \qquad X \equiv X^1 \qquad Z \qquad \theta$

Metric: $ds^2 = \frac{dX^+ dX^- + dX^2 + dZ^2}{Z^2}$

Field: $\Phi(X^+, X^-, X, Z, \theta) = \sum_{s=0}^{\infty} \left(\cos(2s\theta) \frac{A^{XX\cdots XX}}{Z} + \sin(2s\theta) \frac{A^{XX\cdots XZ}}{Z} \right)$

Summary: Higher Spin Gravity

Equation of motion for physical d.o.f.:

$$\left(\frac{\partial}{\partial X^+}\frac{\partial}{\partial X^-}+\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Z^2}\right)\frac{A^{XXZ\cdots ZX}}{Z}=0$$

Repackaged the complete set of physical and independent fields into a single field, which is a function of 5 co-ordinates:

$$\Phi(X^+,X^-,X,Z,\theta)=\sum_{s=0}^{\infty}\left(\cos(2s\theta)\frac{A^{XX\cdots XX}}{Z}+\sin(2s\theta)\frac{A^{XX\cdots XZ}}{Z}\right)$$

Action of conformal symmetry on $\frac{A^{XX\cdots X}}{Z}$, $\frac{A^{ZX\cdots X}}{Z}$ is known: for $L^A\in so(2,3)$

$$L^A_{\oplus}\Phi(X^+,X^-,X,Z,\theta)=\sum_{s=0}^{\infty}\left(\cos(2s\theta)L^A_{2s}\frac{A^{XX\cdots XX}}{Z}+\sin(2s\theta)L^A_{2s}\frac{A^{XX\cdots XZ}}{Z}\right)$$

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Counting degrees of freedom

The spinning currents ($d = 2+1$) are symmetric, traceless, conserved rank $2s$ tensors $J_{\mu_1 \cdots \mu_{2s}}$.

There are $\frac{(2s+1)(2s+2)}{2}$ symmetric rank $2s$ tensors.

There are $4s+1$ symmetric, traceless rank $2s$ tensors.

There are 2 symmetric, traceless, conserved rank $2s$ tensors.

Thus the number of independent components of the spinning primary match the number of physical and independent components of the gauge field.

We have reduced the gravitational theory to its independent and physical fields. To construct the holographic map we will reduce the CFT to its independent fields, and then, at each $2s$ match the 2 components of the gauge field to the 2 components of the spinning current.

Symmetries in the reduced theory

Higher spin currents $J_s^{\mu_1\mu_2\cdots\mu_s}(x^\nu)$, have $\Delta = s+1$, are traceless and conserved

$$\partial_\mu J_s^{\mu\mu_2\cdots\mu_s}(x^\nu) \;=\; 0 \qquad \eta_{\mu\nu} J_s^{\mu\nu\mu_3\cdots\mu_s}(x^\nu) \;=\; 0$$

Represent the higher spin current as

$$|J_s(t,\vec{x},a^\mu)\rangle \;=\; J_s^{\mu_1\mu_2\cdots\mu_s}(x^\nu)a_{\mu_1}\cdots a_{\mu_s}|0\rangle$$

where

$$[\bar{a}^\mu, a^\nu] = \eta^{\mu\nu} \qquad \mu,\nu = 0,1,2 \qquad \bar{a}^\mu|0\rangle = 0$$

Conservation and traceless conditions are now

$$\bar{a}^\nu\partial_\nu|J_s(t,\vec{x},a^\mu)\rangle \;=\; 0 \qquad \bar{a}^\nu\bar{a}_\nu|J_s(t,\vec{x},a^\mu)\rangle \;=\; 0$$

Symmetries in the reduced theory

Conservation and traceless conditions are now

$$\bar{a}^{\nu}\partial_{\nu}|J_s(t,\vec{x},a^{\mu})\rangle = 0 \qquad \bar{a}^{\nu}\bar{a}_{\nu}|J_s(t,\vec{x},a^{\mu})\rangle = 0$$

Conservation eliminates a polarization of current. Eliminating + polarizations we have

$$|J_{(s)}\rangle = \exp\left(-a^{+}\left[\frac{\bar{a}^{+}\partial^{-}+\bar{a}^{b}\partial^{b}}{\partial^{+}}\right]\right)|i_{(s)}\rangle \equiv \mathcal{P}|i_{(s)}\rangle$$

where

$$|i_{(s)}\rangle = j_{(s)}^{i_1 i_2\cdots i_s}a_{i_1}a_{i_2}\cdots a_{i_s}|0\rangle \qquad i_k = -, b$$

Operators O acting on the original currents $|J_{(s)}\rangle$ become $\tilde{O} = \mathcal{P}^{-1}\,O\,\mathcal{P}$.

Symmetries in the reduced theory

Computing $\tilde{O} = \mathcal{P}^{-1}\, O\, \mathcal{P}$ for the symmetry generators we find

$$\tilde{J}^{+-} = \mathcal{P}^{-1} J^{+-} \mathcal{P} = x^{+} \frac{\partial}{\partial x^{+}} - x^{-} \frac{\partial}{\partial x^{-}} - a^{-} \frac{\partial}{\partial a^{-}}$$

$$\tilde{J}^{+i} = \mathcal{P}^{-1} J^{+i} \mathcal{P} = x^{+} \frac{\partial}{\partial x^{i}} - x^{i} \frac{\partial}{\partial x^{-}} - a^{i} \frac{\partial}{\partial a^{-}}$$

$$\tilde{J}^{-i} = \mathcal{P}^{-1} J^{-i} \mathcal{P} = x^{-} \frac{\partial}{\partial x^{i}} - x^{i} \frac{\partial}{\partial x^{+}} + a^{-} \frac{\partial}{\partial a^{i}} + a^{i} \frac{\bar{a}^{b} \partial^{b}}{\partial^{+}}$$

and

$$\tilde{J}^{ij} = \mathcal{P}^{-1} J^{ij} \mathcal{P} = x^{i} \partial^{j} - x^{j} \partial^{i} + a^{i} \bar{a}^{j} - a^{j} \bar{a}^{i} \tag{1}$$

The $+$ polarizations have indeed been eliminated. (No $\frac{\partial}{\partial a^{+}}$ or a^{+} and generators close correct algebra)

Equal time bilocal fields

Phrase the dynamics in terms of a bilocal field. From OPE: bilocal packages the complete set of single trace primary operators

$$\begin{aligned} \sigma(t_1,\vec{x}_1,t_2,\vec{x}_2) &= \sum_{a=1}^{N}\phi^a(t_1,\vec{x}_1)\phi^a(t_2,\vec{x}_2) \\ &= \sum_{s=0}^{\infty}\sum_{d=0}^{\infty} c_{sd}\left(y^\mu\frac{\partial}{\partial x^\mu}\right)^d y_{\mu_1}\cdots y_{\mu_{2s}} j_{(2s)}^{\mu_1\cdots\mu_{2s}}(x) \end{aligned}$$

where we have the coordinates

$$x^\mu = \frac{1}{2}(x_1^\mu + x_2^\mu) \qquad y^\mu = \frac{1}{2}(x_1^\mu - x_2^\mu)$$

Equal x^+ bilocal $\sigma(x^+,x_1^-,x_1,x_2^-,x_2)=\sum_a\phi^a(x^+,x_1^-,x_1)\phi^a(x^+,x_2^-,x_2)$ has $y^+=0$

$\Rightarrow$ packages only currents with $-$ and x polarizations.

Using known transformation rule of scalar field and the OPE, we verify symmetries are implemented as in the reduced theory obtained by eliminating $+$ polarizations.

The equal x^+ bilocal theory provides the reduction of the CFT to independent fields.

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Basic claim: Holography is accomplished by a change to gauge invariant (bilocal) field variables in the CFT. A change of spacetime coordinates is needed to give the bulk interpretation of the bilocal collective field theory.

Das and Jevicki, hep-th/0304093

Starting from the CFT and carrying out these two steps, one obtains the higher dimensional gravitational theory. In this way, bilocal holography is a constructive approach towards demonstrating AdS/CFT.

There is a clear motivation for both steps.

Bilocal Holography

Basic claim: Holography is accomplished by a change to gauge invariant (bilocal) field variables in the CFT. A change of spacetime coordinates is needed to give the bulk interpretation of the bilocal collective field theory.

Das and Jevicki, Phys. Rev. D **68** (2003), 044011

The loop expansion parameter of the original CFT is $\hbar$. After changing to invariant (bilocal) variables the loop expansion parameter is $1/N$ matching the loop expansion parameter of the dual gravity. Example of **collective field theory**.

Jevicki and Sakita, Nucl. Phys. B **165** (1980), 511

The bilocal transforms in a tensor product. The complete collection of higher spin fields transform in a direct sum. The natural change of basis

$$V_{[\frac{1}{2},0]} \otimes V_{[\frac{1}{2},0]} \longrightarrow V_{[1,0]} \oplus \bigoplus_{s=2,4,\cdots} V_{[s+1,s]}$$

determines a map between CFT and bulk coordinates.

(Think of addition of angular momentum: $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$.)

Change to Bilocal Field Variables

To solve QFT "all" we need to do is evaluate a complicated integral.

$$\int d\phi^a e^{-S(\phi^a)} \qquad a = 1, \cdots, N$$

Hard to do when $N \to \infty$, but things simplify when the theory has an $O(N)$ symmetry, so the action is an $O(N)$ invariant.

Suppose the ϕ^a are in vector rep of $O(N)$. Then S is a function of $\sigma = \phi^a \phi^a$, the unique invariant. One integration variable and not N - much simpler!

$$\int d\sigma e^{-N\tilde{S}(\sigma)}$$

N appears because we had a total of N variables. Saddle point approximation produces an expansion with $1/N$ the loop counting parameter.

2+1 Minkowski in light cone coordinates $x^+ \equiv x^0 + x^2$, $x^- \equiv x^2 - x^0$, $x \equiv x^1$. Invariant variables are equal x^+ bilocals

$$\sigma(x^+, x_1^-, x_1, x_2^-, x_2) = \sum_{a=1}^{N} \phi^a(x^+, x_1^-, x_1)\phi^a(x^+, x_2^-, x_2)$$

Change to Bilocal Field Variables

2+1 Minkowski in light cone coordinates $x^+ \equiv x^0 + x^2$, $x^- \equiv x^2 - x^0$, $x \equiv x^1$. Invariant variables are equal x^+ bilocals

$$\sigma(x^+, x_1^-, x_1, x_2^-, x_2) = \sum_{a=1}^{N} \phi^a(x^+, x_1^-, x_1)\phi^a(x^+, x_2^-, x_2)$$

The field $\sigma(x^+, x_1^-, x_1, x_2^-, x_2)$ develops a large N expectation value. It is the fluctuation $\eta(x^+, x_1^-, x_1, x_2^-, x_2)$ about this large N background that maps to bulk AdS fields.

$$\sigma(x^+, x_1^-, x_1, x_2^-, x_2) \quad = \quad \sigma_0(x^+, x_1^-, x_1, x_2^-, x_2) + \frac{1}{\sqrt{N}}\eta(x^+, x_1^-, x_1, x_2^-, x_2)$$

$\sigma_0(x^+, x_1^-, x_1, x_2^-, x_2)$ is the large N two point function.

RdMK, Jevicki, Jin and Rodrigues, Phys. Rev. D **83** (2011) 025006.

Change of Spacetime Co-ordinates

The bilocal transforms in $V_{\frac{1}{2},0} \otimes V_{\frac{1}{2},0}$ $\qquad (L^A \in \mathrm{so}(2,3))$

$$L^A_\otimes \sigma \quad = \quad \Big(L^A \phi^a(x^+, x_1^-, x_1)\phi^a(x^+, x_2^-, x_2) + \phi^a(x^+, x_1^-, x_1) L^A \phi^a(x^+, x_2^-, x_2) \Big)$$

$$V_{\frac{1}{2},0} \otimes V_{\frac{1}{2},0} = V_{1,0} \oplus \bigoplus_{s=2,4,6,\cdots} V_{s+1,s}$$

The complete collection of higher spin fields fill out the reducible representation $V_{1,0} \oplus \bigoplus_{s=2,4,6,\cdots} V_{s+1,s}$

$$L^A_\oplus \Phi(X^+, X^-, X, Z, \theta) = \sum_{s=0}^{\infty} \left(\cos(2s\theta) L^A_{2s} \frac{A^{XX\cdots XX}}{Z} + \sin(2s\theta) L^A_{2s} \frac{A^{XX\cdots XZ}}{Z} \right)$$

We want to change from the natural representation ($L^A_\otimes$) of the CFT to the representation that is natural for the bulk gravity ($L^A_\oplus$).

Change of Spacetime Coordinates

Bilocal field $\eta(x^+, x_1^-, x_1, x_2^-, x_2)$. 5 coordinates in CFT: $x^+, x_1^-, x_1, x_2^-, x_2$.

Higher spin gravity field $\Phi(X^+, X^-, X, Z, \theta)$. 5 coordinates in gravity: X^+, X^-, X, Z, θ.

Symmetry: $X^- \to X^- + a$ in gravity and $x^- \to x^- + b$ in CFT motivates the Fourier transform:

$$\eta(x^+, p_1^+, x_1, p_2^+, x_2) = \int \frac{dx_1^-}{2\pi} \int \frac{dx_2^-}{2\pi} \, \eta(x^+, x_1^-, x_1, x_2^-, x_2) \, e^{-ip_1^+ x_1^- - ip_2^+ x_2^-}$$

5 coordinates in CFT: $x^+, p_1^+, x_1, p_2^+, x_2$.

$$\Phi(X^+, P^+, X, Z, \theta) = \int \frac{dX^-}{2\pi} \, \Phi(X^+, X^-, X, Z, \theta) \, e^{-iP^+ X^-}$$

5 coordinates in gravity: X^+, P^+, X, Z, θ.

Change of Spacetime Coordinates

Bilocal field $\eta(x^+, x_1^-, x_1, x_2^-, x_2)$. 5 coordinates in CFT: $x^+, x_1^-, x_1, x_2^-, x_2$.

Higher spin gravity field $\Phi(X^+, X^-, X, Z, \theta)$. 5 coordinates in gravity: X^+, X^-, X, Z, θ.

$$x_1 = X + Z\tan\left(\frac{\theta}{2}\right) \qquad x_2 = X - Z\cot\left(\frac{\theta}{2}\right) \qquad x^+ = X^+$$

$$p_1^+ = P^+\cos^2\left(\frac{\theta}{2}\right) \qquad p_2^+ = P^+\sin^2\left(\frac{\theta}{2}\right)$$

$$X = \frac{p_1^+ x_1 + p_2^+ x_2}{p_1^+ + p_2^+} \qquad Z = \frac{\sqrt{p_1^+ p_2^+}(x_1 - x_2)}{p_1^+ + p_2^+}$$

$$P^+ = p_1^+ + p_2^+ \qquad \theta = 2\tan^{-1}\left(\sqrt{\frac{p_2^+}{p_1^+}}\right)$$

$$L_\oplus^A \Phi = 2\pi P^+ \sin\theta \; L_\otimes^A \eta$$

RdMK, Jevicki, Jin and Rodrigues, Phys. Rev. D **83** (2011) 025006.

Summary: Bilocal Holography

$$
\begin{aligned}
\sigma(x^+, x_1^-, x_1, x_2^-, x_2) &= \sum_{a=1}^{N} \phi^a(x^+, x_1^-, x_1)\phi^a(x^+, x_2^-, x_2) \\
&= \sigma_0(x^+, x_1^-, x_1, x_2^-, x_2) + \frac{1}{\sqrt{N}}\eta(x^+, x_1^-, x_1, x_2^-, x_2)
\end{aligned}
$$

$$
X = \frac{p_1^+ x_1 + p_2^+ x_2}{p_1^+ + p_2^+} \qquad Z = \frac{\sqrt{p_1^+ p_2^+}(x_1 - x_2)}{p_1^+ + p_2^+} \qquad X^+ = x^+
$$

$$
P^+ = p_1^+ + p_2^+ \qquad \theta = 2\tan^{-1}\left(\sqrt{\frac{p_2^+}{p_1^+}}\right)
$$

$$
\begin{aligned}
\Phi &= \sum_{s=0}^{\infty}\left(\cos(2s\theta)\frac{A^{XX\cdots XX}(X^+, P^+, X, Z)}{Z} + \sin(2s\theta)\frac{A^{XX\cdots XZ}(X^+, P^+, X, Z)}{Z}\right) \\
&= 2\pi P^+ \sin\theta\ \eta(X^+, P^+\cos^2\frac{\theta}{2}, X + Z\tan\frac{\theta}{2}, P^+\sin^2\frac{\theta}{2}, X - Z\cot\frac{\theta}{2})
\end{aligned}
$$

We are studying the free O(N) vector model. Its a UV fixed point.

By perturbing and flowing we can reach an IR fixed point. The bilocal holography for this fixed point has also been worked out. Mulokwe and Rodrigues, JHEP 11 (2018) **047** and Johnson, Mulokwe and Rodrigues, Phys. Lett. B **829** (2022) 137056.

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Bulk Reconstruction

Does the proposed bulk field $\Phi(X^+, P^+, X, Z, \theta)$ obey the correct bulk equation of motion with the correct boundary condition? CFT equation of motion:

$$\left(\frac{\partial}{\partial x^+}\frac{\partial}{\partial x^-} + \frac{\partial^2}{\partial x^2}\right)\phi^a(x^+, x^-, x) = 0$$

implies

$$\left(\frac{\partial}{\partial X^+}\frac{\partial}{\partial X^-} + \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Z^2}\right)\Phi\left(X^+, X^-, X, Z, \theta\right) = 0$$

$$(p_1^+ + p_2^+)^s \cos\left(2s\tan^{-1}\sqrt{\frac{p_2^+}{p_1^+}}\right) = \mathcal{N}\sum_{k=0}^{s}\frac{(-1)^k(p_1^+)^{s-k}(p_2^+)^k}{\Gamma\left(s-k+\frac{1}{2}\right)\Gamma\left(k+\frac{1}{2}\right)k!(s-k)!}$$

implies that ($\mathcal{N} = \Gamma\left(\frac{1}{2}\right)s!\Gamma\left(s+\frac{1}{2}\right)$; recall $\Phi \sim \cos(2s\theta)\frac{A_{x\cdots x}}{Z} + \sin(2s\theta)\frac{A_{x\cdots xz}}{Z}$)

$$\frac{\partial^s}{\partial X^{-s}}\Phi_s(X^+; X^-, X, 0) = 16\pi\mathcal{N}\sum_{k=0}^{s}\frac{(-1)^k\partial_-^{s-k}\phi^a(X^+, X^-, X)\partial_-^k\phi^a(X^+, X^-, X)}{\Gamma\left(s-k+\frac{1}{2}\right)\Gamma\left(k+\frac{1}{2}\right)k!(s-k)!}$$

Consistency with GKPW dictionary

GKPW dictionary is formulated in de Donder gauge: $D^A H_{AA_2\cdots A_{2s}} = 0$. Residual gauge symmetry can be used to make $H_{A_1 A_2\cdots A_{2s}}$ traceless.

From e.o.m. near $Z = 0$ we have (M is a boundary index - does not take Z values)

$$H_{M_1\cdots M_{2s}} \sim Z^{2-2s} B_{M_1\cdots M_{2s}}(X^+, X^-, X) + Z^{1+2s} A_{M_1\cdots M_{2s}}(X^+, X^-, X)$$

$$\begin{aligned} H_{M_1\cdots M_{2s-k}Z\cdots Z} \quad \sim \quad & Z^{2-2s-k} B_{M_1\cdots M_{2s-k}Z\cdots Z}(X^+, X^-, X) \\ & + Z^{1+2s+k} A_{M_1\cdots M_{2s-k}Z\cdots Z}(X^+, X^-, X) \end{aligned}$$

GKPW says:

$$j_{M_1,M_2\cdots M_{2s}} \propto A_{M_1,M_2\cdots M_{2s}}$$

CFT operator is related to component of boundary field falling off as Z^{1+2s} and the relation is local!

Consistency with GKPW dictionary

Bilocal holography is formulated in light cone gauge and not de Donder gauge.

Transform to lightcone gauge: $H'_{A_1A_2\cdots A_{2s}} = H_{A_1A_2\cdots A_{2s}} - D_{(A_1}\Lambda_{A_2\cdots A_{2s})}$

Requiring $H'_{+A_2\cdots A_{2s}} = 0$ fixes the gauge parameter $\Lambda_{A_1\cdots A_{2s-1}}$.

Example: Spin $2s$ After the gauge transformation, GKPW says

$$A'_{XXX\cdots X} = -A'_{ZZX\cdots X} \propto Z\partial_-^{-2s} j_{--\cdots--}$$

$$\Rightarrow \partial_-^{2s}\frac{A'_{XXX\cdots X}}{Z} = -\partial_-^{2s}\frac{A'_{ZZX\cdots X}}{Z} \propto j_{--\cdots--}$$

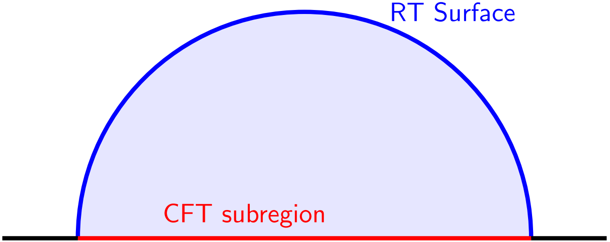
so bilocal holography gives a correct bulk reconstruction.

Mintun and Polchinski, arXiv:1411.3151

Subregion Duality

Which subregion of the CFT (if any) is dual to a given subregion of the bulk?

Entanglement wedge reconstruction claims that everyting from the boundary up to the RT surface can be reconstructed.



Subregion Duality

Which subregion of the CFT (if any) is dual to a given subregion of the bulk?

Consider localized CFT excitations, at time x^+, with the first at (x_1, p_1^+) and the at (x_2, p_2^+), described as wavepackets, tightly peaked at x_1 and x_2 along direction x transverse to the light cone, and smeared along x^-.

$$\left(X - \frac{x_1 + x_2}{2}\right)^2 + Z^2 = \left(\frac{x_1 - x_2}{2}\right)^2$$

The bulk excitation is on a semicircle in the X, Z plane, with radius $(x_1 - x_2)/2$ and center $X = (x_1 + x_2)/2$ and $Z = 0$. To locate the excitation specify angle θ

$$\tan\theta = \frac{Z}{X - \frac{x_1 + x_2}{2}} = \frac{2\sqrt{p_1^+ p_2^+}}{p_1^+ - p_2^+}$$

This angle θ is the angle θ appearing in the map.

Subregion Duality

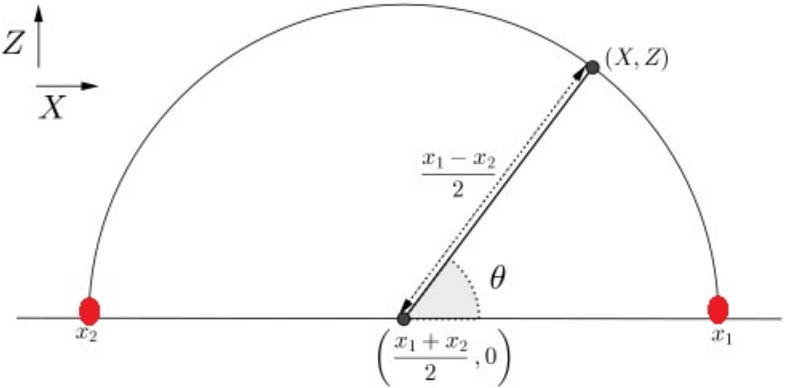


Figure: The bilocal describing excitations localized at (x_1, p_1^+) and (x_2, p_2^+) corresponds to a bulk excitation localized at (X, Z) as shown. This figure lives on a constant $x^+ = X^+$ slice. The angle θ is $\theta \;=\; 2\tan^{-1}\left(\sqrt{\frac{p_2^+}{p_1^+}}\right)$.

Bulk Reconstruction

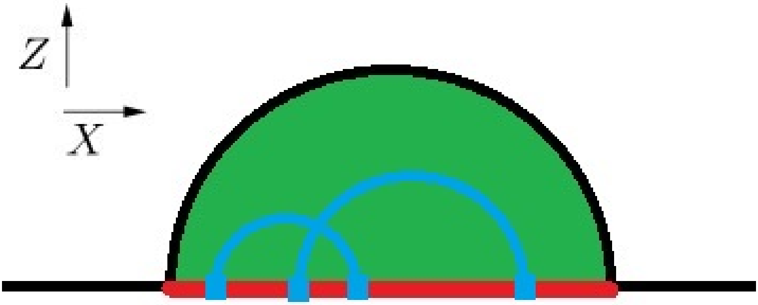


Figure: Using bilocals restricted to the red region of the CFT we are able to reconstruct bulk field living in the area shaded in green. This figure is for fixed time $x^+ = X^+$.

What is the interpretation of the boundary of the green region? The boundary is a geodesic so that the green region is reminiscent of the entanglement wedge.

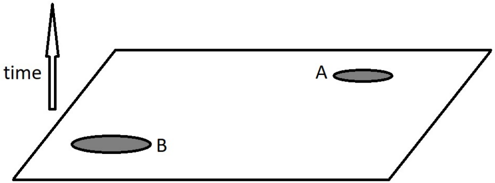
Principle of the holography of information and Locality

Locality is a cherished principle in physics.

Relativistic causality says no information carrying signal propagates faster than light. Implemented by ensuring spacelike separated fields commute.

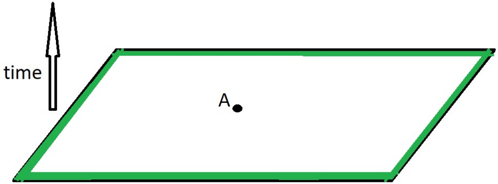
Ensures that laboratories in spacelike separated regions function independently.

Formalized in algebraic formulation of QFT as the split property: we can specify the state of quantum fields independently on different parts of a Cauchy slice.



The Principle of the Holography of Information

In a theory of quantum gravity, a copy of all the information available on a Cauchy slice is also available near the boundary of the Cauchy slice. This redundancy in description is already visible in the low-energy theory.



Laddha, Prabhu, Raju, Shrivastava, arXiv:2002.02448;
Chowdhury, Papadoulaki, Raju, arXiv:2008.01740;
Raju, arXiv:2110.05470.

The Principle of the Holography of Information

In a theory of quantum gravity, a copy of all the information available on a Cauchy slice is also available near the boundary of the Cauchy slice. This redundancy in description is already visible in the low-energy theory.

Changing the values of fields within R changes field values in the green band. The split property fails.

Raju, arXiv:2110.05470.

Goal and Comments

The principle of the holography of information is pointing out an unusual localization of quantum information in quantum gravity.

AdS/CFT gives non-perturbative definition of quantum gravity on negatively curved spacetime, as CFT. Can we see the principle of the holography of information in CFT? Use AdS/CFT to map the principle of the holography of information into a statement about CFT and explore it using only CFT.

In AdS/CFT is the holography of information trivially true? The principle is a statement about quantum gravity itself. The proof does not invoke AdS/CFT and it holds (for example) for flat spacetime.

Location of single trace primaries

Where do single trace primaries map to in the AdS$_4$ bulk? ($Z = \frac{\sqrt{p_1^+ p_2^+}(x_1 - x_2)}{p_1^+ + p_2^+}$)

Scalar primary $= \phi^a(x^+, x^-, x)\phi^a(x^+, x^-, x)$ i.e. located on boundary $Z = 0$.

Conserved currents are

$$
\begin{aligned}
J_s(x^+, x^-, x, \alpha) &= J_{\mu_1\mu_2\cdots\mu_s}(x^+, x^-, x)\alpha^{\mu_1}\alpha^{\mu_2}\cdots\alpha^{\mu_s} \\
&= \sum_{k=0}^{s} \frac{(-1)^k \; :(\alpha\cdot\partial)^{s-k}\phi^a(x^+, x^-, x)\;(\alpha\cdot\partial)^k\phi^a(x^+, x^-, x):}{k!(s-k)!\Gamma(k+\frac{1}{2})\Gamma(s-k+\frac{1}{2})} \\
&= \sum_{k=0}^{s} \frac{(-1)^k (\alpha\cdot\partial_1)^{s-k}\,(\alpha\cdot\partial_2)^k}{k!(s-k)!\Gamma(k+\frac{1}{2})\Gamma(s-k+\frac{1}{2})}\eta(x^+, x_1^-, x_1, x_2^-, x_2)\Big|_{x_1=x_2=x,\, x_1^-=x_2^-=x^-}
\end{aligned}
$$

To construct spinning currents, separate x_1 and x_2 by a small amount ϵ, evaluate the derivatives and then send $x_2 \to x_1$. Take $|x_1 - x_2| < \epsilon$ where ϵ can be arbitrarily small.

Location of single trace primaries

Where do single trace primaries map to in the AdS_4 bulk?

$$Z = \frac{\sqrt{p_1^+ p_2^+}(x_1 - x_2)}{p_1^+ + p_2^+}$$

p_1^+ and p_2^+ are both positive $\Rightarrow$ the ratio $0 < \frac{\sqrt{p_1^+ p_2^+}}{p_1^+ + p_2^+} < 1$.

$$|x_1 - x_2| < \epsilon \qquad \Rightarrow \qquad Z < \epsilon$$

where ϵ can be arbitrarily small.

The complete set of single trace primary operators, after mapping to the dual gravity, are supported in an arbitrarily small neighbourhood of the boundary.

Operators deep in the bulk

Which CFT bilocals map to operators localized deep in the bulk of AdS_4?

$$Z = \frac{\sqrt{p_1^+ p_2^+}(x_1 - x_2)}{p_1^+ + p_2^+}$$

p_1^+ and p_2^+ are both positive $\Rightarrow$ the ratio $0 < \frac{\sqrt{p_1^+ p_2^+}}{p_1^+ + p_2^+} < 1$.

To obtain a large value for Z we must consider a large separation between the two fields in the bilocal, i.e. $x_1 - x_2$ must be large.

Holography of information and OPE

HOI predicts bulk operators can be expressed as elements of the boundary algebra.

Single trace primaries are supported in arbitrarily small neighbourhood of the boundary.

By separating x_1 and x_2 to be arbitrarily distant, the bilocal $\eta(x^+, x_1^-, x_1, x_2^-, x_2)$ is located arbitrarily deep in the bulk ($Z = \frac{\sqrt{p_1^+ p_2^+}(x_1 - x_2)}{p_1^+ + p_2^+}$).

HOI is true if we can replace the bilocal field $\eta(x^+, x_1^-, x_1, x_2^-, x_2)$ by a sum of single trace primaries $\mathcal{O}^{\mu_1 \cdots \mu_{2s}}(x^\mu)$. This is exactly what the OPE does!

$$
\begin{aligned}
\eta(x^+, x_1^-, x_1, x_2^-, x_2) &= \,:\phi^a(x^+, x_1^-, x_1)\phi^a(x^+, x_2^-, x_2): \\
&= \sum_s C(x_1^- - x_2^-, x_1 - x_2)_{\mu_1 \cdots \mu_{2s}} J^{\mu_1 \cdots \mu_{2s}}(x^+, x_1^-, x_1)
\end{aligned}
$$

Burden of the proof is to show the OPE converges.

(Pappadopulo, Rychkov, Espin, Rattazzi, arXiv:1208.6449, Qiao, arXiv:2005.09105)

Discussion and Future Directions

Using bilocal holography (collective field theory) we have constructed a higher dimensional gravitational theory.

The resulting holographic map provides a valid bulk reconstruction, a detailed example of entanglement wedge reconstruction and exhibits the principle of holography of information.

Convincing indications that bilocal holography is indeed constructing the quantum gravity dual to the original conformal field theory.

For matrix theories, many more gauge invariant variables. For this case collective field theory applied to a single matrix quantum mechanics provides a string field theory for the $c = 1$ string. Das, Jevicki, Mod. Phys. Lett. A **5** (1990) 1639-1650.

For the free massless matrix in $2 + 1$ dimensions, bilocal, trilocal and in general multi-local operators appear. OPE can be used to take the product of separated operators and express them in terms of local operators. Non-trivial new examples with which to work out the collective map. (Work in progress with Jaco Van Zyl and Pratik Roy)

Thanks for your attention!