ESI Erwin Schrödinger International Institute for Mathematics and Physics

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Workshop on

Elliptic Hypergeometric Functions in Combinatorics, Integrable Systems and Physics

March 20–24, 2017

ABSTRACTS OF POSTERS

(poster session on Tuesday, March 21, 2017, 15:40–16:40)

Farrokh Atai and Edwin Langmann (KTH Royal Institute of Technology)

Series solutions of the non-stationary Heun equation

ABSTRACT: We present recent results for constructing particular solutions of the non-stationary Heun equation, also known as quantum Painlevé VI, which is a one-parameter generalization of the Heun equation. These solutions can be regarded as an elliptic generalization of the Jacobi polynomials. Our approach is based on kernel functions and yields series representations of these solutions. We also present special cases where these solutions reduce to simple explicit integrals, generalizing known integral representations of Jacobi polynomials to the elliptic case.

Tamás F. Görbe (University of Szeged)

Elliptic Ruijsenaars–Schneider models on the complex projective space

ABSTRACT: We construct elliptic Ruijsenaars–Schneider models whose completed center-of-mass phase space is the complex projective space with the Fubini–Study symplectic form. For n particles, these models are labelled by an integer $p \in \{1, \dots, n-1\}$ relative prime to n and a coupling parameter y varying in a certain punctured interval around $p\pi/n$. Our work extends Ruijsenaars's pioneering study of compactifications that imposed the restriction $0 < y < \pi/n$, and also builds on an earlier derivation of such compactified models with trigonometric potential by Hamiltonian reduction. This is a joint work with László Fehér.

Martin Hallnäs (Chalmers University of Technology)

On the spectra of complex Lamé operators

ABSTRACT: We present recent results (obtained with William Haese-Hill and Alexander Veselov) on the spectra of complex Lamé operators, i.e. Schrödinger operators with a potential given by the Weierstrass $\wp$-function, depending on a period lattice and an integer m (which in the real case corresponds to the number of

bands in the spectrum). We concentrate on the simplest non-trivial case $m = 1$. Under certain non-degeneracy and non-singularity conditions, we prove that the spectrum consists of precisely two non-intersecting regular analytic arcs. We also provide example of singular cases, with intersecting curves, and establish the existence of precisely one degenerate case, in which the spectrum has a tripod shape, and is a union of three rather than the expected two simple regular analytic arcs.

Nalini Joshi and Nobutaka Nakazono (The University of Sydney)

An elliptic Painlevé equation from next-nearest-neighbor translation on the $E_8^{(1)}$ lattice

ABSTRACT: Elliptic Painlevé equations head the list of the differential and discrete Painlevé equations. The well known elliptic Painlevé equation is given by a nearest neighbor vector on the $E_8^{(1)}$ weight lattice. In this poster, we present an elliptic Painlevé equation, which is obtained by a next-nearest-neighbor vector. We also show that its projective reduced equation is the elliptic difference equation found by Ramani, Carstea and Grammaticos in 2009 from the reduction of the discrete analogue of the Krichever–Novikov equation.

Andrew Kels (University of Tokyo)

Elliptic hypergeometric sum/integrals

ABSTRACT: Four-dimensional supersymmetric indices calculated on a lens space, are expressed in the form of a "sum/integral", which are generalisations of elliptic hypergeometric integrals involving extra summation over some discrete variables. This poster will present the corresponding sum/integral generalisation of Spiridonov's elliptic beta integral, which depends on both integer and complex variables m_i, t_i respectively, where $i = 1, \ldots, 6$. This elliptic beta sum/integral (also called rarefied elliptic beta integral) is expressed in terms of the "lens elliptic gamma function", a generalisation of the elliptic gamma function that depends on an extra integer parameter $r = 1, 2, \ldots$ (where $r = 1$ is the usual elliptic gamma function), and an extra integer variable m mod r. This poster will also discuss connections to the Yang–Baxter equation, which in the general case should correspond to a sum/integral analogue of Rains' A_n transformation formula.

Katarina Kukić (University of Belgrade)

Separation of variables based on discriminantly separable polynomials in some integrable generalizations of Kowalevski top

ABSTRACT: In the few recent papers, starting from the notion of discriminantly separable polynomials of degree two in each of three variables, we proposed separation of variables for few well known integrable dynamical systems. These systems can be integrated explicitly in genus two theta-functions in a procedure which is similar to the classical one for the Kowalevski top. The discriminantly separable polynomials play the role of the Kowalevski fundamental equation. The natural examples include the Sokolov systems and the Kowalevski-type top in a double constant force field. Results are joined work with Vladimir Dragović.

Jules Lamers (Chalmers University of Technology)

Constructive method for the elliptic solid-on-solid model with domain walls and a reflecting end

ABSTRACT: We review the constructive approach devised by Galleas and further developed by Galleas and the author. The simplest example concerns the computation of the partition function for the six-vertex model with domain-wall boundaries. At the heart of the method lies a linear functional equation for the partition function. This equation contains Korepin's recurrence relation as a special case. At the same it provides a recipe for obtaining a closed expression in the form of a symmetrized sum, which can be rewritten to recover Izergin's determinant. The method is quite general: it also works for the elliptic solid-on-solid model with domain walls and a reflecting end that was studied by Filali and Kitanine.

Takao Suzuki (Kindai University)

A generalization of the q-Painlevé VI equation from a viewpoint of a basic hypergeometric solution

ABSTRACT: In this poster we propose a generalization of Jimbo–Sakai's q-Painlevé VI equation from a viewpoint of Heine's basic hypergeometric function.

Tomoyuki Takenawa (Tokyo University of Marine Science and Technology)

From the QRT maps to elliptic difference Painlevé equations

ABSTRACT: It is well known that two-dimensional mappings preserving a rational elliptic fibration, like the Quispel–Roberts–Thompson mappings, can be deautonomized to discrete Painlevé equations. However, the dependence of this procedure on the choice of a particular elliptic fiber has not been sufficiently investigated. In this presentation we establish a way of performing the deautonomization for a pair of an autonomous mapping and a fiber. Especially, in the case where the fiber is smooth elliptic, imposing certain restrictions on such non-autonomous mappings, we obtain new and simple elliptic difference Painlevé equations, including examples whose symmetry groups do not appear explicitly in Sakai's classification. (Joint work with A. S. Carstea and A. Dzhamay)
