Revisiting the Okubo–Marshak argument

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*"Even in the true sciences, deep intellectual investment
in what turns out to be fallacy is not easily given up"*

– Robert Conquest

Synopsis

1 The "strong CP problem" and the original counter-argument

2 String-local fields

3 Epstein–Glaser treatment of string-local fields

4 A semantic whose time is gone?

5 On quantum entanglement

6 Chasing (in the) ALPs

An apparent concern in QCD

Experiment, particularly the smallness of the neutron's electric dipole moment, tells us that QCD is **P**-, **C**- and thus **T**-invariant to an extremely good approximation.

45 years ago it was argued that this is **unnatural** and, true to form, in 1977 a paper was received by PRL from F. Wilczek, positing a new particle within the SM framework: the "axion". Till now it has eluded all searches.

The so-called θ-vacuum

The hypothetical axion is directly related to the so-called θ-vacuum, an angle θ describing, according to some, the "true" vacuum of QCD.

I won't tarry on the original arguments, based on the standard "gauge potential" $A^\mu(x)$ for QED or QCD, which is *not* Lorentz-covariant, **not** positive, **not** local – par-for-the-course in attempting to describe a two-degrees-of-freedom particle by means of a four-component object.

The standard orthodoxy

It goes through instantons (Euclidean denizens, first detected by Belavin, Polyakov, Schwartz and Tyupkin) and "vacuum tunnelling" into topologically inequivalent vacua, apparently leading to the degenerate θ-vacuum and, according to some, to the "axion".

The vacuum states in Minkowski space are so defined that instantons become "tunnelling events" between different Minkowski vacua $|m\rangle$, $|m'\rangle$, the winding numbers m, m' satisfying $m' - m = n$.

The "true vacuum" is of the form $|\theta\rangle = \sum_m e^{-im\theta} |m\rangle$, it is then argued. The value of θ is anyone's guess.

A bridge too far

Even within the standard formalism, this is *a bridge too far*. In 1992, on the basis of the rigorous, covariant Kugo–Ojima formalism for Yang–Mills theories, Okubo and Marshak proved that the **BRS charge** "kills" the physical vacuum, which if cyclic must be unique. But that charge and the anti-unitary operator for CPT invariance **commute** – and this demands the zero (or π) value for the θ-parameter of the makeup.

Marshak's (paper's and) book's verdict

In his posthumous book on the SM (late 1992, finished a few months after the article with Okubo), Marshak concluded:

> "It does seem that the odds of finding the 'invisible' axion are rapidly diminishing and that the incentive to carry on the ingenious searches for the 'invisible' axions **is fueled more by astrophysical and cosmological interest** than by any hope of salvaging the Peccei–Quinn-type solution of the 'strong CP problem' in QCD."

A bridge too far

The conclusion in "Conceptual Foundations of Modern Particle Physics" rings even truer 30 years later!

Despite (or because of) this, the Okubo–Marshak (OM) argument is not even noticed by many works on the axion, which prefer to deal with "straw men" – see for instance the otherwise competent review by di Luzio et al, *arXiv:2003.01100*.

What I dwell into

I do **not** intend to go through the details of the OM argument (nor of what could be termed the "axion industrial complex"). I propose to learn what string-local fields (SLF) tell us about QCD, about the OM conclusion, and about the future (or lack of it) for geometric gauge field theory in QFT.

I will call upon for this the Epstein–Glaser theory of renormalization without regularization. What follows is joint work with Christian Gaß and Jens Mund, *arXiv:2108.01792*.

And so the plan is...

- Basic theory of string-local fields (they do many things for us that I have no time to delve into).

- **Deriving QCD** from string-independence in the S-matrix.

- Other consequences of string independence.

- Obtaining anew the OM conclusion.

- Thoughts on the future.

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SLF basics

The string-local (true) vector potential $A_\mu(x, e)$, where e is a spacetime ray, is directly constructed from the field strength:

$$A_\mu(x, e) = \int_0^\infty dt\, F(x + te)_{\mu\nu}\, e^\nu \implies$$
$$\partial_\mu A_\nu(x, e) - \partial_\nu A_\mu(x, e) = F_{\mu\nu}(x).$$

The vector potential $A(x, e)$ fulfils $e^\mu A_\mu(x, e) = 0$ and $\partial^\mu A_\mu(x, e) = 0$. They *reduce* the number of degrees of freedom, as required for on-shell photons or gluons – just as the components of F reduced by the Maxwell equations propagate two degrees of freedom.

A true homomorphism

Anticipating events: since $F^{\mu\nu}(x) \downarrow 0$ moderately fast at spatial infinity already implies $A^\mu(x, e)$ to be bounded, there are no "pure gauge configurations" in QED or QCD in SLF theory.

As well: the field $A(x, e)$ *lives on the same Fock space* as $F(x)$. Thus the equations above are *operator relations* – at variance with the usual framework.

Properties of $A(x, e)$

Note that $A_\mu(x, e)$ is truly covariant: for Λ a Lorentz transformation, c a translation, and U the second quantization of that unirrep pair:

$$U(c, \Lambda) A_\mu(x, e) U^\dagger(c, \Lambda) = (\Lambda^{-1})^\lambda_\mu A_\lambda(\Lambda x + c, \Lambda e);$$
$$\text{moreover,} \quad [A_\mu(x, e), A_\nu(x', e')] = 0,$$

whenever the strings $x + \mathbb{R}^+ e$ and $x' + \mathbb{R}^+ e'$ are space-like separated.

And, as said, A^μ lives on a Hilbert space.

Enter the scattering matrix

A Bogolyubov-type functional scattering operator $\mathbb{S}[g;h]$, dependent on a multiplet g of external fields and a test function h averaging over the string directions, is introduced as a formal power series in g, obeying the conditions of causality, unitarity and covariance:

$$\mathbb{S}[g;h] := \sum_{n=0}^{\infty} \frac{i^n}{n!} \prod_{k=1}^{n} \prod_{l=1}^{m} \int d^4 x_k$$

$$\times \int d\sigma(e_{k,l})\, g(x_k) h(e_{k,l}) S_n(x_1, e_1; \ldots; x_n, e_n).$$

String independence fixes the couplings (!)

Only the first-order model $S_1 = S_1(x, e)$, a Wick polynomial in the free fields, is postulated – already under severe restrictions. It depends on an array $e = (e_1, ..., e_m)$ of string coordinates, with m the maximum number of SLF appearing in a sub-monomial of S_1. For $n \geq 2$, the S_n are time-ordered products that need to be constructed.

The natural hypothesis of interacting SLF is simple: physical observables and closely related quantities, particularly the $\mathbb{S}$-matrix, cannot depend on the strings. This is the quantum **string independence** principle.

First piece of wisdom

For the physics described by S_1 to be string-independent, we require that a vector field $Q^\mu(x, e_i)$ exist such that

$$d_{e_1} S_1^{\text{sym}} = (\partial Q) \equiv \partial_\mu Q^\mu,$$

so that on applying integration by parts in the "adiabatic limit", the covariant $\mathbb{S}[g; h]$ approaches the physical scattering matrix $\mathbb{S}$, all dependence on the strings disappearing.

Now comes the good part

Proposition 1:

Suppose that we are given n massless fields $A_{\mu a}, (a = 1, \ldots, n)$. For their mutual cubic coupling modulo divergences, string independence at first order enables only the Wick product combination:

$$S_1(x, e_1, e_2) = \frac{g}{2} f_{abc} A_{\mu a}(x, e_1) A_{\nu b}(x, e_2) F_c^{\mu\nu}(x), \quad (1)$$

where the f_{abc} are **completely skew-symmetric** coefficients.

Good times roll on

Proposition 2:

String independence of the $\mathbb{S}$-matrix at second order holds iff the Jacobi identity:

$$f_{abc}f_{cdf} + f_{afc}f_{cbd} + f_{adc}f_{cfb} = 0 \tag{2}$$

for the cubic coupling constants in (1) holds, and the quartic term

$$i\frac{g^2}{2} f_{abc}f_{dfc} A_a^\mu(x, e_1) A_b^\nu(x, e_2) A_{\mu d}(x, e_1') A_{\nu f}(x, e_2') \delta(x - x')$$

is present at that order in the $\mathbb{S}$-matrix.

A semantic whose time is gone?

This author currently knows of **five** bottom-up arguments **internal** to perturbative QFT for compact Lie *algebras* to govern interactions.

1. The classical one by Cornwall, Levin and Tiktopoulos from *unitarity bounds* at high energy in electroweak theory.

2. The Aste–Dütsch–Scharf analysis by Epstein–Glaser methods.

3. A powerful one based on the bispinor-helicity machinery for QCD, that not only bypasses Feynman graphs, but (one shouldn't forget!) it allows to derive the interaction, as well.

4. The one presented here.

5. Most charming of them all: Weinberg's "soft limit" reasoning, long ago applied to link helicity 1 particles with charge conservation and helicity 2 particles with universality of gravity.

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From the bottom-up

The "gauge principle", is a top-down, classical-geometrical one that has ruled particle theory for over 60 years. Isn't it foreign to QFT?

Isn't it a defect (the apparent unavailability in conventional QFT of a Hilbert space framework for massless particles) elevated into a doctrine?

At a minimum, now bottom-up, inherently quantum principles for the construction of interactions deserve their place in the sun.

Electroweak interlude I

The above ideas can be and have been used to render new vistas on the electroweak theory ("quantum flavourdynamics", as Marshak liked to say) — "The chirality theorem": JMG-B, J. Mund & J. C. Várilly, Annales HP **19** (2018).

In string-local fields theory it is enough to input the masses and charges of the particles to obtain the whole flavourdynamics, with its nearly maximal **P**-violation, arguing as above.

Electroweak interlude II

That is to say, **chirality** is not introduced by hand, as in the GWS model. Somewhat misteriously, this property depends also on the existence of the scalar particle, although "symmetry breaking" is out of question.

The GWS model can be reproduced – in all its ugliness – by field redefinitions. But this is hardly worthwhile in our context.

On entanglement

This is perhaps a good place to comment on the action-at-a-distance Aharonov–Bohm effect being held as "proof" of the "fundamental character" of the standard electromagnetic gauge potential – since the calculation via the electromagnetic field depends on a region where the test particle is not allowed.

This is surely a misunderstanding: this effects can be computed by means of the SL $A(x, e)$-field – which contains the same information as the F-field.

On entanglement of **observables**

The deep reasons lie in the mind-boggling entanglement properties of QFT, as compared to ordinary quantum mechanics. As pedagogically explained by Edward Witten (*arXiv:1803.04993*), in QFT **not only the states, but also the observables, are entangled**, the concrete aspects of this entanglement being dependent on mass and spin.

In particular, *Haag duality* **fails** for all quantum massless fields with helicity $r \geq 1$, and this accounts for the mentioned effects. That was shown over forty years ago by Leyland, Roberts and Testard.

The bottom line

The contention to the Okubo–Marshak purpose in SLF theory is straightforward: the string-localized vector potentials live on Hilbert space and act cyclically on the vacuum. This is part of the Reeh–Schlieder property. Therefore θ-vacua are not allowed.

Only the $m = 0$ vacuum (so to speak) occurs: there are no instantons, and no θ-vacua, thus the so-called CP problem is solved.

A moral of the story: QFT should stand on its own feet, rather than on classical crutches.

Chasing (in the) ALPs

This analysis only considers gluodynamics at high energy. However, recent work by Nakamura and Schierholz shows confinement to be inimical as well to violation of CP invariance.

The contemporary main selling point for the axion (which should couple to the photon), even more than in the nineties, is the "axiverse" as a component of dark matter. That is, the search for the axion is essentially model-free nowadays. Absence of evidence is not evidence of absence – and so the hunting for "ALPs" is bound to go on.

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