ESI-Programmme on "Modern Trends in Topological Quantum Field Theory"

February 03 to March 28, 2014

Talks & Abstracts (in alphabetical order of speakers):

Dror Bar-Natan
A partial reduction of BF theory to combinatorics

I will describe a nearly-rigorous reduction to computable combinatorics of perturbative BF theory (Cattaneo-Rossi math-ph/0210037), in the case of ribbon 2-links. Also, I will explain how and why my approach may or may not work in the non-ribbon case. Weak this result is, and at least partially already known (Watanabe math/0609742). Yet in the ribbon case, the resulting invariant is a universal finite type invariant, a gadget that significantly generalizes and clarifies the Alexander polynomial and that is closely related to the Kashiwara-Vergne problem. I cannot rule out the possibility that the corresponding gadget in the non-ribbon case will be as interesting.

See also: www.math.toronto.edu/~drorbn/Talks/Vienna-1402

John Barrett
The geometry of matrices and 2d TQFT

This talk discusses two distinct topics: Firstly, the non-commutative geometry of real spectral triples that have as algebra of coordinates the N x N matrices. Secondly, the extension of the Fukuma, Hosono and Kawai state sum models on surfaces to a framework that is sensitive to the spin structure of a surface. Finally I will discuss the question of what a "quantum geometry" is and propose some tentative relation between these two topics.

Christian Becker
Relative differential cohomology and Chern-Simons theory

We introduce two different notions of relative differential cohomology and derive long exact sequences for both. We discuss the module structure and construct fiber iintegration that commutes with the exact sequences. Transgression to loop space is a special case thereof.
As a particular example we obtain the Cheeger-Chern-Simons relative character. In the same way as the Cheeger-Simons character generalizes

the Chern-Simons invariant for closed manifolds, the Cheeger-Chern-Simons character generalizes the Chern-Simons invariant for manifolds with boundary.

Anna Beliakova CANCELLED
Trace as an alternative decategorification functor

Categorification is a lifting of a given mathematical structure to a higher categorical level. Decategorification is the inverse process of simplifying higher structure into the original one. Both procedures are not unique. Usually, the Grothendieck group K_0 is used as a decategorification functor.
In this talk, we illustrate on few examples that the trace or 0th Hochschild homology is an interesting alternative to K_0. We show that duality between trace and center
gives rise to an action of the current algebra Usl(n)[t] on the center of any 2-representation of the categorified quantum sl(n). This was previously observed
by Brundan for t=1.

Ulrich Bunke
Differential cohomology theory

I will describe joint work with Th. Nikolaus and M. Voelkl. We observe that the basic structures of differential cohomology
are consequences of the fact that differential cohomology is represented by a sheaf of spectra on manifolds. We give a classification of differential cohomology theories from this point of view. Further I will explain, how the classical examples fit into this general framework.

Sebastian Burciu
On the¬†Grothendieck groups of equivariantized fusion categories"

We describe a Mackey type decomposition for group actions on abelian categories. In the case of an action by tensor
autoequivalences the Mackey functor at the level of Grothendieck rings has a Green functor structure. As an application we give a description of the Grothendieck rings of equivariantized fusion categories under group actions by tensor autoequivalences on graded fusion categories. It is shown that these Grothendieck rings have a ring structure similar to the (double) Burnside rings of finite groups and some other rings obtained by Bouc and Witherspoon.

Alan Carey
A geometric approach to twisted K-homology

I will describe the approach to constructing twisted K homology of manifolds due to B-L Wang. It uses a generalisation of Baum-Douglas geometric cycles and gives a mathematical interpretation of D-branes.
The proof that all classes in the twisted K-group can be realised this way depends on the manifold structure. For spaces with singularities a different approach is needed. I will explain an alternative idea due to Baum, Wang and myself.
It is sufficiently general to construct the twisted K-homology of CW complexes.

Qingtao Chen
Congruent skein relation and LMOV conjectures

We obtain several very interesting congruent skein relations with the right motivation from studying the LMOV conjectures. When we applied this idea to colored Jones polynomials and su(n) invaraints, we also obtain a set of congruent skein relations for colored Jones polynomials and su(n) invariants.

Kazuo Habiro
Kirby calculus for null-homologous framed links in 3-manifolds

Kirby's calculus on framed links gives a method of constructing 3-manifold invariants using link invariants. I will discuss Kirby type theorems for framed links each of whose components is null-homologous in the ambient 3-manifold. This is joint work with Tamara Widmer.

Bas Janssens
Representation theory of gauge groups

Despite the fact that gauge groups are infinite dimensional, their bounded unitary representations behave remarkably like those of finite dimensional semisimple Lie groups. We give a classification result for these bounded unitary representations and indicate how this generalises to (possibly unbounded) positive energy representations. These turn out to behave much like highest weight representations of affine Kac- Moody algebras.

Theo Johnson-Freyd
Poisson AKSZ theory and homotopy actions of properads

I describe a generalization of the AKSZ construction of topological field theories to allow targets with possibly-degenerate up-to-homotopy

Poisson structure.¬†The construction requires investigating in what sense the chains on an oriented manifold carry a chain-level homotopy Frobenius structure.¬†There are two versions of the construction: a "classical field theory" tree-level version, and a "quantum field theory" graph-level version. The tree-level version is well-behaved for all possible spacetimes and targets.¬†The graph-level version is much more subtle, and intimately connected to the "formality" or "quantization" problem for the operad of little n-dimensional disks.

Rinat Kashaev
Beta pentagon relations

The (quantum) pentagon relation underlies the existing constructions of three dimensional quantum topology in the combinatorial framework of triangulations. Following recent works on constructions of TQFT‚Äôs with infinite state spaces, I will discuss a special type of integral
pentagon relations called beta pentagon relations, and their relationships with the Faddeev type operator pentagon relations.¬†

Sergei Merkulov
Grothendieck-Teichmueller group and exotic automorphisms of the Lie algebra of polyvector fields

Using some new operads of compactified semialgebraic configuration spaces, we show an explicit formula for a universal action of an element of the Grothendieck-Teichmueller group as a Lie-infinity automorphism of the Lie algebra of polyvector fields on an arbitrary smooth manifold.

Catherine Meusburger
Diagrams for Gray categories with duals

Tricategories arise in the description of defects in topological field theories. Examples are topological field theories of Turaev-Viro type in which certain submanifolds are decorated with tricategorical data. As these submanifolds are oriented, this requires a notion of duals to implement orientation reversal.
We introduce a notion of duals for Gray categories, which can be viewed as maximally strict tricategories, and a diagrammatic representation of these duals. This generalises the well-known diagrammatic calculus for pivotal and braided categories and relates the geometry of the diagrams to the structures in the Gray category. We show that the evaluation of diagrams is invariant under certain isotopies of diagrams, introduce a notion of trace, which gives rise to manifold invariants, and discuss their applications. This is joint work with John Barrett and Gregor

Schaumann (arxiv.org/abs/1211.0529).

Scott Morrison
Progress on small fusion categories

The 2-dimensional extended TFTs correspond to fusion categories. We know constructions of fusion categories from finite group data and from quantum groups at roots of unity, but what else is out there? We're very far from having satisfying answers, but have now approached the problem from several different directions. I'll explain some of the notions of a 'small' fusion category, and describe what we've seen so far. Some examples, in particular the fusion categories coming from the Haagerup subfactor, appear over and over again. I suspect that 'eventually' there will be an unmanageable surfeit of sporadic fusion categories, but so far efforts at classification have turned up surprisingly few instances.

Jeffrey Morton
Towards extended TQFT from higher gauge theory

I will describe a general construction which gives "extended" Topological Quantum Field Theories (ETQFT) in codimension 2 from a gauge group, using a quantization functor in the spirit of Freed-Hopkins-Lurie-Teleman. I will discuss prospects for a generalization of this method to a codimension-3 ETQFT higher gauge theory based on categorical groups, and beyond.

Sonia Natale
On weakly group-theoretical non-degenerate braided fusion categories

A fusion category is called weakly group-theoretical if it is tensor Morita equivalent to a nilpotent fusion category. It is conjectured that every fusion category of integer Frobenius-Perron dimension is weakly group-theoretical. We shall present a result concerning the class of a braided non-degenerate weakly group-theoretical fusion category in the Witt group introduced recently by Davydov, M√°ger, Nikshych and Ostrik.¬†
We shall also give some sufficient conditions for a braided fusion category to be weakly group-theoretical or solvable in terms of the factorization of its Frobenius-Perron dimension and the Frobenius-Perron dimensions of its simple objects.

Thomas Nikolaus (Workshop 1)
Twisted differential cohomology

We describe joint work with U. Bunke. We discuss the construction and properties of twisted differential cohomology for an arbitrary multiplicative differential cohomology theory. Examples of interest include K-theory and topological modular forms. We show how to construct and classify all differential twist and how to compute the respective groups. If time permits we explain some of the details related to the higher categorical construction.

Thomas Nikolaus (String Network meeting)
T-duality in K-theory and ellitptic cohomology

Joost Nuiten & Urs Schreiber
Cohomological quantization

We discuss a general natural scheme for formalizing quantization via pull-push in twisted generalized cohomology. We show how this reproduces traditional geometric quantization in a "holographic" way as the boundary field theory of a 2d Poisson-Chern-Simons theory and generalizes it to a geometric quantization of Poisson manifolds that captures for instance the "universal orbit method" of Freed-Hopkins-Teleman. The recent preprint by Hopkins and Lurie turns out to use a special case of this construction.
This is based on Nuiten's MSc thesis
http://ncatlab.org/schreiber/show/master+thesis+Nuiten and on the notes
http://ncatlab.org/schreiber/show/Homotopy-type+semantics+for+quantization.

Pranav Pandit
The topological A-model, spectral networks, WKB-theory and buildings

BPS-branes in an A-model TQFT associated to the Hitchin integrable system can be described in terms of spectral networks, which are certain decorated graphs on a Riemann surface. In this talk, I will introduce the notion of a building, and explain how spectral networks naturally arise from certain harmonic maps to affine buildings. I will discuss the sense in which spectral networks and harmonic maps to buildings control the asymptotic behavior of the Riemann-Hilbert correspondence, and conjecturally, the non-abelian Hodge correspondence.
This is joint work with Ludmil Katzarkov, Alexander Noll and Carlos Simpson.

Ulrich Pennig
An introduction to I-spaces and a conjecture about K(ku)

We give an introduction to I-spaces and commutative I-monoids and show how they can be used to model the units of commutative symmetric ring spectra. As an application we discuss how our operator algebraic model for the units of K-theory fits into this picture. In the second half of the talk we will discuss a conjectural non-commutative geometric model for the algebraic K-theory of topological K-theory K(ku).

Ingo Runkel
Spin from defects in two-dimensional field theory

Studying defects in quantum field theories has many interesting applications.
I will consider two dimensional topological and conformal field theories defined on oriented surfaces in the presence of line defects. It turns out that with the help of a certain type of line defect one can use the original
theory to obtain a new theory which is defined on surfaces with spin structure.
This is joint work with Sebastian Novak.

Thomas Schick
Geometric models for higher twisted of K-theory (joint with Andrei Ershov, Saratov)

Homotopy theory tells us that there are many "exotic" twists for K-theory, corresponding to maps to BBU. The latter has an infinite cyclic homotopy group in each odd degree bigger than 1.
Of this, the "degree 3 part" is well understood geometrically (with a number of different models). E.g. one can describe twists as U(1)-bundle gerbes and (for torsion twists) the twisted K-theory as the Grothendieck group of bundle gerbe modules.
We propose to use higher dimensional "homotopy bundle gerbes" to model the remaining twists. Passage to higher dimensional bundles forces a tensor stabilization and the replacement of strict isos by homotopies and higher homotopies.
In favorable cases, we define the corresponding twisted K-theory by suitably defined homotopy bundle gerbe modules.
We check that the definitions work well for homotopy bundle gerbes with trivializations.

Chris Schommer-Pries
Extended topological field theories and tensor categories

We describe joint work with C. Douglas and N. Snyder which
shows that every (finite) tensor category gives rise to a fully local partially defined 3-dimensional topological field theory. If the tensor category is fusion, then this extends to a fully defined tft.

Urs Schreiber
Homotopy-type semantics for quantization

We discuss in the foundational logical framework of homotopy-type theory a natural formalization of integral kernels in geometric stable homotopy theory. We observe that this yields a process of non-perturbative cohomological quantization of local prequantum field theory. Recalling that traditional linear logic has categorical semantics in symmetric monoidal categories, such as that of Hilbert spaces, and hence serves to axiomatize quantum mechanics, what we consider is its refinement to linear homotopy-type theory which has categorical semantics in stable infinity-categories of bundles of stable homotopy types, hence of generalized cohomology theories. We discuss how this provides a natural formalization of geometric quantization of local prequantum field theory, following Schreiber (2013), Nuiten (2013) and closely related to Hopkins-Lurie (2014).

See also: ncatlab.org/schreiber/show/Type+semantics+for+quantization

Joost Slingerland
Local representations of the loop braid group

I will give an introduction to the "loop braid group". This group governs the topological exchange properties of ring shaped "particles" in three dimensional space. It plays a role similar to that of the braid group in 2 spatial dimensions. Loops can perform a number of nontrivial exchange motions including simple exchanges like those of point particles and "leapfrogging" like smoke rings.
I will introduce the concept of a local representation of the loop braid group; in such a representation, each ring has an internal Hilbert space and exchange motions act only on the internal Hilbert spaces of the rings that are involved in the motion. Examples of such local representations come from gauge theories and are closely related to the anyons that arise in toric code models, or discrete gauge theories. I will argue that, subject to an additional condition, all local representations of the loop braid group are of this type.

Thomas Strobl
Dirac sigma models and gauging

The Dirac Sigma Model is a joint generalization of the Poisson Sigma Model and the G/G Wess-Zumino-Witten Model. We recall its definition, it being associated to Dirac structures (which contain Poisson structures as particular cases), and its basic features like that its field equations up to gauge transformations are Lie algebroid morphisms up to Lie algebroid homotopies. We then show that the Dirac Sigma Model can be obtained from gauging an infinite dimensional rigid symmetry group. We conclude with an outlook on higher dimensional and non-topological generalizations.

Ulrike Tillmann
Commutative K-Theory and other new generalised cohomology theories

Vector bundles over a compact manifold can be defined via transition functions to a linear group. Often one imposes conditions on this structure group. For example for real vector bundles on may ask that all transition functions lie in the special orthogonal group to encode orientability. Commutative K-theory arises when we impose the condition that the transition functions commute with each other whenever they are simultaneously define.
We will introduce commutative K-theory and some natural variants of it, and will show that they give rise to new generalised cohomology theories. This is joint work with Adem, Gomez and Lind building on previous work by Adem, F. Cohen, and Gomez.

Alessandro Valentino
Boundary conditions for 3d TQFTs and Module Categories

I will discuss some aspects of boundary conditions for a 3d TFT of Reshetikhin-Turaev type, and their description in terms of module categories.

Boris Vertman
Combinatorial quantum fIeld theory and gluing formula for determinants

We define the combinatorial Dirichlet-to-Neumann operator and establish a gluing formula for determinants of discrete Laplacians using a combinatorial Gaussian quantum field theory. We relate the combinatorial gluing formula to the corresponding Mayer-Vietoris formula by Burghelea, Friedlander and Kappeler for zeta-determinants of analytic Laplacians, using the approximation theory of Dodziuk.

Alexis Virelizier
3-dimensional HQFTs

Homotopy quantum field theory (HQFT) is a branch of quantum topology concerned with maps from manifolds to a fixed target space. The aim is to define and to study homotopy invariants of such maps using methods of quantum topology. I will focus on 3-dimensional HQFTs with target the Eilenberg,ÄìMacLane space K(G,1) where G is a discrete group. (The case G = 1
corresponds to more familiar 3-dimensional TQFTs.) These HQFTs provide

numerical invariants of principal G-bundles over closed 3-manifolds which can be viewed as ``quantum'' characteristic numbers. To construct such HQFTs, the relevant algebraic ingredients are G-graded categories, which are
monoidal categories whose objects have a multiplicative G-grading.

Christian Voigt
Clifford algebras, fermions, and categorification

We describe a categorification of complex Clifford algebras arising from certain categories of twisted modules over fermionic vertex superalgebras. The product in our categorified Clifford algebra is closely related to fusion of surface defects in 3D topological field theory. The higher categorical structure arises from varying polarisations in the construction of fermionic Fock spaces. We will include some background from the theory of unitary vertex algebras, explain the connection of our setup with infinite Grassmannians, and discuss how the String 2-group fits naturally into the picture.

Michael Völkl
The intrinsic eta-invariant and geometrizations

In this talk we will briefly review the homotopy theoretic content of Bunke's universal eta-invariant.
Then we will discuss a generalization of this eta-invariant. For this we recall Chern-Weil theory and generalise it to so-called geometrizations.
This allows us to define the new eta-invariant.
We close with some easy examples.

Konrad Waldorf
String geometry vs. spin geometry on loop spaces

I will present some recent progress concerning the correspondence between string geometry on a manifold and spin geometry on its free loop space. The ultimate goal of this correspondence is the definition of a Dirac operator on the loop space, and the computation of its index. While this ultimate goal is still far out of reach, some technical requirements for the
definition of a Dirac operator are now better understood, most importantly the distinctive features of spin structures and spin connections on loop space

Kevin Walker
Premodular TQFTs

From a premodular category one can construct a fully extended (all the way down to points) 3+1-dimensional TQFT. In the case where the input is modular (rather than merely premodular), this TQFT contains all the information of the corresponding Witten-Reshetikhin-Turaev invariants. In the general premodular case, one can analyze the TQFT using a higher Morita equivalence between 4+1-dimensional TQFTs related to finite (super) groups. This leads naturally to [de]equivariantization and Turaev's theory of homotopy TQFTs.

Christoph Wockel
Topological group cohomology and Chern-Weil theory

We will explain how the topological group cohomology (equivalently, the Segal-Mitchison cohomology or the measurable cohomology) of a finite-dimensional
Lie group with torus coefficients can be computed via Chern-Weil theory, the smooth group cohomology and the classifying space cohomology. In the end we will also discuss applications to bounded continuous cohomology and some open problems there.

Mahmoud Zeinalian
A concise construction of differential K-theory

We construct a model of differential K-theory and its S1-interation maps that is based on a geometrically defined spectrum and also does not require the data of an additional form. This is joint work with Thomas Tradler and Scott Wilson.