Ultra Unification

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Einige Leute kennen mich vielleicht nicht. Als meine Selbsteinführung.

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Juven **W**ang

Just **W**ondering

Ehrlich, **J**ob **W**anted

Discrimination vs Unification

"Der schwer gefaßte Entschluß. Muß es sein? Es muß sein!"

"The heavy (high energy and gapped) decision. Must it be? It must be!"

String Quartet No. 16 in F major, op.135

Ludwig van **Beethoven** in 1826


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Ultra Unification (UU):

2006.16996: Electroweak+Strong+other forces + topological sector, e.g.,
• 4d topological field theory [Schwarz type unitary **TQFT**] at low energy below an energy gap. A long-range entangled intrinsic topological order,
• 5d Symmetry-Protected Topological state [**SPTs**]. Invertible TQFT. More than a 2nd Chern class θ-term $F \wedge F$.

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Equations/Actions: Weyl spinors + Yukawa-Higgs-Dirac term + U(1) Maxwell + SU(N) or Spin(N) Yang-Mills + unitary 4d non-invertible / 5d invertible or non-invertible TQFT

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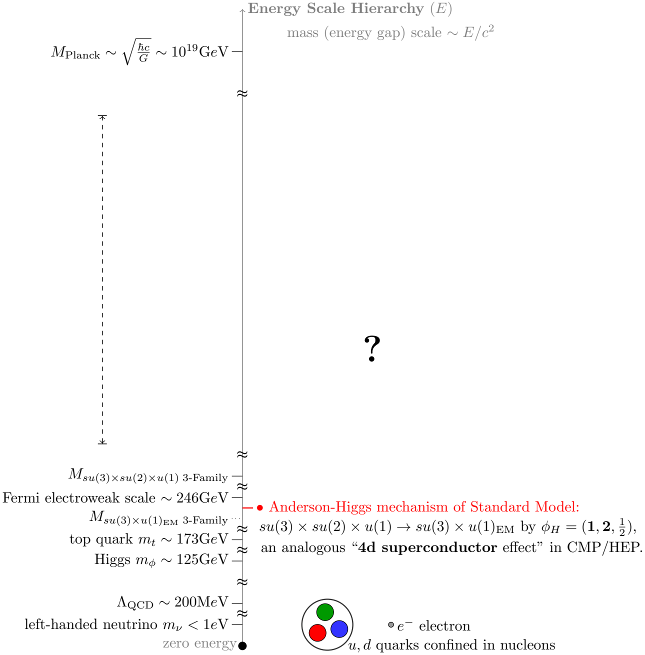
Energy Hierarchy: Known and Unknown (?)



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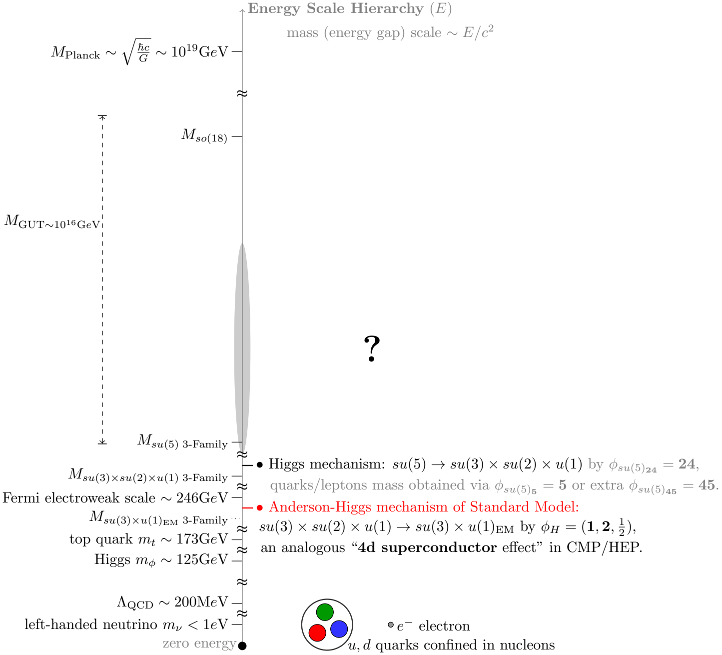
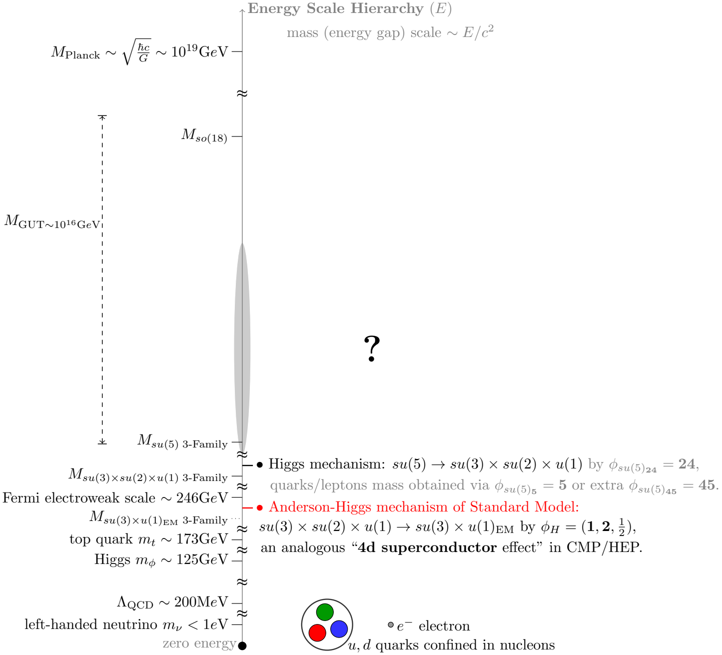


My Aim of this talk:

Are there non-perturbative constraints given the low energy physics at SM, guiding us toward something heavy at higher energy?

1. Global structure of Lie group (more than local Lie algebra)
2. Discrete gauge/global symmetries
3. (Invertible) Anomalies (4d on 5d SPTs):
   Perturbative Local ($\mathbb{Z}$ class)+
   Non-Perturbative Global Anomalies ($\mathbb{Z}_n$ class)
4. Classification of Anomalies via Cobordism
5. Symmetric gapping rule, Kitaev-Wen mechanism, symmetric mass (mass without breaking "chiral symmetry") without 't Hooft anomaly. $\nu = 0 \mod 16$ of $\mathbb{Z}_{16}$.
6. Symmetric anomalous TQFT/topological order with 't Hooft anomaly. $\nu = 1, 2, 3 \neq 0 \mod 16$ of $\mathbb{Z}_{16}$. (e.g. 2+1d Vishwanath-Senthil'12)
7. Energy hierarchy
8. String landscape/swampland + no global sym in quantum gravity

Energy Hierarchy: Who live above us heavily Upstair/Unknown (?)



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Energy Scale Hierarchy (E)

mass (energy gap) scale $\sim E/c^2$

$M_{\text{Planck}} \sim \sqrt{\frac{\hbar c}{G}} \sim 10^{19}\text{GeV}$

$\approx$

- $\Delta_{\text{KW}_1.so(18)}$

$M_{so(18)}$

$M_{so(10)\times so(8)}$

$M_{so(10)\times so(6)}$

(possible dynamical gauge symmetry breaking)

$M_{so(10)\times so(5)}$

$M_{\text{GUT}\sim 10^{16}\text{GeV}}$

- $\Delta_{\text{KW}_2.so(10)}$ (beyond Higgs mechanism, an alternative to Hypercolor)

$M_{so(10)}$ 3-Family

- Higgs mechanism: $so(10) \to su(5)$ by $\phi_{so(10)_{\mathbf{16}}} = \mathbf{16}$ (for Δ_{TQFT}) *or* by $\phi_{so(10)_{\mathbf{126}}} = \mathbf{126}$ (for sterile Majorana mass M_ν).
  $so(10) \to su(3) \times su(2) \times u(1)$ by another $\phi_{so(10)_{\mathbf{45}}} = \mathbf{45}$ *or* $\phi_{so(10)_{\mathbf{54}}} = \mathbf{54}$.
- Δ_{TQFT} **New Scenario**: from **Topological Mass/Energy Gap** of 4d non-invertible TQFT or 5d invertible TQFT

"**4d** $\mathbb{Z}_{4,X}$-symmetry preserving anomalous **topological order** and **5d SPTs/topological superconductor** effects" in CMP/HEP.

$M_{su(5)\times \mathbb{Z}_{4,X}}$ 3-Family

- M_ν **Old Scenario**: Majorana mass of right-handed neutrinos breaks $\mathbb{Z}_{4,X}$.

($\mathbb{Z}_{4,X}$ may become dynamically gauged [preserved] above a scale, but become a "global" symmetry [or broken] below a scale.)

(Δ_{TQFT} is an alternative to **Majorana/Dirac mass** of heavy sterile right-handed neutrinos, alternative to the seesaw mechanism.)

$M_{su(5)}$ 3-Family

$\approx$

- Higgs mechanism: $su(5) \to su(3) \times su(2) \times u(1)$ by $\phi_{su(5)_{\mathbf{24}}} = \mathbf{24}$,
  quarks/leptons mass obtained via $\phi_{su(5)_{\mathbf{5}}} = \mathbf{5}$ or extra $\phi_{su(5)_{\mathbf{45}}} = \mathbf{45}$.

$M_{su(3)\times su(2)\times u(1)}$ 3-Family

$\approx$

Fermi electroweak scale $\sim 246\text{GeV}$

- Anderson-Higgs mechanism of Standard Model:

$su(3) \times su(2) \times u(1) \to su(3) \times u(1)_{\text{EM}}$ by $\phi_H = (\mathbf{1}, \mathbf{2}, \frac{1}{2})$,
an analogous "**4d superconductor** effect" in CMP/HEP.

$M_{su(3)\times u(1)_{\text{EM}}}$ 3-Family

$\approx$

top quark $m_t \sim 173\text{GeV}$

$\approx$

Higgs $m_\phi \sim 125\text{GeV}$

$\approx$

$\Lambda_{\text{QCD}} \sim 200\text{MeV}$

$\approx$

left-handed neutrino $m_\nu < 1\text{eV}$

zero energy

What are known? Spacetime-internal sym $G \equiv (\frac{G_{\text{spacetime}} \ltimes G_{\text{internal}}}{N_{\text{shared}}})$ and Rep.

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$G_{\text{spacetime}}$:
3+1d Weyl Ψ_L, a left-handed (L) chiral spacetime spinor of the $G_{\text{spacetime}}$:
Lorentz group Spin(3,1) (or Euclidean Spin(4)) in Spinor Rep(resentation):

$$
\begin{array}{llll}
3+1\text{d} & \Psi_{\mathrm{L}} & \sim & 2_L \text{ of } \mathrm{Spin}(3,1) = \mathrm{SL}(2,\mathbb{C}), \text{ complex Rep.} \\
4\text{d} & \Psi_{\mathrm{L}} & \sim & 2_L \text{ of } \mathrm{Spin}(4) = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R, \text{ pseudoreal Rep.} \\
4+1\text{d} & \Psi & \sim & 4 \text{ of } \mathrm{Spin}(4,1) = \mathrm{Sp}(1,1), \text{ pseudoreal Rep.} \\
5\text{d} & \Psi & \sim & 4 \text{ of } \mathrm{Spin}(5) = \mathrm{USp}(4) = \mathrm{Sp}(2), \text{ pseudoreal Rep.}
\end{array}
$$

Unitarity of Lorentz $\sim$ Reflection positivity of Euclidean. (Freed-Hopkins '16)
dd invertible TQFT with reflect.pos in Euclidean signature
$\Rightarrow$ anomaly of $(d-1)$d reflect.pos Euclidean QFT.
$\Rightarrow$ anomaly of $(d-2,1)$d unitary Lorentz QFT. Say $d=5$.

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G_{internal}: Standard Model Lie algebra $su(3) \times su(2) \times u(1)$.
$G_{\text{SM}_q} \equiv \frac{\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)}{\mathbb{Z}_q}$, $q = 1, 2, 3, 6$.

Weyl Lorentz spinor in Fund.Rep:$\left(\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad u_R, \quad d_R, \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad e_R \right) \quad \times 3$ family

$((\mathbf{3},\mathbf{2},1/6)_L, (\mathbf{3},\mathbf{1},2/3)_R, (\mathbf{3},\mathbf{1},-1/3)_R, (\mathbf{1},\mathbf{2},-1/2)_L, (\mathbf{1},\mathbf{1},-1)_R) \times 3$ family.
$((\mathbf{3},\mathbf{2},1/6)_L, (\overline{\mathbf{3}},\mathbf{1},-2/3)_L, (\overline{\mathbf{3}},\mathbf{1},1/3)_L, (\mathbf{1},\mathbf{2},-1/2)_L, (\mathbf{1},\mathbf{1},1)_L) \quad \times 3$ family.

Higgs complex scalar $\Phi_H : (1, \mathbf{2}, 1/2)$.

What are known? SO(10) GUT ⊃ SU(5) GUT ⊃ Standard Model

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- Standard Model Lie algebra $su(3) \times su(2) \times u(1)$.
- Lie group: $\mathrm{Spin}(10) \supset \mathrm{SU}(5) \supset \frac{\mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3)}{\mathbb{Z}_6}$.

$$\frac{\mathrm{Spin}(d) \times \mathrm{Spin}(10)}{\mathbb{Z}_2^f} \supset \mathrm{Spin}(d) \times \mathrm{SU}(5) \supset \mathrm{Spin}(d) \times \frac{\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)}{\mathbb{Z}_6}$$

ψ_L **Internal Rep** of G_{internal}:
$((\mathbf{3},\mathbf{2},1/6)_L, (\overline{\mathbf{3}},\mathbf{1},-2/3)_L, (\overline{\mathbf{3}},\mathbf{1},1/3)_L, (\mathbf{1},\mathbf{2},-1/2)_L, (\mathbf{1},\mathbf{1},1)_L) \quad \times 3$ family

Higgs complex scalar $\Phi : (1,2,1/2)$.

$$\begin{array}{rcl} (\overline{\mathbf{3}},\mathbf{1},1/3)_L \oplus (\mathbf{1},\mathbf{2},-1/2)_L & \sim & \overline{\mathbf{5}} \text{ of } \mathrm{SU}(5). \\ (\mathbf{3},\mathbf{2},1/6)_L \oplus (\overline{\mathbf{3}},\mathbf{1},-2/3)_L \oplus (\mathbf{1},\mathbf{1},1)_L & \sim & \mathbf{10} \text{ of } \mathrm{SU}(5). \\ R\text{-handed sterile neutrino } (\mathbf{1},\mathbf{1},0)_R = (\mathbf{1},\mathbf{1},0)_L & \sim & \mathbf{1} \text{ of } \mathrm{SU}(5). \end{array}$$

$$\overline{\mathbf{5}} \oplus \mathbf{10} \oplus \mathbf{1} \text{ of } \mathrm{SU}(5) \quad \sim \quad \mathbf{16}^+ \text{ of } \mathrm{Spin}(10).$$

We may skip gauge bosons (as they are always in Adjoint Rep), e.g.,
$\mathbf{24}$ of $\mathrm{SU}(5) \sim (\mathbf{8},\mathbf{1},0) \oplus (\mathbf{1},\mathbf{3},0) \oplus (\mathbf{1},\mathbf{1},0) \oplus (\mathbf{3},\mathbf{2},-\frac{5}{6}) \oplus (\overline{\mathbf{3}},\mathbf{2},\frac{5}{6})$
$\mathbf{45}$ of $\mathrm{Spin}(10) \sim \mathbf{24} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{1}$ of $\mathrm{SU}(5)$
and Higgs, for the consideration of anomaly.

What are known? SO(10) GUT $\supset$ SU(5) GUT or SM+ discrete X sym.
$\frac{\mathrm{Spin}(d)\times\mathrm{Spin}(10)}{\mathbb{Z}_2^F} \supset \mathrm{Spin}(d) \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times \mathrm{SU}(5) \supset \mathrm{Spin}(d) \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times \frac{\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)}{\mathbb{Z}_6}$.

- $X \equiv 5(\mathbf{B}-\mathbf{L}) - 4Y = 5(\mathbf{B}-\mathbf{L}) - 2Y_W$. **B** baryon number, **L** lepton number.
- Electric charge: $Q_{\mathrm{EM}} = T_3 + Y = T_3 + \frac{1}{2}Y_W = T_3 + \frac{1}{6}\tilde{Y}$.
- A $\mathrm{U}(1)_X$ subgroup sits at Spin(10): $\mathbb{Z}_2^F \subset \mathbb{Z}_{4,X} = Z(\mathrm{Spin}(10)) \subset \mathrm{Spin}(10)$.

SM fermion spinor field	SU(3)	SU(2)	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_{Y_W}$	$\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$	$\mathrm{U}(1)_X$	$\mathbb{Z}_{4,X}$
$\bar{d}_R$	$\bar{\mathbf{3}}$	$\mathbf{1}$	1/3	2/3	-1/3	-3	1
l_L	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-1	-1	-3	1
q_L	$\mathbf{3}$	$\mathbf{2}$	1/6	1/3	1/3	1	1
$\bar{u}_R$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	$-4/3$	$-1/3$	1	1
$\bar{e}_R = e_L^+$	$\mathbf{1}$	$\mathbf{1}$	1	2	1	1	1
$\bar{\nu}_R = \nu_L$	$\mathbf{1}$	$\mathbf{1}$	0	0	1	5	1

Garcia-Etxebarria-Montero'18 arXiv:1808.00009 studies $\mathrm{Spin}(d) \times \mathrm{Spin}(10)$.
JW-Wen '18 1809.11171, Wan-JW'19 arXiv:1910.14668 also studies $\frac{\mathrm{Spin}(d)\times\mathrm{Spin}(10)}{\mathbb{Z}_2^F}$.

Classify 4d Anomalies by 5d iTQFT/SPTs via Cobordism

5d invertible TQFT/SPTs and 4d Anomalies via 5d Cobordism

Kapustin'14, Freed-Hopkins'16 (systematic)
Unitarity of Lorentz $\sim$ Reflection positivity of Euclidean.
dd invertible TQFT with reflect.pos in Euclidean signature
$\Rightarrow$ anomaly of $(d-1)$d reflect.pos Euclidean QFT.
$\Rightarrow$ anomaly of $(d-2,1)$d unitary Lorentz QFT. Take $d=5$.

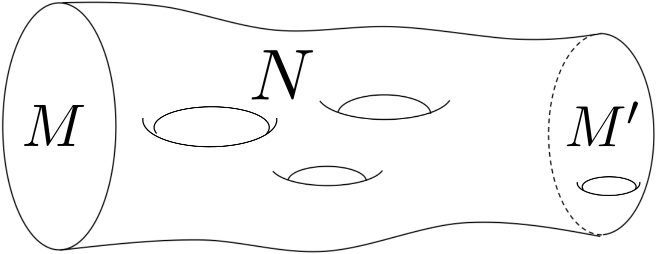
Here we only concern a cobordism group $\Omega_G^d \equiv \mathrm{TP}_d(G)$,
There is also a bordism group Ω_d^G. Note $(\mathrm{TP}_d(G))_{\text{tors}} = (\Omega_d^G)_{\text{tors}}$.
tor: a torsion group (only a finite group part).

Given a G structure, we will later show co/bordism group (abelian group classification) and dd topological terms (invertible TQFT or Symmetry Protected Topological states [SPTs]) and the anomaly of a $(d-2,1)$d unitary Lorentz QFT.

Classify iTQFT/SPTs and Anomalies via Cobordism

Bordism group (abelian): Ω_d^G

- Closure: Disjoint union of manifolds is a manifold.
- Identity: 0 is the empty manifold.
- $+$: the disjoint union.
- Inverse: $[M] + [\bar{M}] = 0$ since $\partial(M \times [0,1]) = M \sqcup \bar{M}$.
- Associativity and commutativity: true for disjoint union.



Spin cobordism:Kapustin-Thorngren-Turzillo-Wang'14 (proposed), Freed-Hopkins'16 (systematic), Wan-JW'18 arXiv:1812.11967: Encode higher-symmetry/classifying space.

Application to SM: Garcia-Etxebarria-Montero'18, JW-Wen'18, Davighi-Gripaios-Lohitsiri'19, Wan-JW'19 arXiv:1910.14668

Classify iTQFT/SPTs and Anomalies via Cobordism

Freed-Hopkins'16 (systematic)

$$\left\{ \begin{array}{c} \text{Deformation classes of the reflection positive} \\ \text{invertible } d\text{-dimensional extended} \\ \text{topological field theories (iTQFT) with} \\ \text{symmetry group } \frac{G_{\text{spacetime}} \ltimes \mathbb{G}_{\text{internal}}}{N_{\text{shared}}} \end{array} \right\}$$

$$\cong [MTG|_{G=(\frac{G_{\text{spacetime}} \ltimes G_{\text{internal}}}{N_{\text{shared}}})}, \Sigma^{d+1} I\mathbb{Z}]_{\text{tors}} = (\Omega_d^G)_{\text{tors}} = (\pi_d(MTG))_{\text{tors}}.$$

MTG is the Madsen-Tillmann-Thom spectrum of the group G, Σ is the suspension, $I\mathbb{Z}$ is the Anderson dual spectrum. tors means a torsion group (only a finite group part). The r.h.s. is the torsion subgroup of homotopy classes of maps from a Madsen-Tillmann-Thom spectrum (MTG) to a suspension shift (Σ^{d+1}) of the Anderson dual to the sphere spectrum ($I\mathbb{Z}$).

$$\Omega_G^d \equiv \Omega^d_{(\frac{G_{\text{spacetime}} \ltimes G_{\text{internal}}}{N_{\text{shared}}})} \equiv \mathrm{TP}_d(G) \equiv [MTG, \Sigma^{d+1} I\mathbb{Z}].$$

$$0 \to \mathrm{Ext}^1(\pi_d(MTG), \mathbb{Z}) \to [MTG, \Sigma^{d+1} I\mathbb{Z}] \to \mathrm{Hom}(\pi_{d+1}(MTG), \mathbb{Z}) \to 0.$$

Below we only concern: $\Omega_G^d \equiv \mathrm{TP}_d(G)$.

Classify 4d Anomalies by 5d iTQFT/SPTs via Cobordism

$G = \text{Spin} \times G_{\text{SM}_{q=6}}$, $\text{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times G_{\text{SM}_{q=6}}$ and $G = \text{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times \text{SU}(5)$:

Cobordism group $\text{TP}_d(G)$ with $G_{\text{SM}_q} \equiv (\text{SU}(3) \times \text{SU}(2) \times \text{U}(1))/\mathbb{Z}_q$ and $q = 1, 2, 3, 6$		
dd	classes	cobordism invariants
		$G = \text{Spin} \times G_{\text{SM}_6}$
5d	$\mathbb{Z}^5$	$\mu(\text{PD}(c_1(\text{U}(2)))) \sim \mu(\text{PD}(c_1(\text{U}(3))))$, $\text{CS}_1^{\text{U}(3)} c_1(\text{U}(3))^2$, $\frac{\text{CS}_1^{\text{U}(3)} c_2(\text{U}(2))}{2} \sim \frac{c_1(\text{U}(3))\text{CS}_3^{\text{U}(2)}}{2}$, $\frac{\text{CS}_1^{\text{U}(3)} c_2(\text{U}(3)) + \text{CS}_5^{\text{U}(3)}}{2} \sim \frac{c_1(\text{U}(3))\text{CS}_3^{\text{U}(3)} + \text{CS}_5^{\text{U}(3)}}{2}$, $\text{CS}_5^{\text{U}(3)}$
		$G = \text{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times G_{\text{SM}_6}$
5d	$\mathbb{Z}^5 \times \mathbb{Z}_2^2 \times \mathbb{Z}_4 \times \mathbb{Z}_{16}$	$\mu(\text{PD}(c_1(\text{U}(3))))$, $c_1(\text{U}(3))^2 \text{CS}_1^{\text{U}(3)}$, $\frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \text{CS}_3^{\text{U}(2)} + \text{CS}_1^{\text{U}(3)} c_2(\text{U}(2))}{2} \sim \frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \text{CS}_3^{\text{U}(2)} + c_1(\text{U}(3))\text{CS}_3^{\text{U}(2)}}{2}$, $\frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \text{CS}_3^{\text{U}(3)} + \text{CS}_1^{\text{U}(3)} c_2(\text{U}(3)) + \text{CS}_5^{\text{U}(3)}}{2} \sim \frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \text{CS}_3^{\text{U}(3)} + c_1(\text{U}(3))\text{CS}_3^{\text{U}(3)} + \text{CS}_5^{\text{U}(3)}}{2}$, $\text{CS}_5^{\text{U}(3)}$, $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\text{U}(3))$, $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\text{U}(2))$, $c_1(\text{U}(3))^2 \eta'$, $\eta(\text{PD}(\mathcal{A}_{\mathbb{Z}_2}))$
		$G = \text{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times \text{SU}(5)$
5d	$\mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_{16}$	$\frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \text{CS}_3^{\text{SU}(3)} + \text{CS}_5^{\text{SU}(3)}}{2}$, $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\text{SU}(5))$, $\eta(\text{PD}(\mathcal{A}_{\mathbb{Z}_2}))$

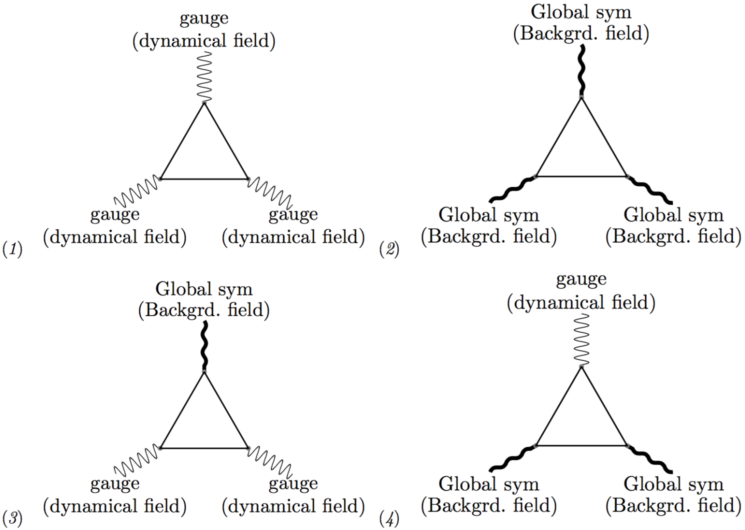
$G = \text{Spin} \times_{\mathbb{Z}_2} \text{Spin}(10)$:

$G = \text{Spin} \times_{\mathbb{Z}_2} \text{Spin}(N)$ for $N \geq 7$, e.g. $\text{Spin}(N) = \text{Spin}(10)$ or $\text{Spin}(18)$ for $\text{SO}(10)$ or $\text{SO}(18)$ GUT		
5d	$\mathbb{Z}_2$	$w_2(TM) w_3(TM) = w_2(V_{\text{SO}(N)}) w_3(V_{\text{SO}(N)})$

JW-Wen '18 1809.11171, Wan-JW'19 1910.14668 uses Adams spectral sequence.

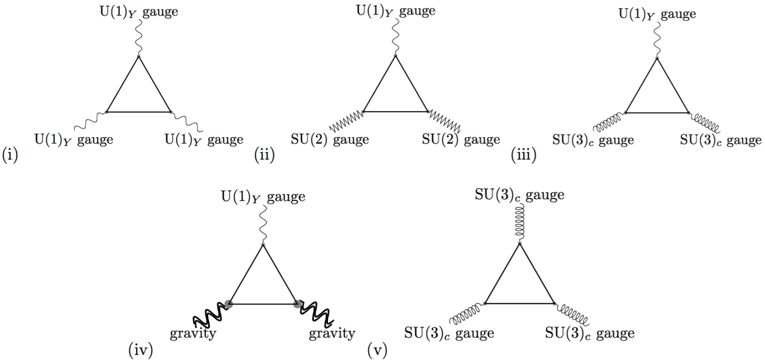
Related work uses Atiyah-Hirzebruch spectral sequence:
Garcia-Etxebarria-Montero'18 1808.00009, Davighi-Gripaios-Lohitsiri'19 1910.11277

The Use of Anomalies



(*1*) Dynamical gauge anomaly. (*2*) 't Hooft anomaly of background (Backgrd.) fields. (*3*) Adler-Bell-Jackiw (ABJ) type of anomalies. (*4*) Anomaly that involves two background fields of global symmetries and one dynamical gauge field.

I. (Local) Anomalies of $\mathrm{Spin}(d) \times G_{\mathrm{SM}_q}\big|_{(\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1))/\mathbb{Z}_q}$

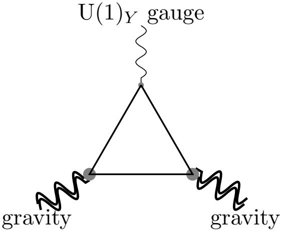


1. $\mathrm{U}(1)_Y^3$: 4d anomaly from 5d $\mathrm{CS}_1^{\mathrm{U}(1)} c_1(\mathrm{U}(1))^2$ and 6d $c_1(\mathrm{U}(1))^3$
2. $\mathrm{U}(1)_Y$-$\mathrm{SU}(2)^2$: 4d anomaly from 5d $\mathrm{CS}_1^{\mathrm{U}(1)} c_2(\mathrm{SU}(2))$, 6d $c_1(\mathrm{U}(1)) c_2(\mathrm{SU}(2))$
3. $\mathrm{U}(1)_Y$-$\mathrm{SU}(3)_c^2$: 4d anomaly from 5d $\mathrm{CS}_1^{\mathrm{U}(1)} c_2(\mathrm{SU}(3))$, 6d $c_1(\mathrm{U}(1)) c_2(\mathrm{SU}(3))$
4. $\mathrm{U}(1)_Y$-(gravity)^2: 4d anomaly from 5d $\mu(\mathrm{PD}(c_1(\mathrm{U}(1))))$, 6d $\frac{c_1(\mathrm{U}(1))(\sigma - \mathrm{F}\cdot\mathrm{F})}{8}$
5. $\mathrm{SU}(3)_c^3$: 4d anomaly from 5d $\frac{1}{2}\mathrm{CS}_5^{\mathrm{SU}(3)}$, 6d $\frac{1}{2} c_3(\mathrm{SU}(3))$
6. 4d global Witten SU(2) anomaly when $q = 1, 3$ from 5d $c_2(\mathrm{SU}(2))\tilde{\eta}$, 6d $c_2(\mathrm{SU}(2))\mathrm{Arf}$. It becomes part of local anomaly in $\mathbb{Z}$ when $q = 2, 6$.

Davighi-Gripaios-Lohitsiri1910.11277, Wan-JW 1910.14668, Davighi-Lohitsiri2001.07731 JW 2006.16996.

$\mathrm{U}(1)_Y$-(gravity)^2: 4d local anomaly from 5d $\mu(\mathrm{PD}(c_1(\mathrm{U}(1))))$ and 6d $\frac{c_1(\mathrm{U}(1))(\sigma - \mathrm{F}\cdot\mathrm{F})}{8}$

Example:

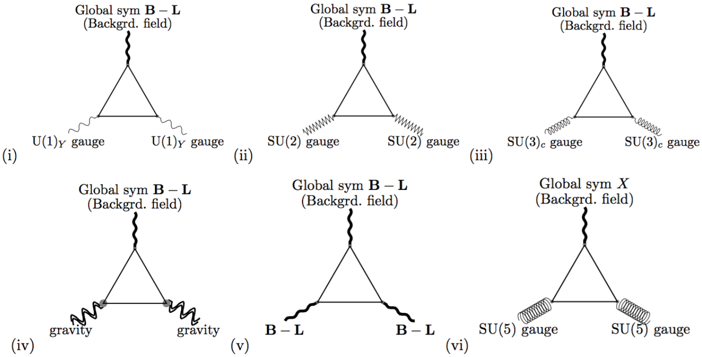


$$
\sum_{\mathrm{q}} \mathrm{Tr}[\hat{Y}_{\mathrm{q}}] = \sum_{\mathrm{q}_L, \mathrm{q}_R} (Y_{\mathrm{q}_L}) - (Y_{\mathrm{q}_R})
$$

$$
= N_{\text{generation}} \cdot \Big(N_c \cdot \big(2 \cdot (1/6) + (-2/3) + (1/3) \big) + 2 \cdot (-1/2) + (1) + (0) \Big)
$$

$$
= N_{\text{generation}} \cdot (0 \cdot N_c + 0) = 0.
$$

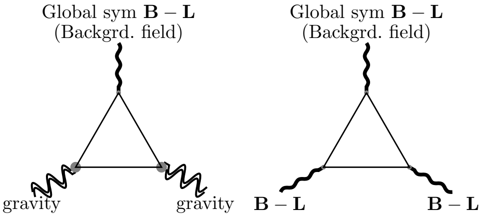
II. (Local) Anomalies of $\mathrm{Spin}^c \times G_{\mathrm{SM}_q}$ and $\mathrm{Spin}^c \times \mathrm{SU}(5)$



1. 4d local anomaly from $\mathrm{TP}_5(\mathrm{Spin}^c \times \frac{\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)}{\mathbb{Z}_q})$ and $\mathrm{TP}_5(\mathrm{Spin}^c \times \mathrm{SU}(5))$.

Wan-JW'19 1910.14668, JW'20 2006.16996, ...
Garcia-Etxebarria-Montero'18 1808.00009, Davighi-Gripaios-Lohitsiri'19 1910.11277

$(\mathbf{B}-\mathbf{L})$-(gravity)^2 and $(\mathbf{B}-\mathbf{L})^3$ as ABJ anomaly or 't Hooft anomaly



$$(\mathbf{B}-\mathbf{L})\text{-(gravity)}^2 \Rightarrow j_{\mathbf{B}} \quad : \quad N_{\text{generation}} \cdot (N_c/3) \cdot \Big(2-1-1\Big) = 0.$$

$$j_{\mathbf{L}} \quad : \quad N_{\text{generation}} \cdot \Big(2-1-N_{\nu_R}\Big) = N_{\text{generation}} \cdot (1-N_{\nu_R}).$$

$$(\mathbf{B}-\mathbf{L})^3 \Rightarrow j_{\mathbf{B}} \quad : \quad N_{\text{generation}} \cdot N_c \cdot (1/3)^3 \cdot \Big(2-1-1\Big) = 0.$$

$$j_{\mathbf{L}} \quad : \quad N_{\text{generation}} \cdot (1)^3 \cdot \Big(2-1-N_{\nu_R}\Big) = N_{\text{generation}} \cdot (1-N_{\nu_R})$$

$\mathrm{d} \star j_{\mathbf{B}} = 0$ but $\mathrm{d} \star (j_{\mathbf{B}} - j_{\mathbf{L}}) = 0$ only when $N_{\nu_R} = 1$.
Require the 16 Weyl fermion or break $(\mathbf{B}-\mathbf{L})$ or?
Do these anomaly survive when we embed: $\mathbb{Z}_{4,X} = Z(\mathrm{Spin}(10)) \subset \mathrm{U}(1)_X$ where $X = \mathbf{B}-\mathbf{L}$ or $5(\mathbf{B}-\mathbf{L}) - 4Y$?

III. (Global+Local) Anomalies: $\mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times G_{\text{internal/gauge}}$

Focus on $\mathbb{Z}_{4,X} = Z(\mathrm{Spin}(10)) \subset \mathrm{U}(1)_X$ where $X = \mathbf{B} - \mathbf{L}$ or $5(\mathbf{B} - \mathbf{L}) - 4Y$.

$G = \mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times G_{\mathrm{SM}_{q=6}}$ and $\mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times \mathrm{SU}(5)$:

	$G = \mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times G_{\mathrm{SM}_6}$	
5d	$\mathbb{Z}^5 \times \mathbb{Z}_2^2 \times \mathbb{Z}_4 \times \mathbb{Z}_{16}$	$\mu(\mathrm{PD}(c_1(\mathrm{U}(3))))$, $c_1(\mathrm{U}(3))^2 \mathrm{CS}_1^{\mathrm{U}(3)}$, $\frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \mathrm{CS}_3^{\mathrm{U}(2)} + \mathrm{CS}_1^{\mathrm{U}(3)} c_2(\mathrm{U}(2))}{2} \sim \frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \mathrm{CS}_3^{\mathrm{U}(2)} + c_1(\mathrm{U}(3)) \mathrm{CS}_3^{\mathrm{U}(2)}}{2}$, $\frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \mathrm{CS}_3^{\mathrm{U}(3)} + \mathrm{CS}_1^{\mathrm{U}(3)} c_2(\mathrm{U}(3)) + \mathrm{CS}_5^{\mathrm{U}(3)}}{2} \sim \frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \mathrm{CS}_3^{\mathrm{U}(3)} + c_1(\mathrm{U}(3)) \mathrm{CS}_3^{\mathrm{U}(3)} + \mathrm{CS}_5^{\mathrm{U}(3)}}{2}$, $\mathrm{CS}_5^{\mathrm{U}(3)}$, $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\mathrm{U}(3))$, $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\mathrm{U}(2))$, $c_1(\mathrm{U}(3))^2 \eta'$, $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$
	$G = \mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4 \times \mathrm{SU}(5)$	
5d	$\mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_{16}$	$\frac{(\mathcal{A}_{\mathbb{Z}_2})^2 \mathrm{CS}_3^{\mathrm{SU}(3)} + \mathrm{CS}_5^{\mathrm{SU}(3)}}{2}$, $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\mathrm{SU}(5))$, $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$

$\mathcal{A}_{\mathbb{Z}_2} = (\mathcal{A}_{\mathbb{Z}_4} \bmod 2) \in \mathrm{H}^1(M, \mathbb{Z}_2)$ is a generator $\mathrm{H}^1(\mathrm{B}(\mathbb{Z}_{4,X}/\mathbb{Z}_2^F), \mathbb{Z}_2)$ of $\mathrm{Spin} \times_{\mathbb{Z}_2^F} \mathbb{Z}_{4,X}$.

1. Witten SU(2) $c_2(\mathrm{SU}(2))\tilde{\eta}$ vs $c_2(\mathrm{SU}(2))\eta'$: 4d $\mathbb{Z}_2$ vs $\mathbb{Z}_4$ anomaly free ($q = 1, 3$)
2. $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\mathrm{SU}(2))$: 4d $\mathbb{Z}_2$ global anomaly free ($q = 2, 6$)
3. $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\mathrm{SU}(3))$: 4d $\mathbb{Z}_2$ global anomaly free
4. $c_1(\mathrm{U}(1))^2 \eta'$: 4d $\mathbb{Z}_4$ global anomaly free
5. $(\mathcal{A}_{\mathbb{Z}_2}) c_2(\mathrm{SU}(5))$: 4d $\mathbb{Z}_2$ global anomaly free
6. $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$: $\Omega_5^{\mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4} = \Omega_4^{\mathrm{Pin}^+} = \mathbb{Z}_{16}$. 4d $\mathbb{Z}_{16}$ global anomaly not matched?

$\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$: 4d $\mathbb{Z}_{16}$ global anomaly – how to match anomaly?

Count odd charge object of $\mathcal{A}_{\mathbb{Z}_2} = (\mathcal{A}_{\mathbb{Z}_4} \mod 2) \in \mathrm{H}^1(M, \mathbb{Z}_2)$ is the generator from $\mathrm{H}^1(\mathrm{B}(\mathbb{Z}_{4,X}/\mathbb{Z}_2^F), \mathbb{Z}_2)$ of $\mathrm{Spin} \times_{\mathbb{Z}_2^F} \mathbb{Z}_{4,X}$.

SM fermion spinor field	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_{Y_W}$	$\mathrm{U}(1)_{\mathbf{B-L}}$	$\mathrm{U}(1)_X$	$\mathbb{Z}_{4,X}$	$\mathbb{Z}_2^F$
$\bar{d}_R$	1/3	2/3	-1/3	−3	1	1
l_L	−1/2	−1	-1	−3	1	1
q_L	1/6	1/3	1/3	1	1	1
$\bar{u}_R$	−2/3	−4/3	−1/3	1	1	1
$\bar{e}_R = e_L^+$	1	2	1	1	1	1
$\bar{\nu}_R = \nu_L$	0	0	1	5	1	1

The anomaly index in $\mathbb{Z}_{16}$ for total $N_{\text{family}}(= 3)$ as $15 \cdot N_{\text{family}} = -1 \cdot N_{\text{family}} \mod 16$.
$N_{\text{family}} \cdot (-1 + N_{\nu_R} + \text{ hidden sector }) = 0 \mod 16$. Anomaly-matching?
(1) R-handed neutrino (16th Weyl) $N_{\nu_R} = 1$. $\mathbb{Z}_{4,X}$
preserved (gapless) vs broken (gap) by **Dirac vs Majorana** mass.

(2) new hidden sectors beyond SM:

1. $\mathbb{Z}_{4,X}$-symmetry-preserving anomalous gapped 4d TQFT (**Topological** Mass).
2. $\mathbb{Z}_{4,X}$-SPTs by 5d cobordism invariant $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$.
3. $\mathbb{Z}_{4,X}$-gauged-5d-(Symmetry)-Enriched Topological state (SETs) + gravity.
4. $\mathbb{Z}_{4,X}$-symmetry-breaking gapped 4d TQFT.

$\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$: 4d $\mathbb{Z}_{16}$ global anomaly – how to match anomaly?

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5d $\mathbb{Z}_{4,X}$-SPTs by a cobordism invariant $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$.
What exactly is the 4d TQFT matching the anomaly?

Based on the Symmetry-Extension method and generalization.

JW-Wen-Witten '17 arXiv:1705.06728. Putrov-Thorngren-JW, ... (in preparation)

Simplified version: a certain non-perturbative global (gauge/gravitational) anomaly in G becomes anomaly-free in $\hat{G}$. via a suitable group extension $1 \to K \to \hat{G} \to G \to 1$.
We can construct a topological K-gauge theory (TQFT) with G anomaly (becomes anomaly-free in $\hat{G}$).

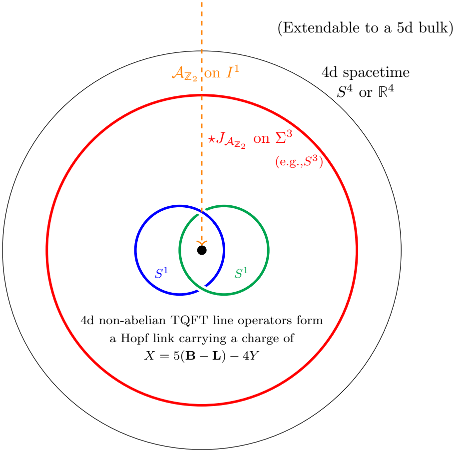
What are physical observables for us (from SM) and for 4d TQFT or 5d SPT?

Quantum communication with "God" from our SM physics?

5d $\mathbb{Z}_{4,X}$-SPTs by a cobordism invariant $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$, and 4d TQFT.

Quantum communication with "God" from our SM physics?

5d $\mathbb{Z}_{4,X}$-SPTs by a cobordism invariant $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$, and 4d TQFT.
One end has SM particles (we), another end has extended objects of TQFT (God)?



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$\bar{e}_R = e_L^+$	1	2	1	1	1	1
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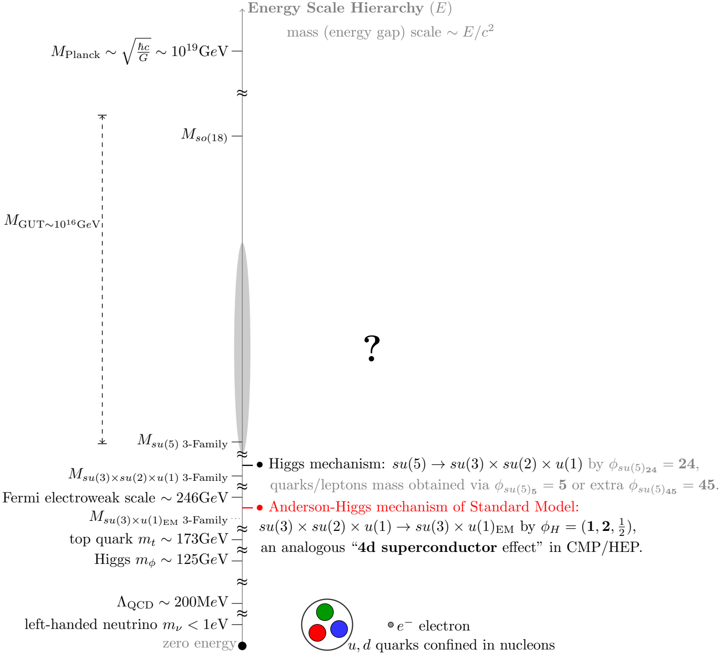
Braiding statistics and link invariants in 4d or 5d.

Energy Hierarchy: Who live above us heavily Upstair/Unknown (?)



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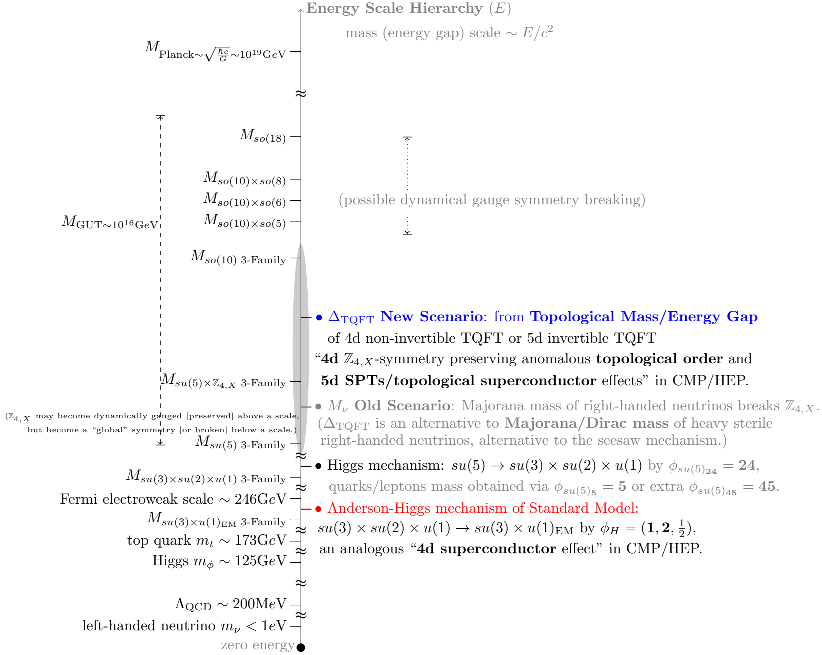


Spacetime-Internal Group Embedding and Cobordism

$$
\begin{tikzcd}
 & \mathrm{Spin} \times_{\mathbb{Z}_2^F} \mathrm{Spin}(18) \arrow[dl] \arrow[dr] & \\
\mathrm{Spin} \times \mathrm{SU}(9) \arrow[d] & & \mathrm{Spin} \times_{\mathbb{Z}_2^F} (\mathrm{Spin}(10) \times_{\mathbb{Z}_2^F} \mathrm{Spin}(8)) \arrow[d] \\
\mathrm{Spin} \times \mathrm{SU}(5) \times \mathrm{SU}(4) \arrow[d] & & \mathrm{Spin} \times_{\mathbb{Z}_2^F} (\mathrm{Spin}(10) \times_{\mathbb{Z}_2^F} \mathrm{Spin}(5)) \arrow[d] \\
\mathrm{Spin} \times \mathrm{SU}(5) \times \mathrm{Spin}(5) \arrow[dr] & & (\mathrm{Spin} \times_{\mathbb{Z}_2^F} \mathrm{Spin}(10))_{\text{3-Family}} \arrow[dl] \\
 & (\mathrm{Spin} \times \mathrm{SU}(5))_{\text{3-Family}} \arrow[d] & \\
 & (\mathrm{Spin} \times \frac{\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)}{\mathbb{Z}_6})_{\text{3-Family}} \arrow[d] & \\
 & (\mathrm{Spin} \times \frac{\mathrm{SU}(3) \times \mathrm{U}(1)_{\mathrm{EM}}}{\mathbb{Z}_3})_{\text{3-Family}} &
\end{tikzcd}
$$

We consider the cobordism classifications of anomalies for these group.

SO(18) GUT Wilczek-Zee, Fujimoto'82. no fermion doubling (vs Nielsen-Ninomiya):
gap: Eichten-Preskill'86, Kitaev,Wen,Wang'13, You-Xu'14, BenTov-Zee'15, Kikukawa'17, JW-Wen'18.
a new SU(2) anomaly: JW-Wen-Witten'18, SU(2)=Spin(3)⊂Spin(10)⊂Spin(18)

[Recall] My Aim of this talk:

Are there non-perturbative constraints given the low energy physics at SM, guiding us toward something heavy at higher energy?

1. Global structure of Lie group (more than local Lie algebra)
2. Discrete gauge/global symmetries
3. (Invertible) Anomalies (4d on 5d SPTs):
   Perturbative Local ($\mathbb{Z}$ class)+
   Non-Perturbative Global Anomalies ($\mathbb{Z}_n$ class)
4. Classification of Anomalies via Cobordism
5. Symmetric gapping rule, Kitaev-Wen mechanism, symmetric mass (mass without breaking "chiral symmetry") without 't Hooft anomaly. $\nu = 0 \mod n$ of $\mathbb{Z}_n$.
6. Symmetric anomalous TQFT/topological order with 't Hooft anomaly. $\nu \neq 0 \mod n$ of $\mathbb{Z}_n$.
7. Energy hierarchy
8. String landscape/swampland + no global symmetry in quantum gravity

Energy Scale Hierarchy (E)

mass (energy gap) scale $\sim E/c^2$

$M_{\text{Planck}} \sim \sqrt{\frac{\hbar c}{G}} \sim 10^{19}\text{GeV}$

$\approx$

• $\Delta_{\text{KW}_1.so(18)}$

$M_{so(18)}$

$M_{so(10)\times so(8)}$

$M_{so(10)\times so(6)}$

(possible dynamical gauge symmetry breaking)

$M_{\text{GUT}\sim 10^{16}\text{GeV}}$

$M_{so(10)\times so(5)}$

$M_{so(10)}$ 3-Family

• Δ_{TQFT} **New Scenario**: from **Topological Mass/Energy Gap**
of 4d non-invertible TQFT or 5d invertible TQFT
"**4d** $\mathbb{Z}_{4,X}$-symmetry preserving anomalous **topological order** and
5d SPTs/topological superconductor effects" in CMP/HEP.

$M_{su(5)\times\mathbb{Z}_{4,X}}$ 3-Family

• M_ν **Old Scenario**: Majorana mass of right-handed neutrinos breaks $\mathbb{Z}_{4,X}$.
(Δ_{TQFT} is an alternative to **Majorana/Dirac mass** of heavy sterile
right-handed neutrinos, alternative to the seesaw mechanism.)

($\mathbb{Z}_{4,X}$ may become dynamically gauged [preserved] above a scale,
but become a "global" symmetry [or broken] below a scale.)

$M_{su(5)}$ 3-Family

$\approx$

• Higgs mechanism: $su(5) \to su(3)\times su(2)\times u(1)$ by $\phi_{su(5)_{\mathbf{24}}} = \mathbf{24}$,
quarks/leptons mass obtained via $\phi_{su(5)_{\mathbf{5}}} = \mathbf{5}$ or extra $\phi_{su(5)_{\mathbf{45}}} = \mathbf{45}$.

$M_{su(3)\times su(2)\times u(1)}$ 3-Family

$\approx$

Fermi electroweak scale $\sim 246\text{GeV}$

• Anderson-Higgs mechanism of Standard Model:
$su(3)\times su(2)\times u(1) \to su(3)\times u(1)_{\text{EM}}$ by $\phi_H = (\mathbf{1},\mathbf{2},\frac{1}{2})$,
an analogous "**4d superconductor** effect" in CMP/HEP.

$M_{su(3)\times u(1)_{\text{EM}}}$ 3-Family

$\approx$

top quark $m_t \sim 173\text{GeV}$

$\approx$

Higgs $m_\phi \sim 125\text{GeV}$

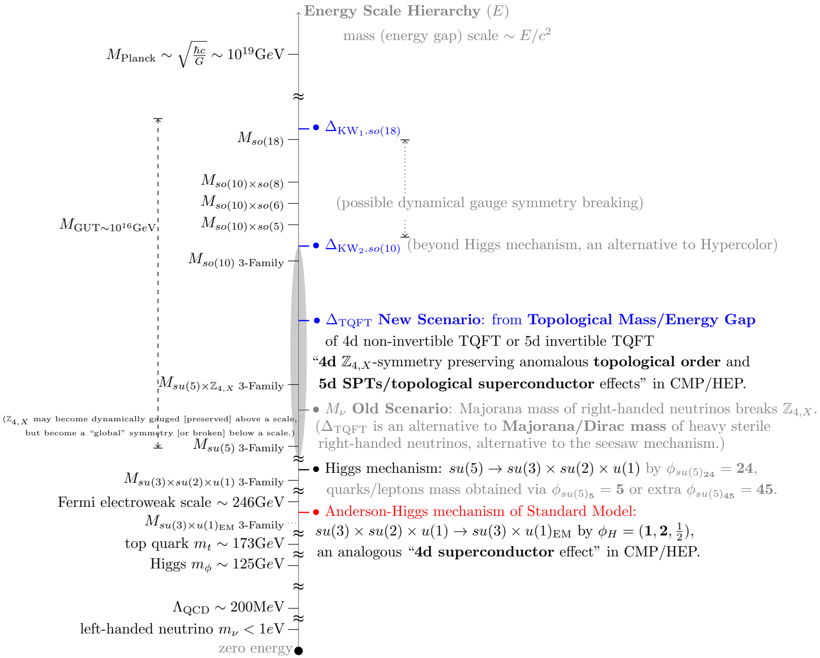
$\approx$

$\Lambda_{\text{QCD}} \sim 200\text{MeV}$

$\approx$

left-handed neutrino $m_\nu < 1\text{eV}$

zero energy



Energy Scale Hierarchy (E)
mass (energy gap) scale $\sim E/c^2$
$M_{\text{Planck}} \sim \sqrt{\frac{\hbar c}{G}} \sim 10^{19}\text{GeV}$
$\bullet\ \Delta_{\text{KW}_1.so(18)}$
$M_{so(18)}$
$M_{so(10)\times so(8)}$
$M_{so(10)\times so(6)}$
(possible dynamical gauge symmetry breaking)
$M_{\text{GUT}\sim 10^{16}\text{GeV}}$
$M_{so(10)\times so(5)}$
$\bullet\ \Delta_{\text{KW}_2.so(10)}$ (beyond Higgs mechanism, an alternative to Hypercolor)
$M_{so(10)}$ 3-Family
$\bullet\ \Delta_{\text{TQFT}}$ **New Scenario**: from **Topological Mass/Energy Gap**
of 4d non-invertible TQFT or 5d invertible TQFT
"**4d** $\mathbb{Z}_{4,X}$-symmetry preserving anomalous **topological order** and
5d SPTs/topological superconductor effects" in CMP/HEP.
$M_{su(5)\times\mathbb{Z}_{4,X}}$ 3-Family
$\bullet\ M_\nu$ **Old Scenario**: Majorana mass of right-handed neutrinos breaks $\mathbb{Z}_{4,X}$.
(Δ_{TQFT} is an alternative to **Majorana/Dirac mass** of heavy sterile
right-handed neutrinos, alternative to the seesaw mechanism.)
($\mathbb{Z}_{4,X}$ may become dynamically gauged [preserved] above a scale,
but become a "global" symmetry [or broken] below a scale.)
$M_{su(5)}$ 3-Family
$\bullet$ Higgs mechanism: $su(5) \to su(3)\times su(2)\times u(1)$ by $\phi_{su(5)_{\mathbf{24}}} = \mathbf{24}$,
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$M_{su(3)\times su(2)\times u(1)}$ 3-Family
Fermi electroweak scale $\sim 246\text{GeV}$
$\bullet$ Anderson-Higgs mechanism of Standard Model:
$su(3)\times su(2)\times u(1) \to su(3)\times u(1)_{\text{EM}}$ by $\phi_H = (\mathbf{1}, \mathbf{2}, \frac{1}{2})$,
an analogous "**4d superconductor** effect" in CMP/HEP.
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top quark $m_t \sim 173\text{GeV}$
Higgs $m_\phi \sim 125\text{GeV}$
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left-handed neutrino $m_\nu < 1\text{eV}$
zero energy

Energy Hierarchy: Who live above us heavily Upstair/Unknown (?)

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(possible dynamical gauge symmetry breaking)

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- $\Delta_{\text{KW}_2 . so(10)}$ (beyond Higgs mechanism, an alternative to Hypercolor)

$M_{so(10)}$ 3-Family

- Higgs mechanism: $so(10) \to su(5)$ by $\phi_{so(10)_{\mathbf{16}}} = \mathbf{16}$ (for Δ_{TQFT}) *or* by $\phi_{so(10)_{\mathbf{126}}} = \mathbf{126}$ (for sterile Majorana mass M_ν).
  $so(10) \to su(3) \times su(2) \times u(1)$ by another $\phi_{so(10)_{\mathbf{45}}} = \mathbf{45}$ *or* $\phi_{so(10)_{\mathbf{54}}} = \mathbf{54}$.
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Spacetime-Internal Group Embedding and Cobordism

$$
\begin{tikzcd}
 & \mathrm{Spin} \times_{\mathbb{Z}_2^F} \mathrm{Spin}(18) \arrow[dl] \arrow[dr] & \\
\mathrm{Spin} \times \mathrm{SU}(9) \arrow[d] & & \mathrm{Spin} \times_{\mathbb{Z}_2^F} (\mathrm{Spin}(10) \times_{\mathbb{Z}_2^F} \mathrm{Spin}(8)) \arrow[d] \\
\mathrm{Spin} \times \mathrm{SU}(5) \times \mathrm{SU}(4) \arrow[d] & & \mathrm{Spin} \times_{\mathbb{Z}_2^F} (\mathrm{Spin}(10) \times_{\mathbb{Z}_2^F} \mathrm{Spin}(5)) \arrow[d] \\
\mathrm{Spin} \times \mathrm{SU}(5) \times \mathrm{Spin}(5) \arrow[dr] & & (\mathrm{Spin} \times_{\mathbb{Z}_2^F} \mathrm{Spin}(10))_{\text{3-Family}} \arrow[dl] \\
 & (\mathrm{Spin} \times \mathrm{SU}(5))_{\text{3-Family}} \arrow[d] & \\
 & (\mathrm{Spin} \times \frac{\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)}{\mathbb{Z}_6})_{\text{3-Family}} \arrow[d] & \\
 & (\mathrm{Spin} \times \frac{\mathrm{SU}(3) \times \mathrm{U}(1)_{\mathrm{EM}}}{\mathbb{Z}_3})_{\text{3-Family}} &
\end{tikzcd}
$$

We consider the cobordism classifications of anomalies for these group.

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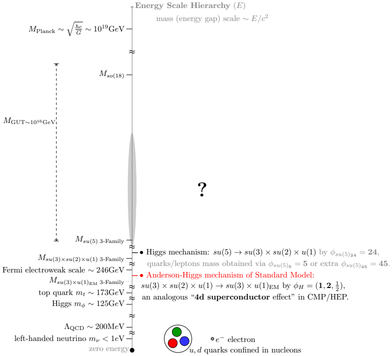
Summary

Maxwell (1865): electric+magnetism, and light unification.
Glashow-Salam-Weinberg [GSW] (1961-1967): Electroweak unification.
GSW+ Strong force: Standard Model (SM) in 3+1d (4d) spacetime.
Grand Unification/Unified Theory (GUT): Georgi-Glashow (1974) SU(5), Georgi or Fritzsch-Minkowski SO(10) GUT (a Spin(10) gauge group). Electroweak+Strong+other forces in a simple Lie group.
Ultra Unification (UU):
2006.16996: Electroweak+Strong+other forces + topological sector, e.g.,
• 4d topological field theory [Schwarz type unitary **TQFT**] at low energy below an energy gap. A long-range entangled intrinsic topological order,
• 5d Symmetry-Protected Topological state [**SPTs**]. Invertible TQFT. More than a 2nd Chern class θ-term $F \wedge F$.
Equations/Actions: Weyl spinors + Yukawa-Higgs-Dirac term + U(1) Maxwell + SU(N) or Spin(N) Yang-Mills + unitary 4d non-invertible / 5d invertible or non-invertible TQFT
Disclaimer: Not directly related to Super/String/M theory. Not directly related to Unification by Qubit or Emergence Principle. So UU is just an extension of GUT from a low energy QFT plus **heavy** topological sector — may be a new direction or a small humble step to literature. No Einstein gravity. No dynamical gravity, until the very end when all spacetime-internal symmetry is gauged.

Summary - Breakout.Discussion.Room 3: 968 3687 3372 pw:008829

Ultra Unification = SM/Grand Unification + Topological Sector (Mass/Force)

Energy Hierarchy: Who live above us heavily Upstair (?)



New Lives at higher energy that we cannot "see/detect" via SM?

"Communicate" via the gauged $\mathbb{Z}_{4,X}$ line/surface operator (topological force).

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Ultra Unification = SM/Grand Unification + Topological Sector (Mass/Force)

Energy Hierarchy: Who live above us heavily Upstair (?)

"Der schwer gefaßte Entschluß. Muß es sein? Es muß sein!"

"The heavy (high energy and gapped) decision. Must it be? It must be!"

String Quartet No. 16 in F major, op.135

Ludwig van **Beethoven** in 1826

New Lives at higher energy that we cannot "see/detect" via SM?

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Vielen Dank. Bleib gesund und sicher.
Thank you for staying tuned. Stay healthy and safe.

Back Up Slides: