Schrödinger model of minimal representations and branching problems

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Minimal Representations and Theta Correspondence:
In honor of Gordan Savin for his 60th birthday

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Global Analysis on “Minimal Representations”

Motif

Our guiding principle*

viewed from “G or $U(\mathfrak{g})$”		
Algebra	small rep	
	$\wr\downarrow$	Geometric Realization
Analysis	large symmetry	
viewed from “function space”		

* T. Kobayashi, Algebraic analysis of minimal representations, Publ. RIMS **47** (2011), 585–611.

Restriction to compact subgroups K'

In 2001 Spring, I gave a course lecture at Harvard. Gordan was returning there. My course intended to elucidate a phenomenon “discrete decomposability” of the restriction to non-compact subgroups.
A key is to prove (i) $\Longrightarrow$ (ii) in Theorem 1 below.
I was asked if the converse (ii) $\Longrightarrow$ (i) holds.

G: real reductive group, K: max compact subgroup.

Theorem 1 (K– 1998*, 2021**) Suppose $\Pi \in \mathrm{Irr}(G)$ and $K' \subset K$. Then (i) $\Longleftrightarrow$ (ii).
(i) $\mathrm{AS}_K(\Pi) \cap C_K(K') = \{0\}$
(ii) $[\Pi|_{K'} : \pi] < \infty \qquad {}^\forall \pi \in \mathrm{Irr}(K')$.

$\mathrm{AS}_K(\Pi)$: asymptotic K-support of Π,
$C_K(K')$: momentum set of $T^*(K/K')$.

Remark. $C_K(K') = \{0\}$ if $K' = K \rightsquigarrow$ HC's admissibility theorem.

* (i) $\Rightarrow$ (ii) Kobayashi, Ann Math 1998; (ii) $\Rightarrow$ (i) Kobayashi, PAMQ 2021 (Kostant memorial issue).

Admissible restriction $\Pi|_{G'}$

$\rightsquigarrow$ Classification of triples (G, G', Π) such that

$$\begin{cases} G \supset G' \text{ reductive symmetric pair} \\ \Pi \in \mathrm{Irr}(G) \text{ is minimal rep}^{*} / \, A_{\mathfrak{q}}(\lambda)^{**} \\ \text{the restriction } \Pi|_{G'} \text{ is } \underline{G'\text{-admissible}}, \end{cases}$$

i.e., discretely decomposable with finite multiplicity.

$$\begin{array}{ccc} G & \supset & G' \\ \cup & & \cup \\ K & \supset & K' \end{array}$$

* Kobayashi–Y. Oshima, Crelles (2015) 201–223; ** Adv Math (2012) 2013–2047.

Definition: Multiplicity of the restriction $\Pi|_{G'}$

G : real reductive Lie group

$\mathcal{M}(G)$: smooth admissible reps of G of finite length
with moderate growth (defined on Fréchet spaces)
$\mathrm{Irr}(G)$: irreducible objects

$G \supset G'$: real reductive Lie groups

Def (multiplicity) For $\Pi \in \mathrm{Irr}(G)$ and $\pi \in \mathrm{Irr}(G')$, we set

$$[\Pi|_{G'} : \pi] = \dim_{\mathbb{C}} \mathrm{Hom}_{G'}(\Pi|_{G'}, \pi) \in \mathbb{N} \cup \{\infty\}$$

symmetry breaking operators

Introduction 1: Multiplicity in tensor product

Let G be a non-compact simple Lie group.

Fact 1* (K- '95) (i) $\Longleftrightarrow$ (ii) holds.

(i) $[\Pi_1 \otimes \Pi_2 : \Pi] < \infty \quad {}^\forall\Pi_1, {}^\forall\Pi_2, {}^\forall\Pi \in \mathrm{Irr}(G)$.

(ii) $\mathfrak{g} \simeq \mathfrak{so}(n, 1)$.

$\rightsquigarrow$ Tensor product $\Pi_1 \otimes \Pi_2$ is "usually" of infinite multiplicity!

T. Kobayashi, Introduction to harmonic analysis on real spherical homogeneous spaces, 1995, 22–41.

Introduction 2: Restriction for symmetric pairs

More generally,

Fact 2[*] For a pair $G \supset G'$ of real reductive group, (i) $\iff$ (ii).

(i) (Rep) $[\Pi|_{G'} : \pi] < \infty \quad {}^\forall\Pi \in \mathrm{Irr}(G), {}^\forall\pi \in \mathrm{Irr}(G')$.

(ii) (Geometry) $(G \times G')/\mathrm{diag}(G')$ is real spherical.

Even for symmetric pairs (G, G'), this condition may fail.

Example[**] (1) $(G, G') = (SL(n, \mathbb{R}), SO(p, q)) \quad p + q = n$

(i) $\iff$ $p = 0$, $q = 0$, or $p = q = 1$

(2) $(G, G') = (O(p, q), O(p_1, q_1) \times O(p_2, q_2))$.

(i) $\iff$ $p_1 + q_1 = 1$, $p_2 + q_2 = 1$, $p = 1$, $q = 1$, or G' compact.

$\rightsquigarrow$ Multiplicity of the restriction $\Pi|_{G'}$ may be infinite even when G' is maximal in G.

* K–T. Oshima, Adv. Math, (2013). ** K–Matsuki, Transformation Group, (2014) (special issue for Dynkin).

Question: Bounded multiplicity $\Pi|_{G'}$ for “small” Π

Question Given a reductive symmetric pair $G \supset G'$.

Does there exist at least one infinite-dim'l $\Pi \in \mathrm{Irr}(G)$ with the following property?

(finite) $$[\Pi|_{G'} : \pi] < \infty \quad {}^\forall \pi \in \mathrm{Irr}(G'),$$

or more strongly

(bounded) $$\sup_{\pi \in \mathrm{Irr}(G')} [\Pi|_{G'} : \pi] < \infty.$$

Uniformly bounded multiplicities

$\Omega \subset \mathrm{Irr}(G)$, $\quad G \supset G'$

<u>Question</u> Find a criterion for the triple (G, G', Ω) such that

$$\sup_{\Pi \in \Omega} \sup_{\pi \in \mathrm{Irr}(G')} [\Pi|_{G'} : \pi] < \infty.$$

Answer in terms of geometric condition (spherical/visible action):

$\Omega = \mathrm{Irr}(G)$	$G_{\mathbb{C}} \times G'_{\mathbb{C}} \curvearrowright (G_{\mathbb{C}} \times G'_{\mathbb{C}})/\operatorname{diag} G'_{\mathbb{C}}$	*
$\Omega = \{H\text{-distinguished reps}\}$	$G'_{\mathbb{C}} \curvearrowright G_{\mathbb{C}}/B_{G/H}$	**
$\Omega = \{\mathrm{Ind}_P^G(\mathbb{C}_\lambda)\}$	$G'_{\mathbb{C}} \curvearrowright G_{\mathbb{C}}/P_{\mathbb{C}}$	***

$\rightsquigarrow$ classification

* Kobayashi–T. Oshima, Adv. Math. 2014, ** Kobayashi, Adv. Math. 2021, *** Kobayashi, J. Lie Theory (2022) 197–238.

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or more strongly

(bounded) $\sup_{\pi \in \mathrm{Irr}(G')} [\Pi|_{G'} : \pi] < \infty$.

Bounded multiplicity theorems

Let G be a 1-connected real non-compact simple Lie group.

Theorem A (K–) There exist $C > 0$ and infinite-dimensional irreducible reps Π_1, Π_2 of G such that

$$\sup_{\Pi \in \mathrm{Irr}(G)} [\Pi_1 \otimes \Pi_2 : \Pi] \leq C.$$

Theorem B (K–) There exist $C > 0$ and an infinite-dimensional irreducible rep Π of G such that

$$\sup_{\pi \in \mathrm{Irr}(G')} [\Pi|_{G'} : \pi] \leq C$$

for *all* symmetric pairs $G \supset G'$.

Review: Complex minimal nilpotent orbit

$\mathfrak{g}_{\mathbb{C}}$: simple Lie algebra $/\mathbb{C}$
$\mathfrak{g}_{\mathbb{C}}^* \supset \mathbb{O}_{\min,\mathbb{C}}$: $^{\exists 1}$ minimal coadjoint orbit ($\neq \{0\}$).

$n(\mathfrak{g}_{\mathbb{C}}) :=$ half the complex dimension of $\mathbb{O}_{\min,\mathbb{C}}$

$\mathfrak{g}_{\mathbb{C}}$	A_n	B_n	C_n	D_n	$\mathfrak{g}_2^{\mathbb{C}}$	$\mathfrak{f}_4^{\mathbb{C}}$	$\mathfrak{e}_6^{\mathbb{C}}$	$\mathfrak{e}_7^{\mathbb{C}}$	$\mathfrak{e}_8^{\mathbb{C}}$
$n(\mathfrak{g}_{\mathbb{C}})$	n	$2n-2$	n	$2n-3$	3	8	11	17	29

Remark Let G be a Lie group such that $\mathfrak{g}_{\mathbb{C}}$ is simple.
$\Longrightarrow \mathrm{DIM}(\Pi) \geq n(\mathfrak{g}_{\mathbb{C}})$ $\quad {}^{\forall}$ infinite-dim'l $\Pi \in \mathrm{Irr}(G)$.

Example $\mathrm{DIM}(\Pi) = n(\mathfrak{g}_{\mathbb{C}})$ if Π is a minimal rep.

Review: Minimal representation (Definition)

$\mathcal{J}$: Joseph ideal $\cdots$ completely prime two-sided primitive ideal whose associated variety is $\mathbb{O}_{\min,\mathbb{C}} \cup \{0\}$.

<u>Definition</u> $\Pi \in \mathrm{Irr}(G)$ is <u>minimal representation</u> if the annihilator of Π in $U(\mathfrak{g}_{\mathbb{C}})$ is the Joseph ideal.

<u>Example</u> The two irreducible components of the Segal–Shale–Weil rep are min reps of $G = Mp(n, \mathbb{R})$.

Classification: Gan–Savin*, Tamori**.

* W. T. Gan, G. Savin, On minimal representations definitions and properties, Represent. Theory **9** (2005), 46–93.

** H. Tamori, Classification of minimal representations of real simple Lie groups. Math. Z. **292** (2019), 387–402.

Bounded multiplicity property for tensor product

Theorem A (K–) There exist $C > 0$ and infinite-dimensional irreducible reps Π_1, Π_2 of G such that
$$\sup_{\Pi \in \mathrm{Irr}(G)} [\Pi_1 \otimes \Pi_2 : \Pi] \leq C.$$

Theorem A'(K-)* $\Pi_1, \Pi_2 \in \mathrm{Irr}(G)$ with $\mathrm{DIM}(\Pi_1) = \mathrm{DIM}(\Pi_2) = n(\mathfrak{g}_{\mathbb{C}})$
$\Longrightarrow$ ${}^{\exists}C > 0$ such that $[\Pi_1 \otimes \Pi_2 : \Pi] \leq C$ ${}^{\forall}\Pi \in \mathrm{Irr}(G)$

Example (Tensor product of two Weil reps)

$$L^2(\mathbb{R}^n) \otimes L^2(\mathbb{R}^n) \underset{\text{Wigner transform}}{\simeq} L^2(\mathbb{R}^{2n})$$

$$Mp(n,\mathbb{R}) \times Mp(n,\mathbb{R}) \underset{\text{diag}}{\hookleftarrow} Mp(n,\mathbb{R}) \to Sp(n,\mathbb{R})$$

* T. Kobayashi, Multiplicity in restricting minimal representations, PROMS (2022). Available also at arXiv:2204.05079

Bounded multiplicity theorem

Let G be a simple Lie group, not complex.

Theorem B'(K-)* If $\Pi \in \mathrm{Irr}(G)$ satisfies $\mathrm{DIM}(\Pi) = n(\mathfrak{g}_{\mathbb{C}})$, then $^{\exists}C > 0$ such that

$$[\Pi|_{G'} : \pi] \leq C \quad {}^{\forall}\pi \in \mathrm{Irr}(G')$$

for all symmetric pairs (G, G').

* T. Kobayashi, Multiplicity in restricting minimal representations, PROMS, (2022). Available also at ArXiv:2204.05079

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Algebra small rep

$\wr$ Geometric Realization

Analysis large symmetry

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* T. Kobayashi, Algebraic analysis of minimal representations, Publ. RIMS **47** (2011), 585–611.

Example 2. $O(p,q) \downarrow O(p',q') \times O(p'',q'')$

ϖ: minimal representation of $G = O(p,q)$ ($p + q \geq 8$, even)

Example 2.* (Branching law $\varpi|_{G'}$ using conformal geometry)

Suppose that $p' + p'' = p$, $q' + q'' = q$, and $p + q$ even

$$\begin{array}{ccc} G & = & O(p,q) \\ \cup & & \cup \\ G' & = & O(p',q') \times O(p'',q'') \end{array}$$

- Conformal construction of the min rep ϖ by the Yamabe operator
- Geometric construction of discrete spectrum of the restriction $\varpi|_{G'}$

  $\cdots$ conformal group v.s. isometry group

* Kobayashi–Ørsted, Analysis on minimal representations I,II,III, Adv. Math., (2003) 486–595.

Example 3. Plancherel-type theorem for the restriction $\Pi|_{G'}$

Joseph ideal is not defiend for $\mathfrak{sl}(n, \mathbb{C})$. But Theorem B' still applies.

Example 3.* Let $G = SL(n, \mathbb{R})$ and $\Pi_\lambda = \mathrm{Ind}_P^G(\mathbb{C}_\lambda)$ be a unitarily induced rep from a mirabolic subgroup P of G. The Plancherel theorem for $\Pi_\lambda|_{G'}$ is proved for all symmetric pairs (G, G'):

let $n = p + q$; $n = 2m$ (n even)

- $G' = SO(p, q)$ — cont spectrum (multiplicity 2) + discrete spectrum (mult. free)
- $G' = Sp(m, \mathbb{R})$ — almost irreducible
- $G' = SL(m, \mathbb{C}) \cdot \mathbb{T}$ — discretely decomposable (mult. free)
- $G' = S(GL(p, \mathbb{R}) \times GL(q, \mathbb{R}))$ — no discrete spectrum (mult. free)

* Kobayashi–Ørsted–Pevzner, Geometric analysis on small unitary representations of $GL(N, \mathbb{R})$, J. Funct. Anal. **260** (2011), 1682–1720.

Sketch of Proof for Theorems A and B

Let G be a 1-connected real non-compact simple Lie group.

<u>Theorem A</u> (K–) There exist $C > 0$ and infinite-dimensional irreducible reps Π_1, Π_2 of G such that

$$\sup_{\Pi \in \mathrm{Irr}(G)} [\Pi_1 \otimes \Pi_2 : \Pi] \leq C.$$

<u>Theorem B</u> (K–) There exist $C > 0$ and an infinite-dimensional irreducible rep Π of G such that

$$\sup_{\pi \in \mathrm{Irr}(G')} [\Pi|_{G'} : \pi] \leq C$$

for *all* symmetric pairs $G \supset G'$.

smallest GK dim

$\longleftarrow$ Theorem A', $+\,\alpha$

$\longleftarrow$ Theorem B', $+\,\alpha$

(i) finite-dim'l reps *vs* (ii) infinite-dim'l reps

$$\begin{array}{ccccc} P & \subset & G & \supset & G' \\ \cap & & \cap & & \cap \\ P_{\mathbb{C}} & \underset{\text{parabolic}}{\subset} & G_{\mathbb{C}} & \underset{\text{reductive}}{\supset} & G'_{\mathbb{C}} \end{array}$$

<u>Theorem A"(K-)</u>* (i) $\iff$ (ii) on (G, P_1, P_2)

(i) $\mathcal{O}(G_{\mathbb{C}}/P_{1,\mathbb{C}}, \mathcal{L}_{\lambda_1}) \otimes \mathcal{O}(G_{\mathbb{C}}/P_{2,\mathbb{C}}, \mathcal{L}_{\lambda_2})$ is multiplicity-free $^\forall \lambda_1, {}^\forall \lambda_2$.

(ii) $\sup_{\pi \in \mathrm{Irr}(G')} [\mathrm{Ind}_{P_1}^{G}(\mathbb{C}_{\lambda_1}) \otimes \mathrm{Ind}_{P_2}^{G}(\mathbb{C}_{\lambda_2}) : \pi] < \infty$.

<u>Theorem B"(K-)</u>* (i) $\iff$ (ii) on (G, P, G')

(i) $\mathcal{O}(G_{\mathbb{C}}/P_{\mathbb{C}}, \mathcal{L}_{\lambda})|_{G'_{\mathbb{C}}}$ is multiplicity-free λ.

(ii) $\sup_{\pi \in \mathrm{Irr}(G')} [\mathrm{Ind}_{P}^{G}(\mathbb{C}_{\lambda})|_{G'} : \pi] < \infty$.

* T. Kobayashi, Bounded multiplicity theorems for induction and restriction, J. Lie Theory, **32** (2022), 197–238.

Sketch of Proof for Theorems A and B

	smallest GK dim	degenerate ps	
Theorem A ⟵	Theorem A'*,	Theorem A'', **	⋯
Theorem B ⟵	Theorem B'*,	Theorem B''**,	⋯
Geometry in proof	coisotropic action on associated variety	visible action (or spherical action) on generalized flag variety	

* Kobayashi, ArXiv:2204.05079; ** Kobayashi, J. Lie Theory, **32** (2022) 197–238.

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Schrödinger model of minimal reps

$\mathfrak{g} = \mathfrak{n}^- + \mathfrak{l} + \mathfrak{n}^+ \qquad \mathfrak{n}^\pm$abelian

$G \overset{\pi}{\curvearrowright} L^2(\Xi) \qquad \mathbb{O}_{\min,\mathbb{C}} \cap \mathfrak{g}^* \underset{\text{Lagrangean}}{\supset} \Xi := \mathbb{O}_{\min,\mathbb{C}} \cap \mathfrak{n}^+.$

$\mathcal{F}_\Xi = \pi(w)$ unitary inversion operator. cf. $P^+ w P^+ \underset{\text{open}}{\subset} G$

Example $G = Mp(N, \mathbb{R}) \curvearrowright L^2(\Xi) \simeq L^2(\mathbb{R}^N)_{\text{even}}$

$\Xi = \{X \in \mathrm{Sym}(N, \mathbb{R}) : \mathrm{rank}\, X = 1\} \overset{2:1}{\twoheadleftarrow} \mathbb{R}^N \setminus \{0\}$

$\mathcal{F}_\Xi \cdots$ Fourier transorm $\quad f \mapsto \int f(x) e^{\sqrt{-1}\langle x, \zeta \rangle} dx$

Example* $G = O(p, q) \quad p + q$ even $\quad \curvearrowright L^2(\Xi)$

$\Xi = \{(x, y) \in \mathbb{R}^{p+q-2} \setminus \{0\} : |x|^2 - |y|^2 = 0\}$

$\mathcal{F}_\Xi \cdots$ explicit kernel by “Bessel distribution”

More general case (without explicit formula of $\mathcal{F}_\Xi$) **,***

* Kobayashi–Mano, Memoirs of AMS 1000 (2011); ** Hilgert–Kobayashi–Möllers J. Math. Soc. Japan (2014).

*** Kobayashi–Savin, Global uniqueness of small representations, Math. Z., **281** (2015), 215–231.

Interpolation of Fourier transform $\mathcal{F}_{\mathbb{R}^N}$

$\mathcal{F}_{\Xi}$	$\cdots$	unitary inversion on $\Xi \quad \subset \mathbb{R}^{p-1,q-1}$
$\mathcal{F}_{\mathbb{R}^N}$	$\cdots$	Fourier transform on $\mathbb{R}^N$

Interpolation of Fourier transform $\mathcal{F}_{\mathbb{R}^N}$

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Assume $q = 2$. Set $p = N + 1$.

$\mathbb{R}^{N,1} \supset \Xi =$  $= \mathbb{R}^N$

Interpolation of Fourier transform $\mathcal{F}_{\mathbb{R}^N}$

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Assume $q = 2$. Set $p = N + 1$.

$\mathbb{R}^{N,1} \supset \Xi =$  $= \mathbb{R}^N$

$\mathcal{F}_{\Xi}$	$\mathcal{F}_{\mathbb{R}^N}$
$O(N+1,2)$	$Mp(N,\mathbb{R})$

Interpolation of Fourier transform $\mathcal{F}_{\mathbb{R}^N}$

$\mathcal{F}_{\Xi}$ $\cdots$ unitary inversion on $\Xi \subset \mathbb{R}^{p-1,q-1}$

$\mathcal{F}_{\mathbb{R}^N}$ $\cdots$ Fourier transform on $\mathbb{R}^N$

Assume $q = 2$. Set $p = N + 1$.

$\mathbb{R}^{N,1} \supset \Xi =$  $= \mathbb{R}^N$

$\mathcal{F}_{\Xi}$ interpolate $\cdots\cdots\cdots$ $\mathcal{F}_{\mathbb{R}^N}$

$a = 1$ $\qquad\qquad$ $a = 2$

$a \cdots$ deformation parameter > 0

Unitary inversion operators

- Fourier transform $\cdots Mp(N, \mathbb{R})$

self-adjoint op. on $L^2(\mathbb{R}^N)$

$$\mathcal{F}_{\mathbb{R}^N} = c \exp\left(\frac{\pi i}{4}(\Delta - |x|^2)\right)$$

phase factor $= e^{\frac{\pi i N}{4}}$ Laplacian

$\rightsquigarrow$ Hermite semigp

- Unitary inversion* on $\Xi \cdots O(N+1, 2)^{\sim}$

self-adjoint op. on $L^2(\mathbb{R}^N, \frac{dx}{|x|})$

$$\mathcal{F}_{\Xi} = c \exp\left(\frac{\pi i}{2}(|x|\Delta - |x|)\right)$$

phase factor $= e^{\frac{\pi i (N-1)}{2}}$ Laplacian

$\rightsquigarrow$ "Laguerre semigp"

* K–Mano, The inversion formula and holomorphic extension of the minimal representation $\cdots$, 2007, pp. 159–223.

(k,a)-generalized Fourier transform $\mathcal{F}_{k,a}$

self-adjoint op. on $L^2(\mathbb{R}^N, \vartheta_{k,a}(x)dx)$

$$\mathcal{F}_{k,a} = c \exp\left(\frac{\pi i}{2a}(|x|^{2-a}\Delta_k - |x|^a)\right)$$

phase factor $= e^{i\frac{\pi(N+2\langle k\rangle+a-2)}{2a}}$

Dunkl Laplacian

(k,a)-deformation of Hermite semigroup *

$$\mathcal{I}_{k,a}(t) := \exp \frac{t}{a}(|x|^{2-a}\Delta_k - |x|^a) \qquad \operatorname{Re} t > 0$$

Deformation parameter k: multiplicity on root system $\mathcal{R}$, $a > 0$

* Ben Saïd–Kobayashi–Ørsted, Compositio Math (2012)

Deformation theory of Fourier transform

Observation (branching laws)

Schrödinger model

$O(N+1,2)\widetilde{\ } \curvearrowright L^2(\Xi) \quad \mathcal{F}_{\Xi}$

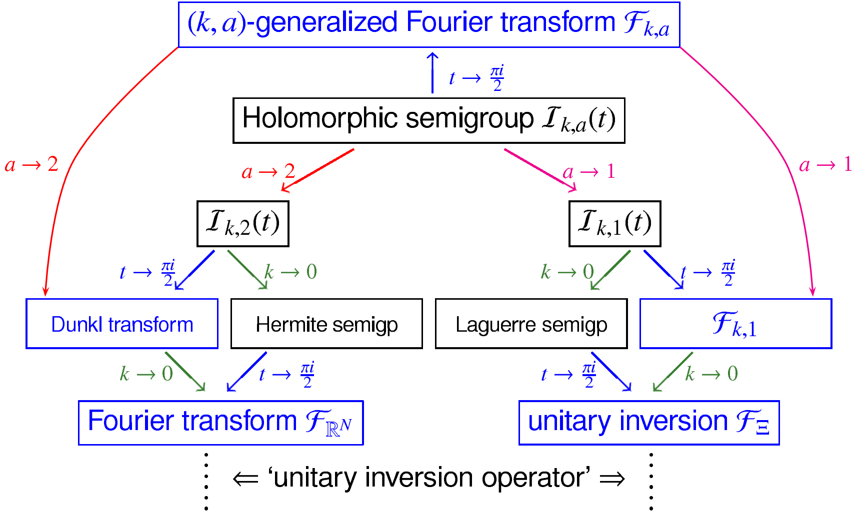
symmetric pair $\nearrow$

$O(N) \times SL(2,\mathbb{R})\widetilde{\ }$

dual pair $\searrow$

$Mp(N,\mathbb{R}) \quad \curvearrowright L^2(\mathbb{R}^n) \quad \mathcal{F}_{\mathbb{R}^n}$

Special values of holomorphic semigroup $\mathcal{I}_{k,a}(t)$



the Weil representation of
the metaplectic group $Mp(N,\mathbb{R})$

the minimal representation of
the conformal group $O(N+1,2)$

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