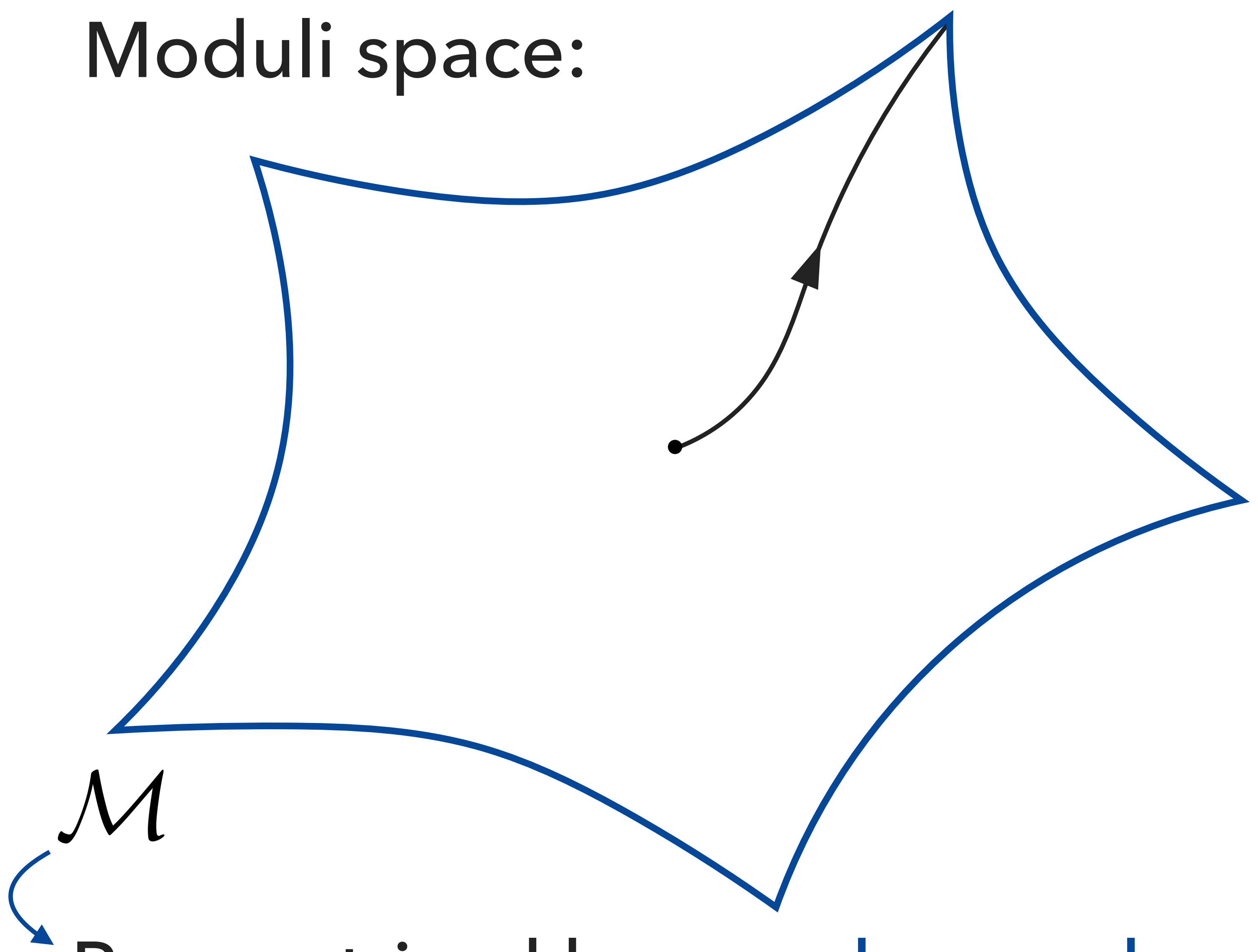
Tensionless Strings Limits in 4d Conformal Manifolds

José Calderón Infante

Based on ongoing work with Irene Valenzuela

The Landscape vs the Swampland, ESI Vienna, 18/07/2024

The Swampland Distance Conjecture



[Ooguri, Vafa '06]

Swampland Distance Conjecture (SDC)

There is an infinite tower of states becoming light at infinite-distance points in moduli space

$$M_{tower} \sim e^{-\alpha \Delta\phi} \quad \text{as} \quad \Delta\phi \to \infty \qquad (M_{Pl} = 1)$$

Distance parameter
(today's main protagonist!)

Lots of top-down evidence!

- String theory:
  [Grimm, Palti, Valenzuela '18] [Lee, Lerche, Weigand '18-'19]
  + many many more!
- AdS/CFT: [Baume, JCI '20+'23] [Ooguri, Wang '24]
  [Perlmutter, Rastelli, Vafa, Valenzuela '20]

+ Bottom-up motivations
[Hamada, Montero, Vafa, Valenzuela '21]
[Stout '21+'22]
[JCI, Castellano, Herráez, Ibáñez '23]

+ connections to other conjectures, pheno implications, … … …

The Distance Conjecture in AdS/CFT

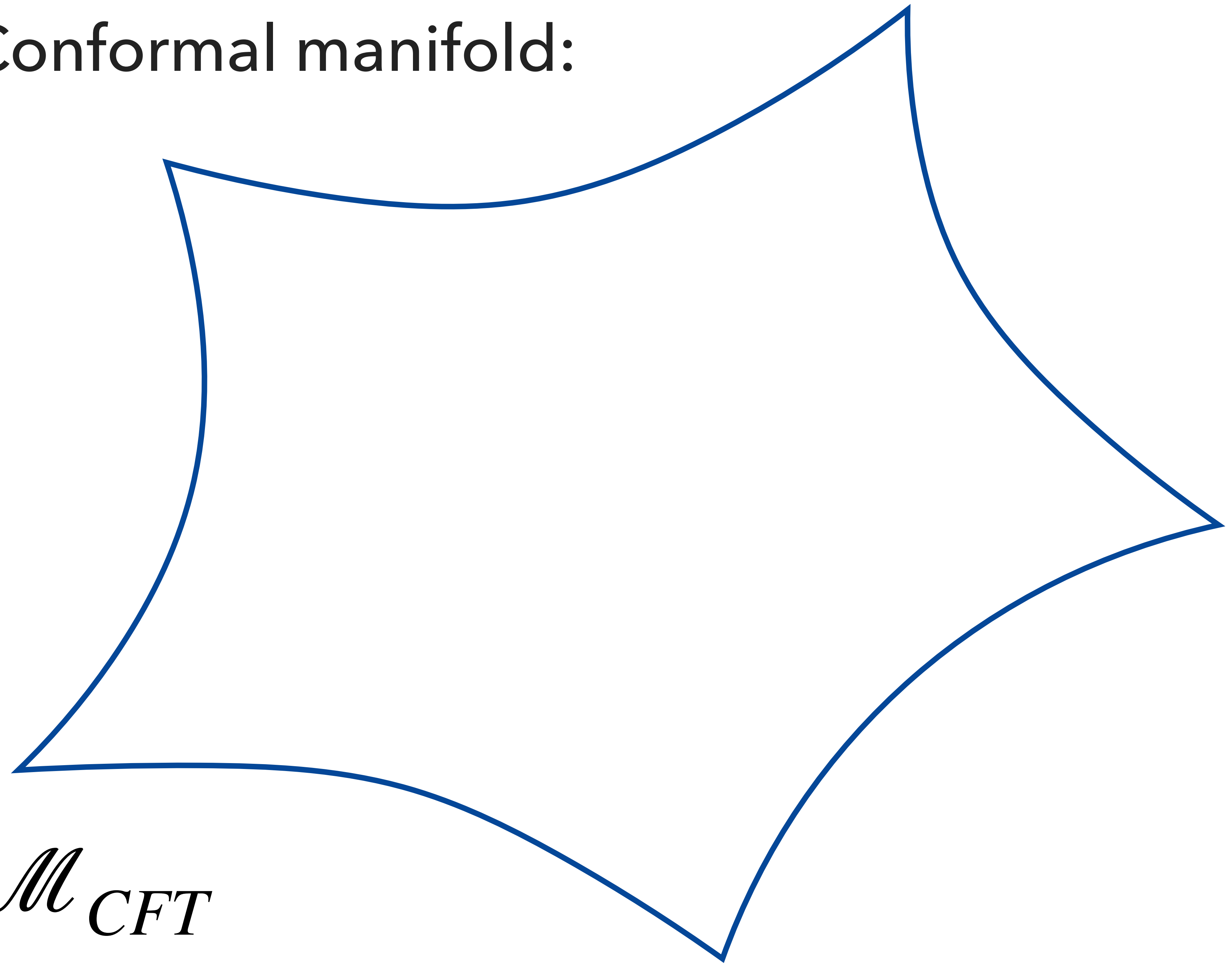
[Baume, JCI '20] [Perlmutter, Rastelli, Vafa, Valenzuela '20]

How does the SDC look like in AdS/CFT ?

AdS/CFT basics:

AdS CFT → Moduli space ⟷ Conformal manifold:

$(\phi, m) \longleftrightarrow (\mathcal{O}, \Delta)$



$$\boxed{(\mathcal{M}, G_{ij}) \longleftrightarrow (\mathcal{M}_{CFT}, \chi_{ij})}$$

Moduli space metric

$$\mathcal{L} \supset M_{Pl}^{d-1} \frac{1}{2} G_{ij} \partial_\mu \phi^i \partial^\mu \phi^j$$

Zamolodchikov metric

$$\left\langle \mathcal{O}_i \mathcal{O}_j \right\rangle = \frac{\chi_{ij}}{|x-y|^{2d}}$$

$\mathcal{M}_{CFT}$

Parametrized by exactly marginal couplings

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At infinite distance:

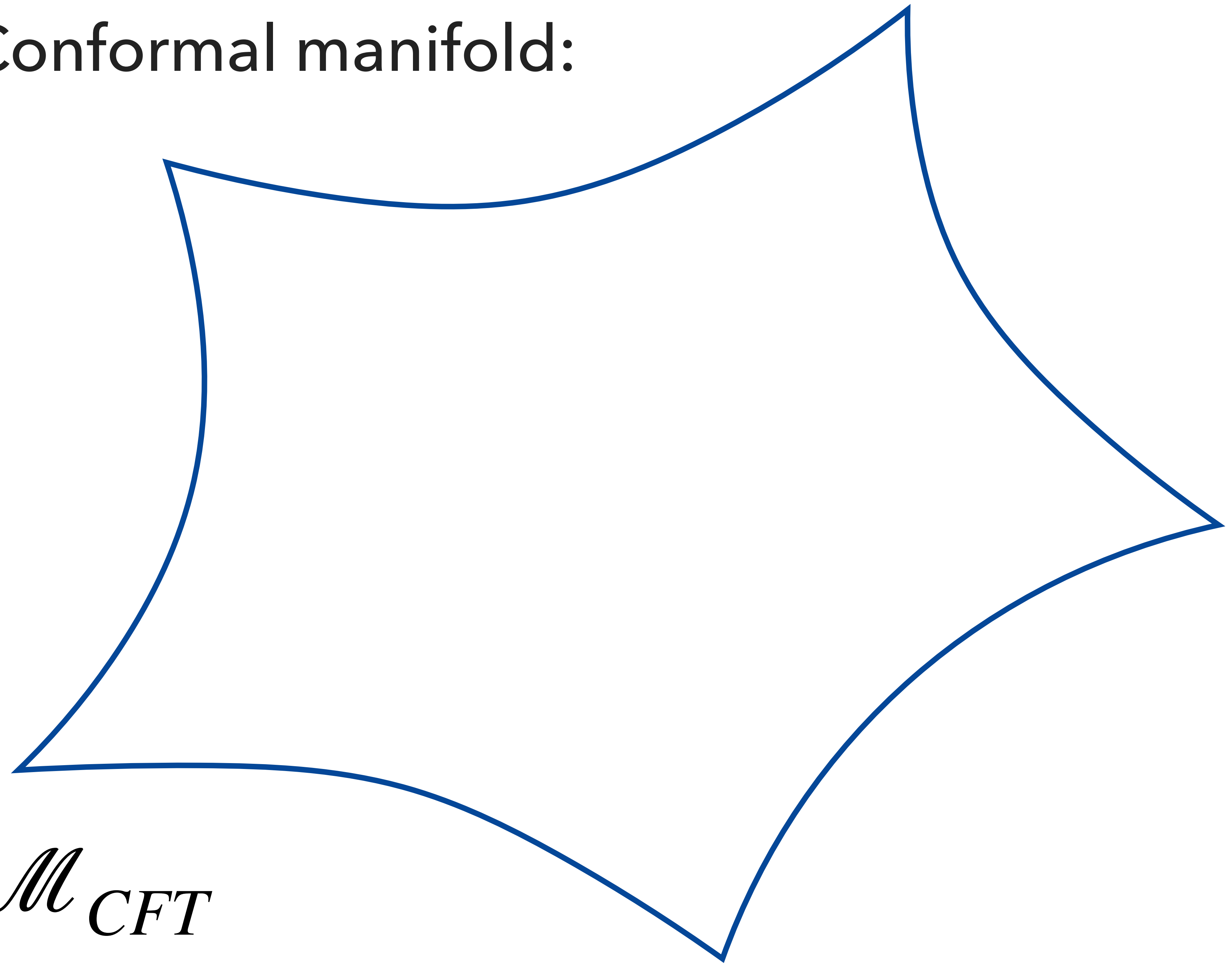
Tower of operators with

$$\Delta - \Delta_{unitarity} \sim e^{-\alpha_{CFT}\, t}$$

Question: Which operators?

(e.g. unitarity bound depend on spin!)

→ **Higher-spin operators** !



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CFT Distance Conjecture:

Conformal manifold of local CFT in d>2

I. HS point $\longrightarrow$ Infinite distance

II. Infinite distance $\longrightarrow$ HS point

III. $\gamma_\ell = \Delta_\ell - (\ell + d - 2) \sim e^{-\alpha_\ell t}$

Zamolodchikov distance

Local CFT: Posses stress tensor

$\Rightarrow$ Dynamical gravity in the bulk!

The Distance Conjecture in AdS/CFT

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Local CFT: Posses stress tensor

➡ Dynamical gravity in the bulk!

Proven ✓ [Baume, JCI '23]

! No extra assumption, e.g., no supersymmetry

+ existence of stress tensor is crucial!

Moreover:

Analogous statement for 2d CFTs

Proven ✓ [Ooguri, Wang '24]

Today: Stringy origin of HS points ? [JCI, Valenzuela '24]

Strings in the Conformal Manifold

Inspiration: Emergent String Conjecture [Lee, Lerche, Weigand '19]

Leading tower → KK modes → Decompactification

Leading tower → Excitations of weakly-coupled string

KK tower → No HS fields ✗

String tower → HS fields ✓

➡ **Expectation:** HS point ↔ tensionless string

Problem: $T_s \lesssim R_{AdS}^{-2}$ ⟶ String in a highly-curved background… **hard to study!**

➡ Rely on CFT results and extract clues !

A Distance Conjecture Approach

In flat space: Value of $\alpha \rightarrow$ Nature of the tower

$$\alpha = \sqrt{\frac{d-2+n}{n(d-2)}}$$ ➡ Decompactification of n extra dimensions

$$\alpha = \frac{1}{\sqrt{d-2}}$$ ➡ Emergent string limit

Caveat: Different values found for decompactification to running solution
[Etheredge, Heidenreich, McNamara, Rudelius, Ruiz, Valenzuela '23]

From the CFT: Restrict to zoo of 4d SCFTs with simple gauge group (Lagrangian) admitting large N

➡ Three different values: $\alpha = \left\{ \sqrt{\frac{2}{3}}, \sqrt{\frac{7}{12}}, \frac{1}{\sqrt{2}} \right\}$ [Perlmutter, Rastelli, Vafa, Valenzuela '20]

Out of 21 theories!

Suggests three different strings in AdS!

But... $\alpha \neq \frac{1}{\sqrt{3}}$ for all of them?

Actually... Match $n = \{3,4,6\}$
$\rightarrow$ Decompactification to $D = \{8,9,11\}$?

So... What is going on?!

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E.g. $\mathcal{N} = 4$ SYM ⟷ Type IIB on $AdS_5 \times S^5$

Goal: Understand this case!

Convex Hull for AdS5xS5

Type IIB on an 5-sphere

Controls 5-sphere radius

$$S = \frac{M_{Pl}^3}{2} \int d^5x \sqrt{-g} \left(R - (\partial \hat{\Phi})^2 - (\partial \hat{R})^2 - V(\hat{\Phi}, \hat{R}) \right)$$

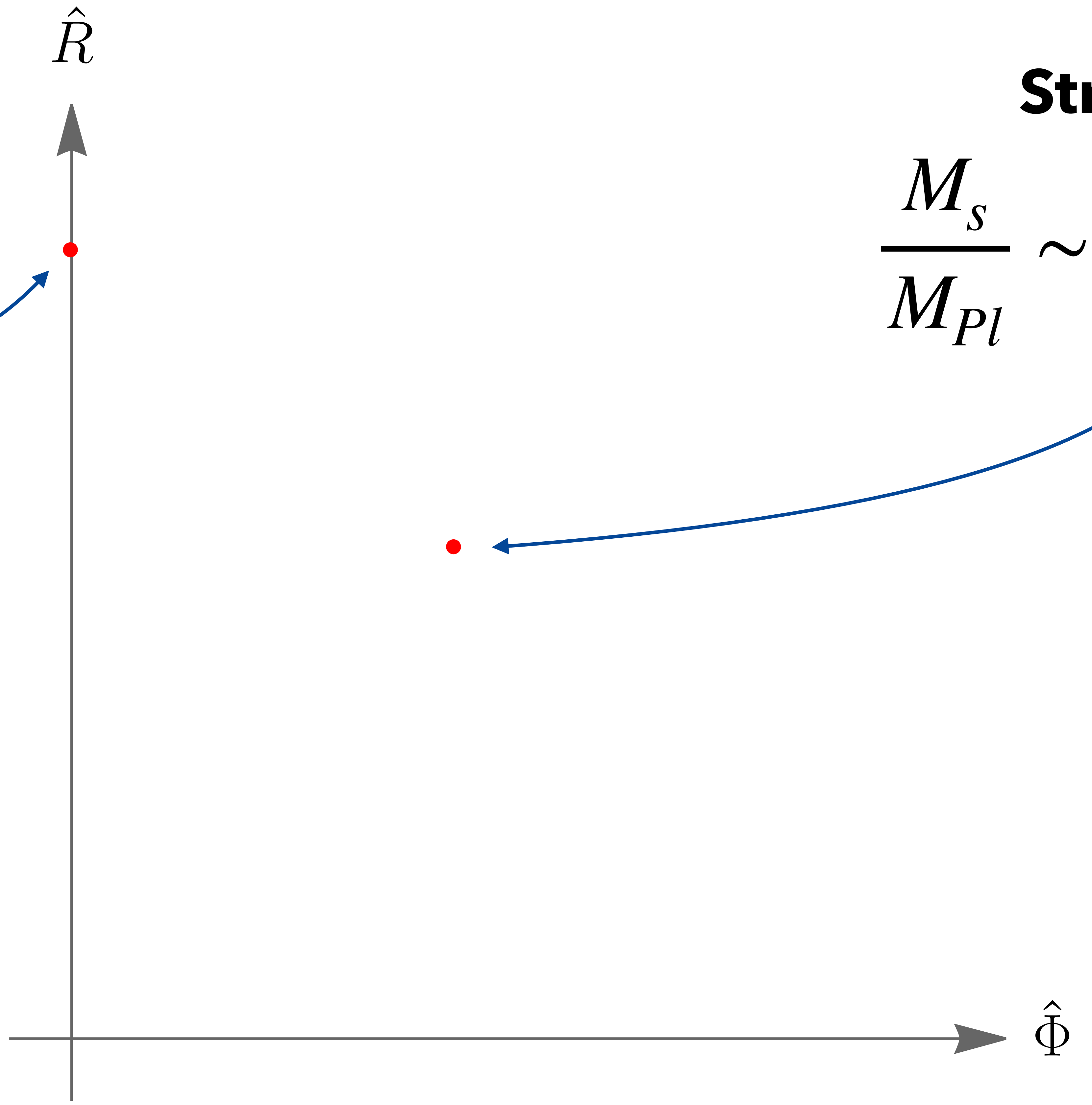
Controls string coupling

KK tower

$$\frac{M_{KK}}{M_{Pl}} \sim e^{-\sqrt{\frac{8}{15}} \hat{R}}$$

String tower

$$\frac{M_s}{M_{Pl}} \sim e^{-\frac{1}{2\sqrt{2}} \hat{\Phi} - \sqrt{\frac{5}{24}} \hat{R}}$$



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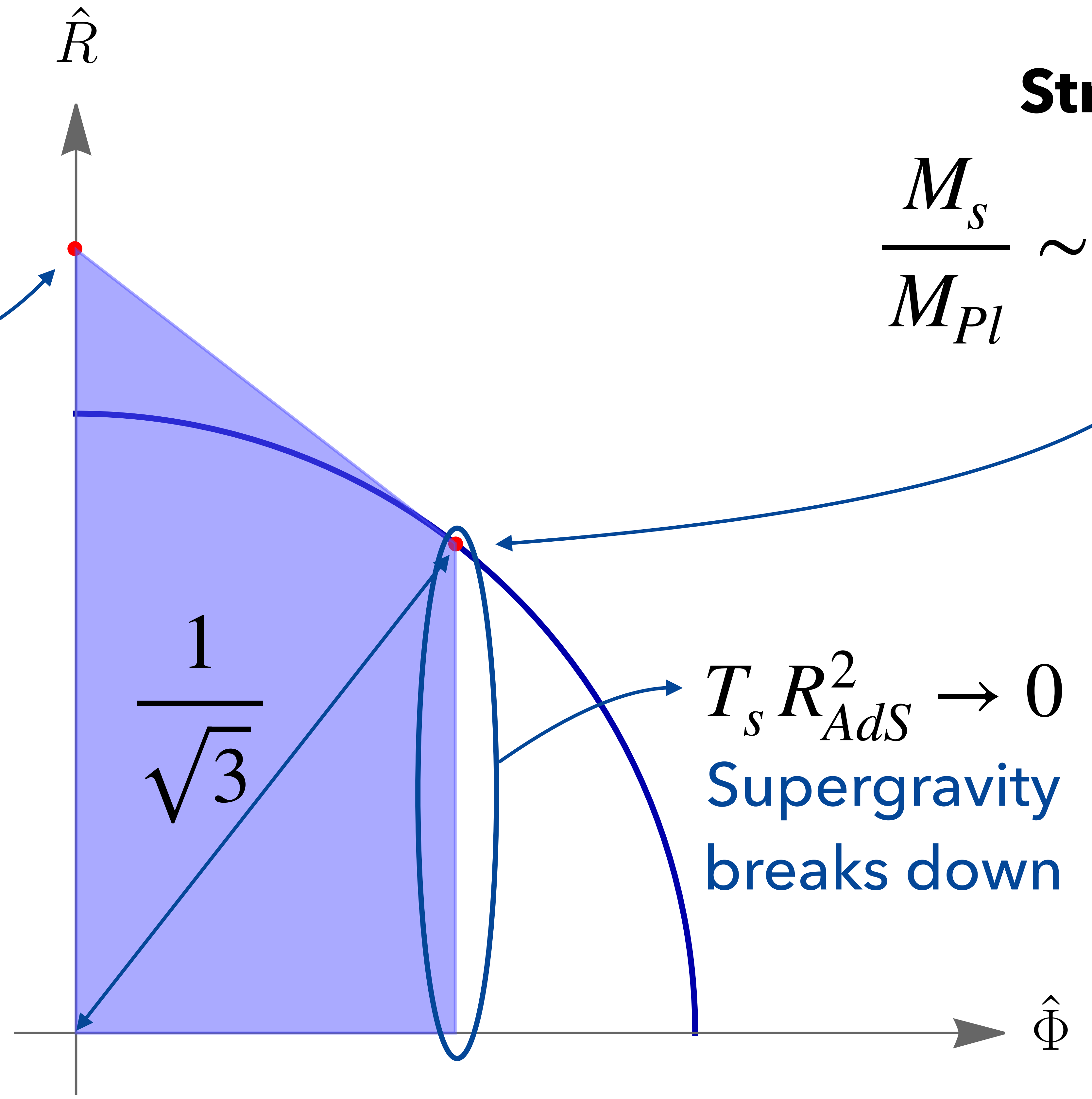


Convex Hull SDC

[JCI, Uranga, Valenzuela '20]

Sharpened SDC

[Etheredge, Heidenreich, Kaya, Qiu, Rudelius '22]

Violated!?

$T_s R_{AdS}^2 \to 0$

Supergravity breaks down

$AdS_5 \times S^5$ moduli space

$$\alpha = \frac{1}{2\sqrt{2}} \neq \frac{1}{\sqrt{2}} \text{ (fix } \hat{R}\text{)}$$

CFT value not reproduced!

Convex Hull for N=4 SYM

$\mathcal{N} = 4$ SU(N) gauge theory in 4d

KK tower ↔ BPS operators

$$\Delta_{BPS} \sim \mathcal{O}(1)$$

$$\frac{M_{KK}}{M_{Pl}} \sim \frac{\mathcal{O}(1)}{R_{AdS} M_{Pl}} \sim N^{-2/3} \sim e^{-\sqrt{\frac{8}{15}}\hat{R}}$$

Supergravity input!

String tower ↔ HS conserved currents

$$\gamma_{HS} \sim \lambda = g_{YM}^2 N \quad \text{(valid for } \lambda \ll 1\text{)}$$

$$\frac{M_s}{M_{Pl}} \sim \frac{\sqrt{\gamma_{HS}}}{R_{AdS} M_{Pl}} \sim N^{-1/6} g_{YM} \sim e^{-\frac{1}{\sqrt{2}}\hat{\Phi} - \frac{1}{\sqrt{30}}\hat{R}}$$

Supergravity input!

Problem:

No CFT distance in the N-direction **:(**

See [Stout '21+'22] and [Basile, Montella '23] for progress in this direction

Need supergravity input!

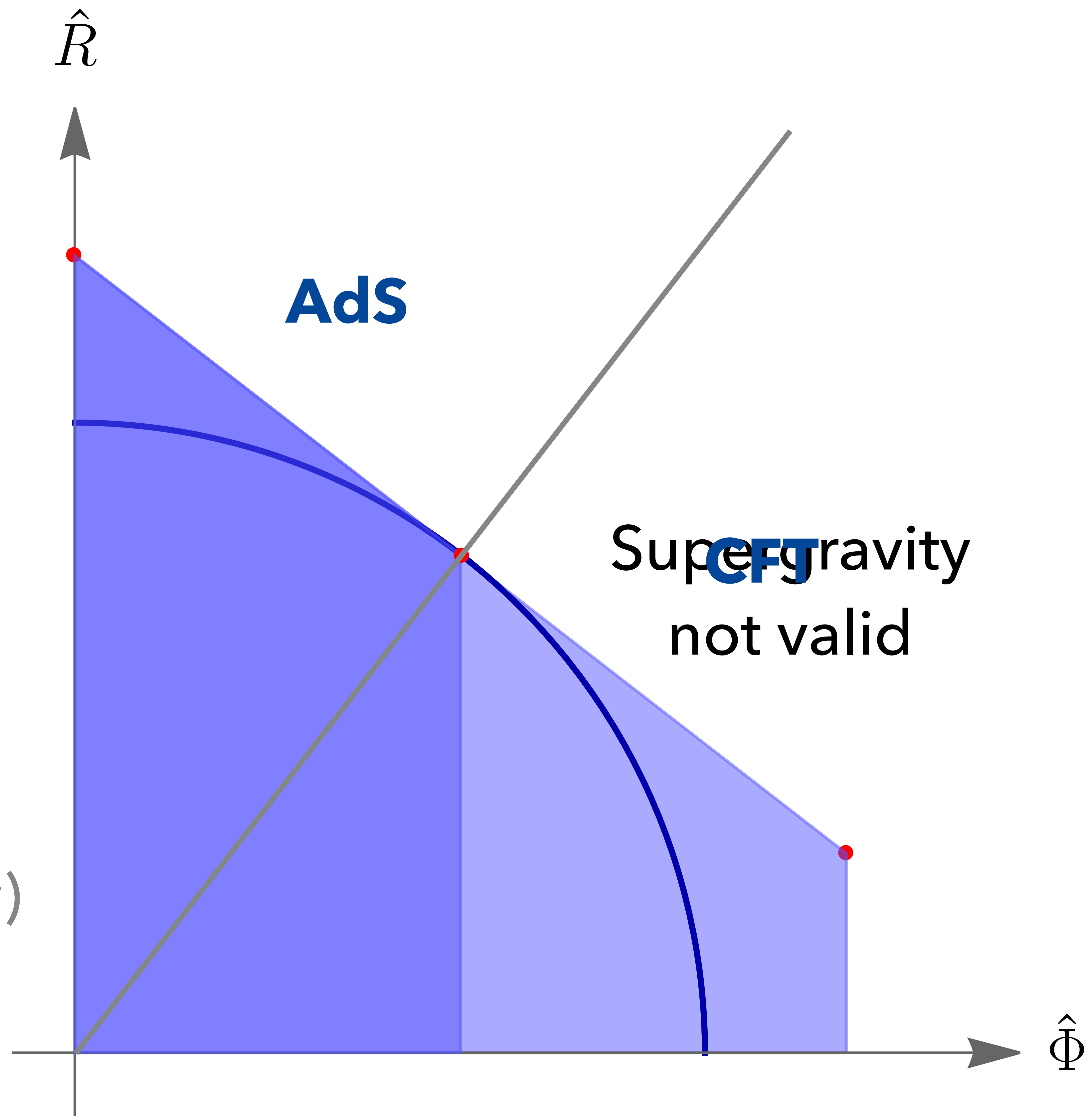
$$\boxed{N \sim e^{\frac{\sqrt{30}}{5}\hat{R}}}$$

Recent works on generalized notions of distance:
[(Li), Palti, Petri '23+'24]
[Mohseni, Montero, Vafa, Valenzuela '24]
[Debusschere, Tonioni, Van Riet '24]
+ talk by Thomas

Convex Hulls Comparison

Notice:
Convex hulls for AdS and CFT glue nicely together!
(see later)

For convex hull connoisseurs:
The string vector slides!
(c.f. talks by Tom, Nacho and Muldrow)



Summary

A Detour: Scale Separation vs Sharpened SDC

KK tower ↔ BPS operators

$\Delta_{BPS} \sim \mathcal{O}(1) \longleftrightarrow M_{KK} \sim R_{AdS}^{-1}$

No scale separation from the CFT!

Relax condition

$\underline{M_{KK} \sim R_{AdS}^{-2\beta}} \longleftrightarrow \Delta_{BPS} \sim N^{\frac{2}{3}(1-2\beta)}$ Weird BPS spectrum

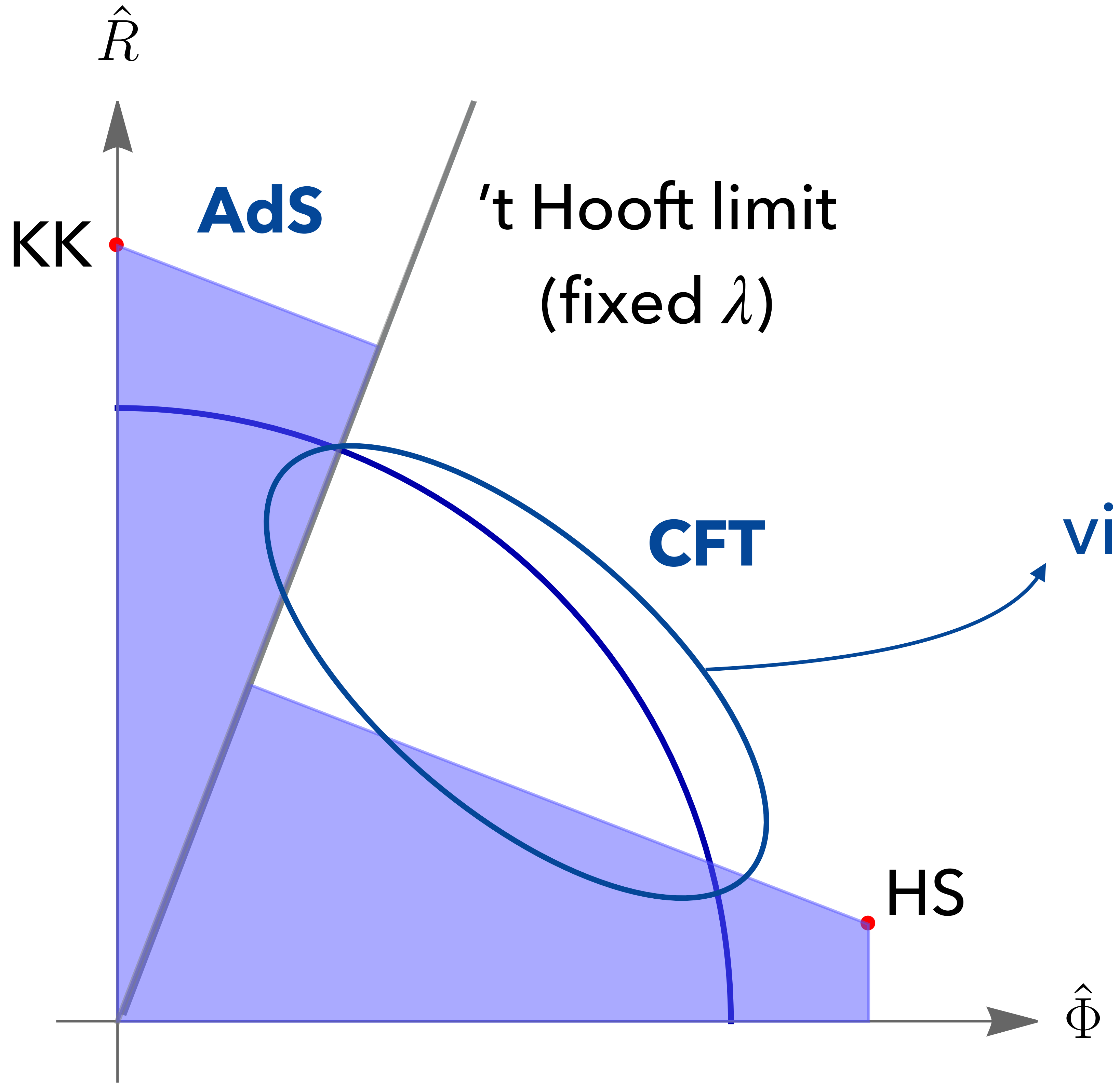
Weird S^5 stabilization

Long story short

Anti-separation of scales: $\beta > 1/2 \rightarrow M_{KK} \ll R_{AdS}^{-1}$

Notice:

Convex hulls for AdS and CFT do not glue nicely together!



Sharpened SDC violation in the CFT !

A Detour: Scale Separation vs Sharpened SDC

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Weird BPS spectrum

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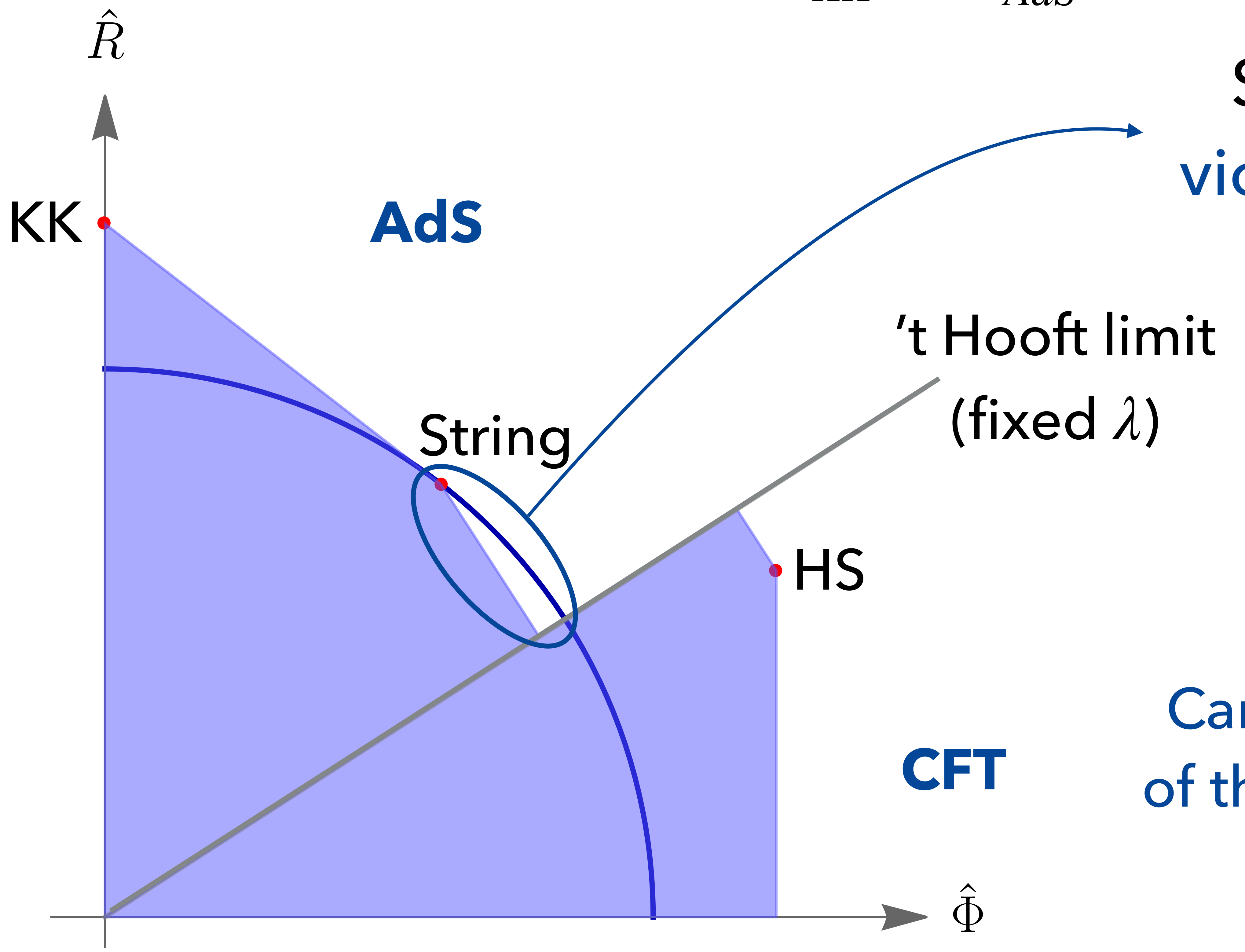
Long story short

Separation of scales: $\beta < 1/2 \rightarrow M_{KK} \gg R_{AdS}^{-1}$

Notice:

Convex hulls for AdS and CFT do not glue nicely together!



Sharpened SDC violation in the AdS !

Link between Sharpened SDC and no scale separation

Caveat:

$$R_{AdS}^{-2} \ll T_s \ll R_{S^5}^{-2}$$

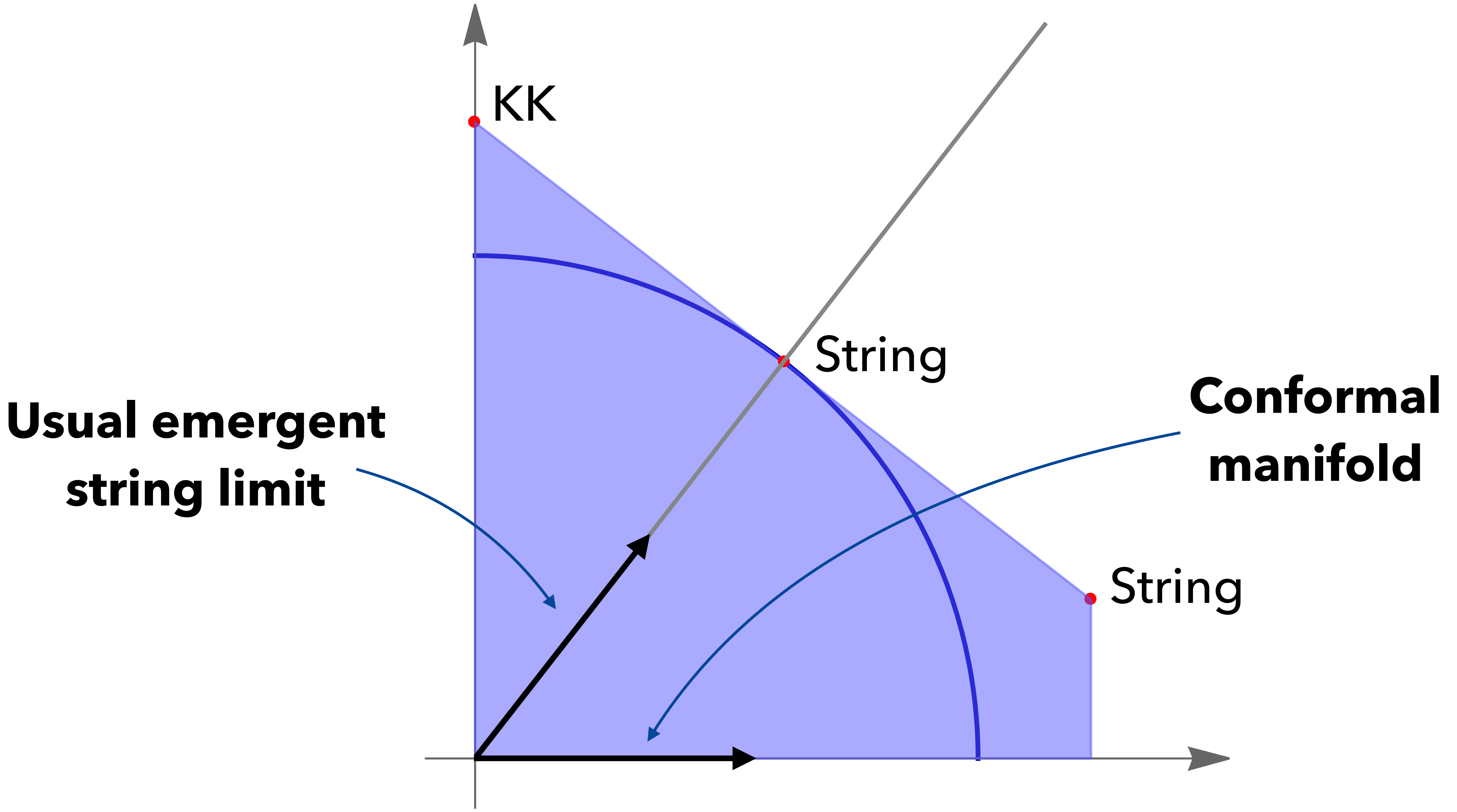
Can we trust quantization of the string in this regime?

Recap

Why $\alpha = \frac{1}{\sqrt{2}} \neq \frac{1}{\sqrt{3}}$ in $\mathcal{N} = 4$ SYM ?

Reason 1:

Flat space emergent string: $\frac{M_s}{M_{Pl}} \to 0 \, + \, M_s \sim M_{KK}$ **Here:** $\frac{M_s}{M_{Pl}} \to 0 \, + \, M_s \ll M_{KK} \sim O(1)$



Recap

Why $\alpha = \frac{1}{\sqrt{2}} \neq \frac{1}{\sqrt{3}}$ in $\mathcal{N} = 4$ SYM ?

Reason 2:

$M_s \ll R_{AdS}^{-1}$ $\rightarrow$ Weakly curved approximation breaks down!

What goes wrong when computing α?

1. Moduli space metric for g_s ✓ 2. String excitation modes with g_s ✗

Weakly curved: $M_s \sim \sqrt{T_s} \sim M_{Pl}\, g_s^{1/4}$

CFT: $M_s \sim M_{Pl}\, g_s^{1/2}$ $\neq$

$\boxed{M_s \sim T_s R_{AdS}}$ Universal?

Food for thought!

CFT prediction for part of string spectrum in highly-curved AdS !

What about the others?

Recap: 4d SCFTs with simple gauge group (Lagrangian) admitting large N

$$\alpha = \left\{ \underbrace{\sqrt{\frac{2}{3}}, \sqrt{\frac{7}{12}}}_{?}, \frac{1}{\sqrt{2}} \right\}$$ [Perlmutter, Rastelli, Vafa, Valenzuela '20]

E.g. $\mathcal{N} = 4$ SYM ⟷ Type IIB on $\mathrm{AdS}_5 \times \mathrm{S}^5$ ✔ 🍇

New strings**?** Or same string, weirder background**?**

Problem: How to detect a string from the CFT**?**

Instead, look for physical properties that are controlled only by α **!**

1. Ratio between a and c central charges

2. Hagedorn temperature at large N

CFT Distances vs Einstein Gravity

[Perlmutter, Rastelli, Vafa, Valenzuela '20]

$$\alpha = \sqrt{\frac{2c}{\dim G}} \xrightarrow{\dim G = f(a,c)} \alpha = \frac{1}{\sqrt{4\frac{a}{c} - 2}}$$

$$\boxed{\frac{a}{c} = \frac{1}{2} + \frac{1}{4\alpha^2}}$$

Depends on α only **!**

Physical meaning**?**

Relevant for various aspects of low energy EFT**!**

[Henningson, Skenderis '98]

Most notably: $a \neq c$ (at large N) $\leftrightarrow$ No weakly-coupled Einstein gravity at low energies

Only theories with $\alpha = \frac{1}{\sqrt{2}}$ have Einstein gravity duals **!**

CFT Distances vs Hagedorn Temperature

$$Z(T) = \sum_{states} e^{-E/T} = \int \rho(E)\, e^{-E/T} dE \xrightarrow{T \to T_H} \infty \Longleftrightarrow \rho(E) \sim e^{E/T_H}$$ **Stringy!** ✓

Hagedorn temperature: $T_H \longrightarrow$ Controls exponential density of states at high energies!

➡ **Expectation:** Hagedorn temperature should only depend on α!

4d $\mathcal{N} = 1$ SU(N) gauge theory $\longrightarrow$ 7 parameters: $\{n_{Ad}, n_F, n_{\bar{F}}, n_A, n_{\bar{A}}, n_S, n_{\bar{S}}\}$ # chiral multiplets

Controls Hagedorn temperature

Long story short… $Z(T) \to \infty \leftrightarrow$ **Hagedorn condition:** $z_v(T_H) + \left\{ \boxed{n_{Ad} + \frac{1}{2}(n_S + n_{\bar{S}} + n_A + n_{\bar{A}})} \right\} z_c(T_H) = 1$

$\mathcal{N} = 1$ vector

$\mathcal{N} = 1$ chirals

Nice … but not enough!

CFT Distance Parameter: $12\,\alpha^2 - 3 = \boxed{n_{Ad} + \frac{1}{2}\left(n_S + n_{\bar{S}} + n_A + n_{\bar{A}}\right)}\boxed{+ n_F + n_{\bar{F}}}$:(

⊕ **Conformal manifold** $\to \beta_{1-loop} = 0$ ➡ $\boxed{n_F + n_{\bar{F}}} = 6 - 2\left(\boxed{n_{Ad} + \frac{1}{2}\left(n_S + n_{\bar{S}} + n_A + n_{\bar{A}}\right)}\right)$

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$\mathcal{N} = 1$ vector

Nice… and enough!

$\mathcal{N} = 1$ chirals

CFT Distance Parameter + $\beta_{1-loop} = 0$: $3\left(3 - 4\alpha^2\right) = \boxed{n_{Ad} + \frac{1}{2}\left(n_S + n_{\bar{S}} + n_A + n_{\bar{A}}\right)}$:)

➡ **Hagedorn condition:** $\boxed{z_v(T_H) + 3\left(3 - 4\alpha^2\right) z_c(T_H) = 1}$ **Expectation confirmed** ✓

CFT Distances vs Hagedorn Temperature

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Hagedorn temperature: $T_H \longrightarrow$ Controls exponential density of states at high energies!

➡ **Expectation:** Hagedorn temperature should only depend on α !

4d $\mathcal{N} = 1$ USp(2N)/SO(N) gauge theory $\longrightarrow$ 3 parameters: $\{n_F, n_A, n_S\}$ # chiral multiplets

Controls Hagedorn temperature

Long story short… $Z(T) \to \infty \leftrightarrow$ **Hagedorn condition:** $z_v(T_H) + \{\boxed{n_S + n_A}\}\, z_c(T_H) = 1$

$\mathcal{N} = 1$ vector

Nice!

$\mathcal{N} = 1$ chirals

CFT Distance Parameter + $\beta_{1-loop} = 0$: $3\left(3 - 4\alpha^2\right) = \boxed{n_S + n_A}$:)

➡ **Hagedorn condition:** $\boxed{z_v(T_H) + 3\left(3 - 4\alpha^2\right) z_c(T_H) = 1}$ **Expectation confirmed** ✓

Same as for SU(N) !

CFT Distances vs Hagedorn Temperature

$$Z(T) = \sum_{states} e^{-E/T} = \int \rho(E)\, e^{-E/T} dE \xrightarrow{T \to T_H} \infty \Longleftrightarrow \rho(E) \sim e^{E/T_H}$$ **Stringy!**

Hagedorn temperature: $T_H \longrightarrow$ Controls exponential density of states at high energies**!**

➡ **Expectation:** Hagedorn temperature should only depend on α**!** **Confirmed**

Possible caveat: Trouble with large numbers of flavors at large N

[Gadde, Pomoni, Rastelli '09] → Restrict to flavor singlets!

Free theory restricted to singlets of **free** flavor group

Hagedorn condition

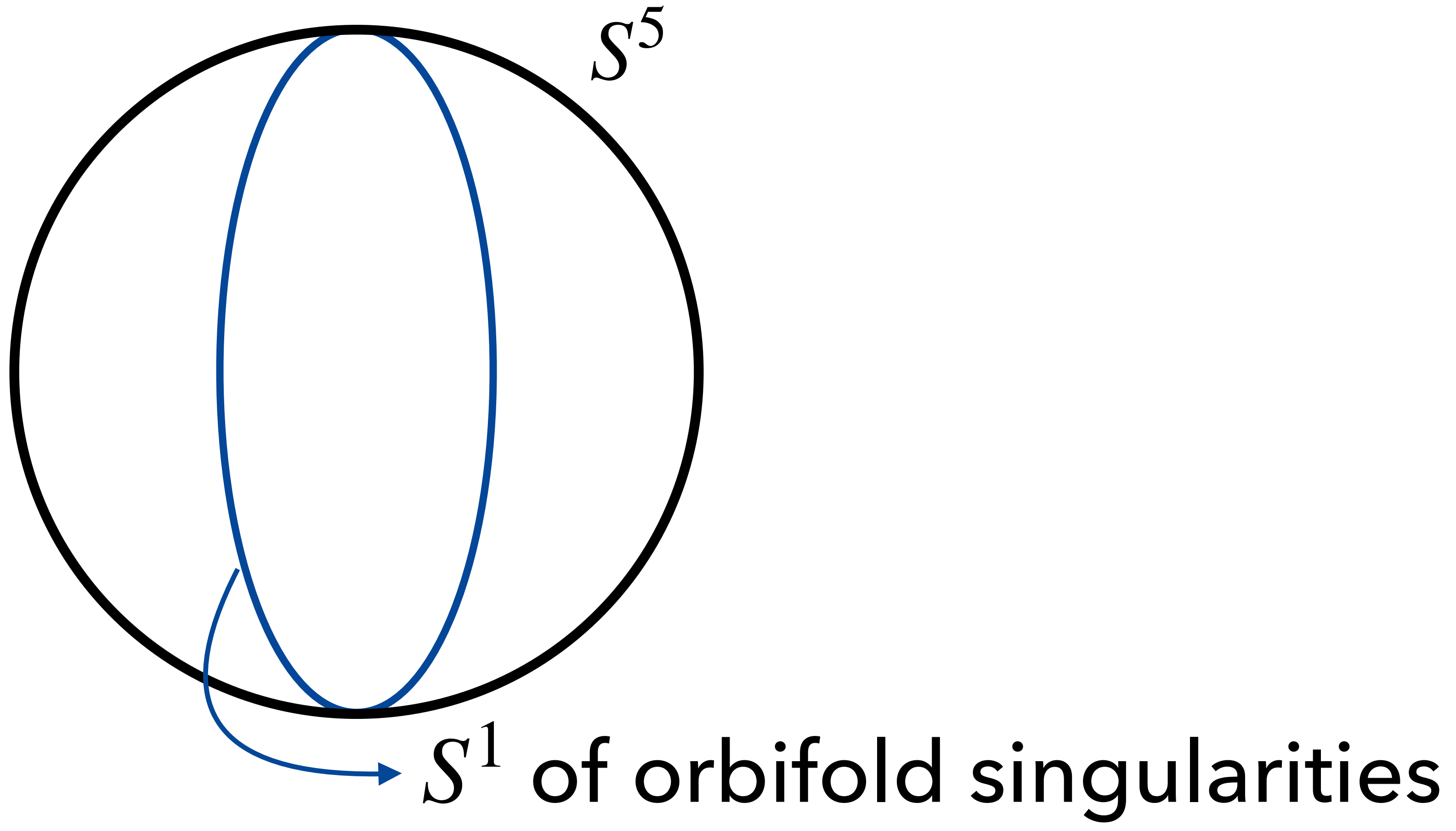
$$\boxed{z_v(T_H) + 3\left(3 - 4\alpha^2\right) z_c(T_H) + \frac{1}{2} z_c(T_H)^2 = 1}$$

Still works!

Stay tuned!

Bonus Track: A New AdS String from Top-down?

Setup: Type IIB on $\mathrm{AdS}_5 \times \mathrm{S}^5/\mathrm{Z}_k \leftrightarrow \mathcal{N} = 2$ necklace quivers



A very peculiar limit:

Driven by only axions $\rightarrow$ Typically finite distance

But! CFT predicts infinite distance + HS conserved currents [Aharony, Berkooz, Rey '15]

Stringy origin?

Fundamental string remains tensionful…

D3 wrapping blow-up 2-cycle become tensionless! [Aharony, Berkooz, Rey '15]

String propagating in $\mathrm{AdS}_5 \times \mathrm{S}^1$! Candidate for new emergent string in AdS? [Baume, JCI '20]

Conclusions and More Questions

There is much to learn about/from the Distance Conjecture in AdS/CFT !

CFT side

Prove rest of CFT Distance Conjecture ?

CFT Distance in N-direction ?

Stringy side

New strings in AdS ?

Building them: D3 wrapping blow-ups in AdS ?

Thank you for your attention!