Theta correspondence and wave front set

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Minimal Representations and Theta Correspondence
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Notation

- $\mathbb{F}_q$ finite field with q elements, $q = p^m$, p prime number.
- $\overline{\mathbb{F}}_q$ algebraic closure of $\mathbb{F}_q$.
- $\mathbf{G}$ connected reductive group defined over $\overline{\mathbb{F}}_q$, defined over $\mathbb{F}_q$.
- $F\colon \mathbf{G} \to \mathbf{G}$ Frobenius endomorphism.
- $\mathbf{G}^F := \{g \in \mathbf{G} \,:\, F(g) = g\} =: \mathbf{G}(q) =: G$ is a finite group of Lie type.
- $\mathrm{Irr}(\mathbf{G}^F)$ set of (isomorphism classes of) complex irreducible representations of G^F.
- C an F-stable unipotent conjugacy class of $\mathbf{G}$. Then C^F is a union of conjugacy classes of $\mathbf{G}^F$.

Theorem [Lusztig (1992)]

Let $\pi \in \mathrm{Irr}(G)$. If p and q are sufficiently large, then

1. there is a unique unipotent class C_π in $\mathbf{G}$ which has the property that
$$\mathrm{AV}(\pi, C_\pi) := \sum_{g \in C_\pi^F} \pi(g) \neq 0,$$
and has maximal dimension among classes with this property ; the class C_π is called the unipotent support of π ;
2. if $g \in G$ is such that $\pi(g) \neq 0$ then the unipotent part of g lies in C_π or in a conjugacy class of dimension $< \dim(C_\pi)$.

Thus there is a canonical map

$$\begin{aligned} \mathrm{Irr}(G) &\to \{F\text{-stable unipotent classes of } \mathbf{G}\} \\ \pi &\mapsto C_\pi. \end{aligned} \tag{1}$$

Small characteristics cases :

The characteristic p is said to be *good* for $\mathbf{G}$ if

- no condition for $\mathbf{G}$ of type A_n ;
- $p \neq 2$ for $\mathbf{G}$ of type B_n, C_n or D_n ;
- $p \neq 2, 3$ for $\mathbf{G}$ of type G_2, F_4, E_6, E_7 ;
- $p \neq 2, 3, 5$ for $\mathbf{G}$ of type E_8.

1. The existence of the unipotent support was established for p good by Geck (1996), and for any p by Geck and Malle (2000).
2. For $\mathbf{G}$ of type G_2 and $p = 3$, there exist unipotent representations of G, which are non-zero on some element in C_{reg}^F but whose average value on C_{reg}^F is zero, where C_{reg} the class of regular unipotent elements.

Kawanaka has shown that, assuming p is good for $\mathbf{G}$, one can associate to any unipotent element $u \in G$ a so-called generalized Gelfand-Graev representation Γ_u (GGR for short), obtained by inducing certain representations from unipotent radicals of parabolic subgroups of G.

Examples

- Γ_1 is the regular representations of G.
- Γ_u is the ordinary Gelfand-Graev representation if $u \in C_{\mathrm{reg}}$.

Notation

- $\mathbf{B} = \mathbf{TU}$ Borel subgroup of $\mathbf{G}$, where $\mathbf{T}$ is maximal torus and $\mathbf{U}$ the unipotent radical of $\mathbf{B}$. We suppose $\mathbf{B}$ and $\mathbf{T}$ F-stable.
- $\Phi \subset \mathrm{Hom}(\mathbf{T}, \overline{\mathbb{F}}_q^\times)$ root system of $\mathbf{G}$, with Φ^+ set of positive roots and Δ set of simple roots.
- $\mathbf{G} = \langle \mathbf{T}, \mathbf{U}_\alpha \ : \ \alpha \in \Phi \rangle$ and $\mathbf{U} = \prod_{\alpha \in \Phi^+} \mathbf{U}_\alpha$.

Definitions

- To each unipotent class of $\mathbf{G}$, is attached a *weighted Dynkin diagram*, that is, an additive map $d\colon \Phi \to \mathbb{Z}$ such that $d(\alpha) \in \{0, 1, 2\}$ for any $\alpha \in \Delta$.
- We write

$$L_d := \langle \mathbf{T}, \mathbf{U}_\alpha \,:\, \alpha \in \Phi, d(\alpha) = 0 \rangle \quad \text{and} \quad \mathbf{U}_{d,i} := \prod_{\alpha \in \Phi^+, d(\alpha) \geq i} U_\alpha,$$

for $i = 1, 2, 3, \ldots$ Then $\mathbf{P}_d := \mathbf{L}_d \mathbf{U}_{d,1}$ is a parabolic subgroup of $\mathbf{G}$, with Levi subgroup $\mathbf{L}_d$ and unipotent radical $\mathbf{U}_{d,1}$. There is a unique unipotent class C in $\mathbf{G}$ such that $C \cap \mathbf{U}_{d,2}$ is dense in $\mathbf{U}_{d,2}$. Moreover, $C \cap \mathbf{U}_{d,2}$ is a single $\mathbf{P}_d$-conjugacy class, and $\mathrm{C}_{\mathbf{G}}(u) \subset \mathbf{P}_d$ for any $u \in C \cap \mathbf{U}_{d,2}$. The class C is called the *unipotent classe associated to* d.

- $\psi\colon \mathbb{F}_q \to \mathbb{C}$ a fixed non-trivial additive character.
- $\mathfrak{g}$ Lie algebra of $\mathbf{G}$. It is defined over $\mathbb{F}_q$ and we still denote by $F\colon \mathfrak{g} \to \mathfrak{g}$ the corresponding Frobenius map.
- $\mathfrak{t} \subset \mathfrak{g}$ Lie algebra of $\mathbf{T}$.
- $\kappa\colon \mathfrak{g} \times \mathfrak{g} \to \overline{\mathbb{F}}_q$ Killing form (G-invariant, non-degenerate bilinear form).
- We have $\mathfrak{g} = \mathfrak{t} \oplus \overline{\mathbb{F}}_q e_\alpha$, where $F(t) = t$ and $F(e_\alpha) = e_\alpha$ for any $\alpha \in \Phi$.
- $x \mapsto x^*$ anti-$\mathbb{F}_q$-automorphism of $\mathfrak{g}$ such that $\mathfrak{t}^* = \mathfrak{t}$ and $e_\alpha^* \in \mathbb{F}_q e_\alpha$ for any $\alpha \in \Phi$.

Let $u \in C \cap U_{d,2}$. Write

$$u = \left(\prod_{\alpha \in \Phi^+, d(\alpha)=2} u_\alpha(a_\alpha) \right) \cdot U_{d,3} \quad \text{where } a_\alpha \in \mathbb{F}_q.$$

Define $\varphi_u \colon U_{d,2} \to \mathbb{C}^\times$ by

$$\varphi_u \left(\prod_{\alpha \in \Phi^+, d(\alpha) \geq 2} u_\alpha(y_\alpha) \right) := \psi \left(\sum_{\alpha \in \Phi^+, d(\alpha)=2} \kappa(e_\alpha^*, e_\alpha) a_\alpha y_\alpha \right),$$

where $y_\alpha \in \mathbb{F}_q$.

Definition of the GGR

The map φ_u is a linear character of U and $[U_{d,1} : U_{d,2}]$ is a power of q^2. We have

$$\mathrm{Ind}_{U_{d,2}}^{G}(\varphi_u) := [U_{d,1} : U_{d,2}]^{1/2} \cdot \Gamma_u,$$

where Γ_u is the generalized Gelfand-Graev representation of G.

Definition [Kawanaka, 1987]

For any unipotent element $v \in \mathbf{G}$, let C_v denote the $\mathbf{G}$-conjugacy class containing v. Let $\pi \in \mathrm{Irr}(G)$. The Kawanaka wave front set of π, if it exists, is an F-stable unipotent conjugacy class C of $\mathbf{G}$ satisfying :

1. $\langle \Gamma_u, \pi \rangle \neq 0$ for some $u \in C$;
2. if v is a unipotent element of $\mathbf{G}$ such that $\langle \Gamma_v, \pi \rangle \neq 0$ then $C_v \subset \overline{C}$, where $\overline{C}$ denotes the Zariski closure of C.

Remark

If the Kawanaka wave front set exists, it is clearly unique.

Theorem [Lusztig, 1992]

We suppose that p and q are sufficiently large. For every $\pi \in \mathrm{Irr}(G)$ there exists an F-stable unipotent conjugacy class C of $\mathbf{G}$ satisfying :

1. $\langle \Gamma_u, \pi \rangle \neq 0$ for some $u \in C$;
2. if v is a unipotent element of $\mathbf{G}$ such that $\langle \Gamma_v, \pi \rangle \neq 0$ then $\dim C_v \leq \dim C$.

Remark

Lusztig's result was extended to

- the case p good for $\mathbf{G}$ assuming that the center of $\mathbf{G}$ is connected [Shoji, 1996] ;
- the case p "acceptable for $\mathbf{G}$", with $\mathbf{G}$ arbitrary [Taylor, 2016]. We have : p very good for $\mathbf{G} \Rightarrow p$ acceptable for $\mathbf{G} \Rightarrow p$ good for $\mathbf{G}$.

Theorem [Achar-A, 2007] for p, q large ; [Taylor, 2016], p acceptable

The Kawanaka wave front set $\mathrm{WF}(\pi)$ exists for every $\pi \in \mathrm{Irr}(G)$.

Relation between Kawanaka wave front set and unipotent support

The wave front set of any irreducible representation of G coincides with the unipotent support of its Alvis-Curtis dual, that is :

$$\mathrm{WF}(\pi) = C_{\pi^*}, \quad \text{for any } \pi \in \mathrm{Irr}(G),$$

where $\pi^* := \pm D_G(\pi) \in \mathrm{Irr}(G)$, with $D_G := \sum_{I \subset \Delta} (-1)^{|I|} \mathrm{i}_{L_I}^G \circ \mathrm{r}_{L_I}^G$.

Dual pairs

There are pairs (G, G') of subgroups of $\mathrm{Sp}_{2d}(q)$ for some integer $N \geq 1$ such that each of them is the centralizer of the other in $\mathrm{Sp}_{2d}(q)$, where q is odd.

Irreducible pairs :

- Type I :
  - $(\mathrm{Sp}_{2n}(q), \mathrm{O}_{m'}(q))$ with $nm' = d$;
  - $(\mathrm{U}_m(q), \mathrm{U}_{m'}(q))$ with $mm' = 2d$;
- Type II :
  - $(\mathrm{GL}_m(q), \mathrm{GL}_{m'}(q))$ with $nn' = 2d$.

A pair (G, G') is in the stable range (with G' smaller) if the defining vector space for G has a totally isotropic subspace of dimension greater or equal than the dimension the defining vector space for G', e.g. the pairs $(\mathrm{Sp}_{2n}(q), \mathrm{O}_{2n'}(q))$ such that $n \geq 2n'$.

Definition of the Theta correspondence

The restriction of the Weil representation ω^ψ of $\mathrm{Sp}_{2N}(q)$ to $G \times G'$ is

$$\omega_{G,G'} = \sum_{\substack{\pi \in \mathrm{Irr}(G) \\ \pi' \in \mathrm{Irr}(G')}} \mathrm{mult}_{\pi,\pi'}\, \pi \otimes \pi', \text{ where } m_{\pi,\pi'} \in \mathbb{Z}_{\geq 0}.$$

Define $\Theta_{G'} : \mathbb{Z}\,\mathrm{Irr}(G(q)) \to \mathbb{Z}\,\mathrm{Irr}(G')$ by

$$\Theta_{G'}(\pi) := \{\pi' \in \mathrm{Irr}(G') \,:\, \mathrm{mult}_{\pi,\pi'} \neq 0\}\,, \quad \text{for } \pi \in \mathrm{Irr}(G).$$

From now on, we suppose m' is even. We write $2n' := m'$ and $(G_n, G_{n'}) := (G, G')$.

Definition

The occurrence of a irreducible representation π of G_n in the Theta correspondence for $(G_n, G'_{n'_\pi})$ with n' minimal (i.e., such that $\Theta_{G'_{n'}}(\pi) = 0$ for any $n' < n'_\pi$) is referred to as the first occurrence.

Between members of a dual pair, the only ones having cuspidal unipotent representations are : $\mathrm{GL}_1(q)$, $\mathrm{Sp}_{2k(k+1)}(q)$, $\mathrm{U}_{(k^2+k)/2}$ (which have a unique such representation, say σ_k), and $\mathrm{O}_{2k^2}(q)$ (which has two : σ_k^{I} and $\sigma_k^{\mathrm{II}} = \sigma_k^{\mathrm{I}} \otimes \mathrm{sign}$). From now on, we will only consider pairs formed by a symplectic group and an orthogonal group.

Theorem [Adams-Moy, 1993]

1. If π is a cuspidal irreducible representation of G_n, then $\Theta_{G'_{n'_\pi}}(\pi)$ is a singleton $\{\pi'\}$ with π' cuspidal irreducible.
2. If $\pi \in \mathrm{Irr}(G)$ is unipotent then any $\pi' \in \Theta_{G'}(\pi)$ is unipotent.
3. The representation σ_k of $\mathrm{Sp}_{2k(k+1)}(q)$ corresponds to σ_k^{II} if ϵ is the sign of $(-1)^k$ and to $\sigma_{k+1}^{\mathrm{I}}$ otherwise.

Corollary

The Theta correspondence between cuspidal unipotent representations is describe by the function $\theta\colon \mathbb{N} \to \mathbb{N}$, defined by $\sigma_{\theta(k)} := \theta(\sigma_k)$.

Theorem [A-Michel-Rouqier, 1996]

The Theta correspondence for unipotent representations induces a correspondence between parabolically induced representations $\mathrm{i}_{\mathrm{Sp}_{2k(k+1)}(q)\otimes T}^{\mathrm{Sp}_{2n}(q)}(\sigma_k \otimes 1)$ and $\mathrm{i}_{\mathrm{O}^{\epsilon}_{2\theta(k)^2}(q)\otimes T'}^{\mathrm{O}^{\epsilon}_{2n'}(q)}(\sigma'_{\theta(k)} \otimes 1)$, where $\sigma'_{\theta(k)} \in \{\sigma^{\mathrm{I}}_{\theta(k)}, \sigma^{\mathrm{II}}_{\theta(k)}\}$, and T, T' are products of $\mathrm{GL}_1(q)$'s.

Corollary

It induces a correspondence Ω_{N_k,N'_k} between irreducible representations of Weyl groups of types B_{N_k} and $\mathrm{B}_{N'_k}$, where $N_k := n - k(k+1)$ and $N'_k := n - \theta(k)^2$.

Conjectural explicit description of Ω_{N_k,N'_k} for pairs $(\mathrm{Sp}_{2n}(q), \mathrm{O}^{\epsilon}_{2n}(q))$

- Formulated in [A-Michel-Rouquier, Duke Math. J. 1996].
- Established by Pan in 2019.
- New proof by Ma-Qiu-Zhou (see Jiajun's talk on Thursday).

Remark

In general, there exist $\pi \in \mathrm{Irr}(G)$ such that $\Theta_{G'}(\pi)$ contains more than one element. Hence, a natural question : can we extract a one-to-one correspondence ?

Several approaches

- Definition of the η correspondence for dual pairs $(\mathrm{Sp}_{2n}(q), \mathrm{O}_{N'}(q))$ in the stable range,[Gurevich-Howe, 2017 and 2020].
- Construction of a one-to-one correspondence for unipotent representations of pairs of type II and of pairs in stable rangle of the form $(\mathrm{O}^{\epsilon}_{2k^2+2}, \mathrm{Sp}_{2(k^2+k+N)}(q))$ or $(\mathrm{Sp}_{2(k^2+k+2)}(q), \mathrm{O}^{\epsilon}_{2k^2+N}(q))$ [A-Kraskiewicz-Przebinda, 2016].
- Construction of a one-to-one correspondence θ for unipotent representations of irreducible pairs of type I in stable rangle [Epequin, 2019].
- Extension of both η and θ correspondences to all irreducible pairs of type I [Pan, 2020].

Partitions :

- $\lambda := (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_l)$ where $\lambda_i \in \mathbb{Z}_{\geq 0}$ is called a partition of n if $\lambda_1 + \lambda_2 + \cdots + \lambda_l = n$.
- $\mathcal{P}(n)$ set of partitions of n.
- $\lambda \cup \mu$ partition of $n + m$ with parts $\lambda_1, \ldots, \lambda_l, \mu_1, \ldots, \mu_l$.
- Usual order on $\mathcal{P}(n)$:

$$\lambda \leq \lambda' \quad \text{if and only if} \quad \lambda_1 + \cdots + \lambda_i \leq \lambda'_1 + \cdots + \lambda'_i, \text{ for all } i \in \mathbb{N}.$$

- Another order on partitions : for λ, λ' partitions of possibly different integers, we write

$$\lambda \preceq \lambda' \quad \text{if and only if} \quad \lambda'_{i+1} \leq \lambda_i \leq \lambda'_i, \text{ for all } i \in \mathbb{N}.$$

From now on, we suppose that $2n' \leq n$. We write $N_k(\zeta) := N_k - |\zeta|$ if ζ is a partition such that $|\zeta| \leq N_k$.

Then we have :

1. Cases $(\mathrm{Sp}_{2n}, \mathrm{O}^+_{2n'})$ with k even and $(\mathrm{Sp}_{2n}, \mathrm{O}^-_{2n'})$ with k odd :

$$\Omega_{N_k, N'_k} = \sum_{r=0}^{\min(N_k, N'_k)} \sum_{(\xi,\zeta)\in\mathcal{P}_2(r)} \sum_{\eta,\eta'} \rho_{\xi,\eta} \otimes \rho_{\xi,\eta'},$$

   where the third sum is over the partitions $\eta \dashv N_k(\xi)$ and $\eta' \dashv N'_k(\xi)$ such that $\zeta \preceq \eta$ and $\zeta \preceq \eta'$.

2. Cases $(\mathrm{Sp}_{2n}, \mathrm{O}^+_{2n'})$ with k odd and $(\mathrm{Sp}_{2n}, \mathrm{O}^+_{2n'})$ with k even :

$$\Omega_{N_k, N'_k} = \sum_{r=0}^{\min(N_k, N'_k)} \sum_{(\xi,\zeta)\in\mathcal{P}_2(r)} \sum_{\xi',\eta'} \rho_{\xi',\eta} \otimes \rho_{\xi,\eta'},$$

   where the third sum is over the partitions $\xi' \dashv N_k(\eta)$ and $\eta' \dashv N'_k(\xi)$ such that $\xi \preceq \xi'$ and $\eta \preceq \eta'$.

Maximal representations

Let $(\xi', \eta') \in \mathcal{P}_2(N'_k)$.

- We write

$$\Theta_{\xi',\eta'} := \left\{(\xi, \eta) \in \mathcal{P}_2(N_k) \; : \; \rho_{\xi,\eta} \otimes \rho_{\xi',\eta'} \text{ occurs in } \Omega_{N_k, N'_k}\right\};$$

- We say that $\rho_{\xi_{\mathrm{M}},\eta_{\mathrm{M}}} \in \Theta_{\xi',\eta'}$ is maximal if

$$\lambda_{\xi,\eta} \leq \lambda_{\xi_{\mathrm{M}},\eta_{\mathrm{M}}} \quad \text{for all } (\eta, \xi) \in \Theta_{\xi',\eta'}.$$

Remark

Since the order is not total, it is not clear a priori that a maximal representation exists, and if so, that it is unique.

Similarly, we say that $\rho_{\xi_{\mathrm{m}},\eta_{\mathrm{m}}} \in \Theta_{\xi',\eta'}$ is minimal if

$$\lambda_{\xi_{\mathrm{m}},\eta_{\mathrm{m}}} \leq \lambda_{\xi,\eta} \quad \text{for all } (\eta,\xi) \in \Theta_{\xi',\eta'}.$$

A representation is said to be extremal if it is either maximal or minimal.

Theorem [Epequin, 2019]

Let $(\xi',\eta') \in \mathcal{P}_2(N'_k)$.

1. There exists a unique maximal representation $\rho_{\xi_{\mathrm{M}},\eta_{\mathrm{M}}} \in \Theta_{\xi',\eta'}$, it is given by
$$\xi_{\mathrm{M}} := \xi' \quad \text{and} \quad \eta_{\mathrm{M}} := (N_k - N'_k + \eta'_1 + \eta'_2, \eta'_3, \ldots, \eta'_l).$$
2. There exists a unique minimal representation $\rho_{\xi_{\mathrm{m}},\eta_{\mathrm{m}}} \in \Theta_{\xi',\eta'}$, it is given by
$$\xi_{\mathrm{m}} := \xi' \quad \text{and} \quad \eta_{\mathrm{m}} := (N_k - N'_k) \cup \eta'.$$

Theorem [A., 2022]

Let $\pi' \in \mathrm{Irr}^{\mathrm{u}}(G')$ that belongs to the principal series of G'. Then there is a unique representation π_{pref} of G such that :

$$\pi_{\mathrm{pref}} \in \Theta_G(\pi') \quad \text{and} \quad \mathrm{WF}(\pi) \leq \mathrm{WF}(\pi_{\mathrm{pref}}) \quad \text{for any } \pi \in \Theta_G(\pi').$$

Remark

In the stable range case, $\pm\pi_{\mathrm{pref}}$ is the image by the Alvis-Curtis duality D_G of the representation of G which is parametrized by the minimal representation $\rho_{\xi_{\mathrm{m}},\eta_{\mathrm{m}}}$.

Thank you very much for your attention

Happy Birthday Gordan !

ORBITES UNIPOTENTES ET REPRÉSENTATIONS

Titres des conférences

ADAMS J.	L-Functoriality for Dual Pairs.
ARTHUR J.	Global motivation for the Unitary Dual (I,II,III,IV).
	L^2-cohomology and Hecke operators
ASAI T.	On the irreducible representations of the finite classical groups with non-connected centers.
AUBERT A.M.	Représentation métaplectique et sous-groupes d'Iwahori.
BARBASCH D.	Unipotent representations for semi-simple Lie group (I,II).
BARLET D.	Fundamental class and intersection cohomology
BENOIST Y.	On the $\mathfrak{n}$-cohomology of $\mathfrak{n}$-locally nilpotent $\mathfrak{g}$-modules.
BIEN F.	Unipotent representations of Diff S^1.
BOUAZIZ A.	Relèvement des caractères d'un groupe endoscopique pour le changement de base C/R.
BRYLINSKI J.L.	Hochschild homology and orbital integrals.
	Twisted differential operators and $\mathfrak{g}$-finite endomorphisms
CARTIER P.	Representations of Hecke algebras of type A_n.
CLOZEL L.	Howe's conjecture.
CURTIS C.W.	Representations of Hecke Algebras (I, II, III).
	The Gelfand-Graev representation of a finite Chevalley group
DIGNE F.	Shintani descent and Hecke algebras
FLICKER Y.	A simple trace formula.
GARFINKLE D.	Cells in Weyl groups.
GUEMES J.	On the homology classes for the components of some fibres of Springer's resolution.
GUPTA H.	Translation actions and limits of functions on adjoint orbits
HALES T.	Germs and transfer for subregular unipotent classes.
HECKMAN G.	Monodromy for the hypergeometric function ${}_nF_{n-1}$
HOWE R.	Minimal K-types, Hecke algebras and the classification of representations of GL(n).
JACQUET H.	Spherical functions and trace formula.
JANTZEN J.C.	Support varieties for restricted Lie algebras.
JOHNSON J.	Base change C/R.
JOSEPH A.	Primitive ideals (I,II,III).
	Scale factors in Goldie rank polynomials.
KAC V.	Moduli of curves and representation theory.
KASHIWARA M.	D-modules and representation theory (I,II,III,IV,V).
	Character formula and Matsuki correspondence.
KAWANAKA N.	Orbits and stabilizers of nilpotent elements of a graded semi-simple Lie algebra.

TITRES DES CONFÉRENCES

KNAPP A.W.	A construction of unitary representations in parabolic rank two.
KOTTWITZ R.	On Tamagawa numbers.
KRAFT H.	Normality and non-normality of closures of conjugacy classes
LAUMON G.	Un analogue global du cône nilpotent.
LEHRER G.I	Actions of Coxeter groups on certain cohomology groups.
	The Kazhdan-Lusztig polynomials and reflection subgroups in Coxeter groups.
LUSZTIG G.	Affine Hecke algebras (I, II, III, IV).
	Fixed point varieties on affine flag manifolds.
MACDONALD I.G.	Symmetric functions and spherical functions.
MATHIEU O.	Weyl formula for general Kac-Moody Lie algebras.
MIRKOVIC I.	Characteristic varieties of character sheaves.
MOY A.	Isomorphisms of Hecke algebras.
POLO P.	A good filtration for tensor products of modules associated with Schubert varieties.
PROCESI M.	Cohomology of compactifications of symmetric varieties.
ROSSMANN W.	Equivariant multiplicities.
ROSSO M.	Algèbres de Hecke et groupes quantiques.
SAVIN G.	Limit multiplicities of cusp forms.
SCHMID W.	Local cohomology and the duality theorem.
SHELSTAD D.	Regular unipotent germs and transfer.
SHI J.	Some recent developments on cells of affine Weyl groups.
SHOJI T.	Geometry of orbits and Springer correspondence (I,II,III). A remark on the Shintani descent for finite algebraic groups.
SOTO-ANDRADE J.	Generalized Weil representations.
SPALTENSTEIN N.	Nilpotent orbits and conjugacy classes in the Weyl group.
SPEH B.	Automorphic representations for complex semisimple Lie groups, and Lefschetz numbers.
SPRINGER T.	Character Sheaves (I, II, III, IV).
	Some properties of Kazhdan-Lusztig polynomials.
TADIC M.	Geometry of dual spaces of p-adic reductive groups.
TANISAKI T.	Hodge modules and Hecke algebras.
UZAWA T.	Equivariant compactifications of symmetric spaces.
VERGNE M.	On Zuckerman's functor.
VILONEN K.	A good category of $(\mathfrak{g},K)$-modules.
VOGAN D.	Unitary dual for real reductive groups (I, II, III, IV).
	On the definition of Arthur's characters.
WALDSPURGER J.L.	Les intégrales orbitales pour le groupe linéaire sur un corps p-adique.
WOLF J.	Harmonic analysis on general semi-simple Lie groups.
ZUCKER S.	L^2-cohomology of arithmetic varieties and intersection homology of their Baily-Borel-Satake compactification.