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Workshop on

"New Trends in the Variational Modeling of Failure Phenomena"

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organized by

Elisa Davoli (U Vienna), Manuel Friedrich (U Münster), Riccardo Scala (CMAF - CIO, U Lisbon)

Abstracts

Lucia De Luca (SISSA)

A minimization approach to the wave equation on time-dependent domains

Abstract: We prove the existence of weak solutions to the homogeneous wave equation on a suitable class of time-dependent domains. Using the approach suggested by De Giorgi and developed by Serra and Tilli, such solutions are approximated by minimizers of suitable functionals in space-time. Joint work with Gianni Dal Maso.

Rita A. Ferreira (King Abdullah University of Science and Technology)

Homogenization in BV of a model for layered composites in finite crystal plasticity

Abstract: In this talk, we address the study of the asymptotic behavior of a two-dimensional variational model within finite crystal plasticity for high-contrast bilayered composites. Precisely, we consider materials arranged into periodically alternated thin horizontal strips of an elastically rigid component and a softer one with one active slip system. The energies arising from our modeling assumptions are of integral form, featuring linear growth and non-standard differential constraints.

Our asymptotic analysis is based on Gamma-convergence. A key step in this analysis is the characterization of rigidity properties of limits of admissible deformations in the space BV of functions of bounded variation. In particular, in certain cases, we prove that the two-dimensional body may split horizontally into finitely many pieces, each one of these undergoing through a globally rotated shear deformation. These results are BV generalizations of recent ones established in [1]. However, different arguments need to be developed to accommodate the jumps of the admissible limit deformations.
This is a joint work with Elisa Davoli and Carolin Kreisbeck.

References

F. Christowiak and C. Kreisbeck, Homogenization of layered materials with rigid components in single-slip finite crystal plasticity. *Calc. Var. Partial Differential Equations* **56** (2017), no. 3, Art. 75, 28 pp.

Irene Fonseca (Carnegie Mellon University)

Mathematical Analysis of Novel Advanced Materials

Abstract: Quantum dots are man-made noncrystalline of semiconducting materials. Their formation and assembly patterns play a central role in nanotechnology, and in particular in the optoelectronic properties of semiconductors. Changing the dots' size and shape gives rise to many applications that permeate our daily lives, such as the new Samsung QLED TV monitor that uses quantum dots to turn light into perfect colour"! Quantum dots are obtained via the deposition of a crystalline overlayer (epitaxial film) on a crystalline substrate. When the thickness of the film reaches a critical value, the profile of the film becomes corrugated and islands (quantum dots) form. As the creation of quantum dots evolves with time, materials defects appear. Their modeling is of great interest in materials science since material properties, including rigidity and conductivity, can be strongly influenced by the presence of defects such as dislocations.

In this talk we will use methods from the calculus of variations and partial differential equations to model and mathematically analyze the onset of quantum dots, the regularity and evolution of their shapes, and the nucleation and motion of dislocations.

Gilles Francfort (Laboratoire Analyse, Géométrie et Applications, Université Paris-Nord)

The ubiquitous role of stability in solids with defects

Abstract: Adjudicating the correct model for the behavior of solids in the presence of defects is not straightforward. In this, solid mechanics lags way behind its more popular and attractive sibling, fluid mechanics. In this talk, I propose to describe the ambiguity created by the onset and growth of material defects in solids. Then, I will put forth a notion of structural stability that helps in securing meaningful evolutions. I will illustrate how such a notion leads us from the good to the bad, and then to the ugly when going from plasticity to fracture, and then damage. The only conclusion to be drawn is that much of the mystery remains.

Alessandro Giacomini (Università degli Studi di Brescia)

Local minimality results for the Mumford-Shah functional via monotonicity

Abstract: Let $\Omega \subset \mathbb{R}^2$ be an open bounded set, and let

$$MS(u) := \int_\Omega |\nabla u|^2 \, dx + \alpha \mathcal{H}^1(J_u) + \beta \int_\Omega |u - g|^2 \, dx$$

denote the Mumford-Shah functional on $SBV(\Omega)$, with $\alpha, \beta > 0$ and $g \in L^\infty(\Omega)$.
We prove that if Ω is sufficiently regular, then the solution of

$$\begin{cases} -\Delta u + \beta u = \beta g \\ u \in H^1(\Omega) \end{cases}$$

is a local minimizer for MS in $SBV(\Omega)$ with respect to the L^1-topology. This is obtained by employing a boundary monotonicity formula for a suitable notion of *quasi-minimizers* of the Mumford-Shah energy, in the spirit of

that proposed recently by Bucur and Luckhaus. This is a joint work with D. Bucur and I. Fragalà.

Peter Gladbach (University of Leipzig)

Optimal transport with dynamic density constraint

Abstract: We study minimal curves in Wasserstein space with density bounded from above by some function h(x). In particular, we find the conservation laws of geodesic flow, which feature a phase transition between a non-interacting photonic phase and an incompressible inviscid fluid, and study geodesic branching and failure conditions.

Martin Jesenko (Albert-Ludwigs-University Freiburg)

Non-trivial pinning threshold for an evolution equation involving long range interactions

Abstract: We will consider an evolution equation with the fractional Laplace-operator and containing a random term. Therefore, we will shortly present this non-local differential operator and state the examples where such problems arise. Then we will focus on finding a suitable supersolution of the related stationary version. This is a joint work with Patrick Dondl (Freiburg).

Dorothee Knees (University of Kassel)

Convergence analysis of time-discretization schemes for rate-independent systems

Abstract: It is well known that rate-independent systems involving nonconvex stored energy functionals in general do not allow for time-continuous solutions even if the given data are smooth in time. Several solution concepts are proposed to deal with these discontinuities, among them the meanwhile classical global energetic approach and the more recent vanishing viscosity approach. Both approaches generate solutions with a well characterized jump behavior. However, the solution concepts are not equivalent. In this context, numerical discretization schemes are needed that efficiently and reliably approximate directly that type of solution that one is interested in. For instance, in the vanishing viscosity context it is reasonable to couple the viscosity parameter with the time-step size. Other approaches rely on different versions of local minimization or on alternate minimization strategies. The aim of this lecture is to discuss different time-discretization schemes proposed in literature, to present convergence results and to characterize as detailed as possible the limit curves as the discretization parameters tend to zero. Finally, an application to damage mechanics will be discussed.

References:

D. Knees, Convergence analysis of time-discretization schemes for rate-independent systems, CVGMT-Preprint, submitted, 2017.

D. Knees, and M. Negri, Convergence of alternate minimization schemes for phase field fracture and damage, Mathematical Models & Methods in Applied Sciences, vol. 27(9), pp. 1743–1794, 2017.

Carolin Kreisbeck (University of Utrecht)

Asymptotic rigidity of layered structures and its applications in homogenization theory

Abstract: Rigidity results in elasticity are powerful statements that allow to derive global properties of a deformation from local ones. The classical Liouville theorem states that every local isometry of a domain corresponds to a rigid body motion. If connectedness of the set fails, clearly, global rigidity can no longer be true. In this talk, I will present a new type of asymptotic rigidity theorem, which shows that if an elastic body contains sufficiently stiff connected components arranged into fine parallel layers, then strict global constraints of anisotropic nature occur in the limit of vanishing layer thickness. The optimality of the scaling relation between layer thickness and stiffness can be identified with the help of explicit bending constructions. Besides its theoretical interest, the result constitutes a useful tool for the homogenization of variational problems modeling high-contrast bilayered composite materials. We will discuss two models in nonlinear elasticity and finite crystal plasticity and show how to determine their homogenized Gamma-limits in terms of explicit formulas. This is joint work with Fabian Christowiak (Universität Regensburg).

Martin Kružík (Institute of Information Theory and Automation, Czech Academy of Sciences (Prague))

A phase-field approach to Eulerian interfacial energies

Abstract: We analyze a phase-field approximation of a sharp-interface model for two phase materials which consists of a polyconvex bulk strain energy density and interface-polyconvex energy assigned to interfaces separating phases. The distinguishing feature of the model rests in the fact that the interfacial term is Eulerian in nature, and it is defined on the deformed configuration. We discuss a functional frame allowing for existence of phase-field minimizers and Γ-convergence to the sharp-interface limit. As a by-product, we provide additional details on the admissible sharp-interface configurations with respect to the analysis. This is a joint work with D. Grandi (Ferrara), E. Mainini (Genova), and U. Stefanelli (Vienna).

Giuliano Lazzaroni (Università di Napoli Federico II)

On the 1d wave equation in time-dependent domains and the problem of debond initiation

Abstract: Motivated by a debonding model for a thin film peeled from a substrate, we analyse the one-dimensional wave equation, in a time-dependent domain which is degenerate at the initial time. First we prove existence for the wave equation when the evolution of the domain is given; in the second part of the paper, the evolution of the domain is unknown and is governed by an energy criterion coupled with the wave equation. Our existence result for such coupled problem is a contribution to the study of crack initiation in dynamic fracture.

Giovanni Leoni (Carnegie Mellon University)

A new class of fractional Sobolev spaces

Abstract: We present a new class of fractional Sobolev spaces which are related to the trace spaces of homogeneous Sobolev spaces in infinite strip-like domains.

Ilaria Lucardesi (IECL, Université de Lorraine)

Energy release rate and strass intensity factors in planar elasticity in presence of smooth cracks

Abstract: In this talk, we first analyze the singular behavior of the displacement of a linearly elastic body in dimension 2 close to the tip of a smooth crack, extending the well-known results for straight fractures. As conjectured by Griffith, the displacement behaves as the sum of an H^2-function and a linear combination of two singular functions, whose profile is similar to the square root of the distance from the tip. The coefficients of the linear combination are the so called stress intensity factors. Afterwards, we prove the differentiability of the elastic energy with respect to an infinitesimal fracture elongation and we compute the energy release rate, enlightening its dependence on the stress intensity factors. This is a joint work with S. Almi (TUM, Munich).

Edoardo Mainini (University of Genova)

Atomistic potentials and the Cauchy-Born rule for carbon nanotubes

Abstract: Carbon nanotubes are modeled as point configurations and investigated by minimizing Tersoff interaction potentials. Optimal configurations are identified with local minima and their fine geometry is fully characterized in terms of lower-dimensional problems. Under moderate tension, such local minimizers are proved to be periodic. This is a joint work with Manuel Friedrich, Paolo Piovano, and Ulisse Stefanelli.

Marco Morandotti (Technische Universität München)

Discrete-to-continuum limits for dislocations: results in one and two dimensions.

Abstract: In this seminar I will present some recent results about upscaling of dislocations. In the one-dimensional case, a discrete-to-continuum limit passage including an annihilation rule is studied. Considering both positive and negative particles, the empirical measures of the discrete dynamics satisfy a continuum evolution equation, whose solution can be identified with the limiting particle density under a mild separation assumption. In the two-dimensional case, a system of screw dislocations confined in a domain by an external strain is considered. The energies associated with the empirical measures of the discrete system are shown to Gamma-converge to an energy associated with a measure which is absolutely continuous with respect to the $\mathcal{H}^1$ measure restricted to the boundary of the domain. These results are from joint works with Ilaria Lucardesi, Patrick van Meurs, Riccardo Scala, and Davide Zucco.

Matteo Negri (Università degli Studi di Pavia)

Alternate minimizing schemes for the evolutions of phase-field fractures

Abstract: We take into account different discrete schemes, in time and/or space, based on alternate minimizing movements and used in crack propagation: either for gradient flows or for BV-evolutions. We will discuss different ways of modeling irreversibility, in particular in terms of monotonicity of the phase-field variable, and possible ways of modeling non-interpenetration, by means of splitting the elastic energy.

In the case of evolutions of gradient flow type in L^2, we will study the continuum limits, in time and/or space, using in particular both one-step and multi-step algorithms, and both strong and weak irreversibility constraints. On the numerical side, we will compare the numerical simulations obtained with different schemes; even if they all converge, multi-step schemes with weak irreversibility constraint show better adaptivity, in catastrophic regimes, and more robustness, as far as the choice of time discretization.

In the case of BV-evolutions, the alternate minimizing scheme does not introduce any auxiliary norm or dissipation, by contrast to the gradient flow. However, a norm is somehow needed in the analysis and its choice will indeed play an important role. On a first stage we will see that intrinsic energy norms provide a very natural and useful tool when the energy is separately quadratic. Unfortunately, this is no longer the case when, modeling non-interpenetration, positive and negative parts of the volumetric strain have different behavior. This technical issue is by-passed recasting the incremental problems as suitable gradient flows. For both the choices of the elastic energy, we will describe the time-continuous limit in terms of (parametrized) BV-evolutions.

Technically, we will highlight a difficulty, somehow common in all the above problems, related to the energy identity and to the chain rule. The content of this talk is based on joint works with S. Almi and S. Belz.

Gianluca Orlando (Technische Universität München)

A model for damage subject to elastic fatigue

Abstract: The result presented in this talk concerns the existence of quasistatic evolutions for a material model where damage is affected by fatigue, i.e., the process of weakening as a consequence of repeated cycles of loading and unloading. The main feature of this model is the fact that damage is favoured in regions where the cumulation of the elastic strain is higher. To prove the existence of a quasistatic evolution, we follow a vanishing viscosity approach based on two steps: we first let the time-step of the time-discretisation and later the viscosity parameter go to zero. As the time-step goes to zero, we find approximate viscous evolutions; then, as the viscosity parameter vanishes, we find an evolution satisfying an energy-dissipation balance with a weight which is in general different from the cumulation of the elastic strain. The result presented here has been obtained in collaboration with R. Alessi and V. Crismale.

Marcello Ponsiglione (Università La Sapienza, Roma)

Gamma-convergence of the Heitmann-Radin sticky disc energy to the crystalline perimeter and other variational models for polycrystals

Abstract: We will introduce and analyze the Heitmann-Radin sticky disc functional, in the limit of diverging number of discs. For configurations whose energy scales like the perimeter, we prove a compactness result which shows the emergence of polycrystalline structures: The empirical measure converges to a set of finite perimeter, while a microscopic variable, representing the orientation of the underlying lattice, converges to a locally constant function. Whenever the limit configuration is a single crystal, i.e., it has constant orientation, we show that the Gamma-limit is the anisotropic perimeter, corresponding to the Finsler metric determined by the orientation of the single crystal. We will also discuss some related tessellation problems, some partial results concerning polycrystals, and some open problems. Finally, we will present a continuous variational model for polycrystals derived by homogenizing edge dislocations. The results are in collaboration with L. De Luca, S. Fanzon, M. Novaga, M. Palombaro.

Riccarda Rossi (University of Brescia)

Visco-Energetic solutions to rate-independent systems in finite-strain plasticity and brittle fracture

Abstract: Visco-Energetic solutions to rate-independent systems have been recently obtained by passing to the time-continuous limit in a time-incremental scheme, akin to that for Energetic solutions, but perturbed by a "viscouscorrection term, as in the case of Balanced Viscosity solutions. However, for Visco-Energetic solutions this viscous correction is tuned by a fixed parameter. The resulting solution notion is characterized by a stability condition and an energy balance analogous to those for Energetic solutions, but, in addition, it provides a fine description of the system behavior at jumps as Balanced Viscosity solutions do. Visco-Energetic evolution can be thus thought as "in-betweenËnergetic and Balanced Viscosity evolution.
We will explore these aspects in a general metric framework and then illustrate the application of the Visco-Energetic concept in two case-studies: finite-strain plasticity and brittle fracture.
Joint work with Gianni Dal Maso, Giuseppe Savaré, and Rodica Toader.

Tomáš Roubíček (Mathematical Institute, Charles University. Institute of Thermomechanics and Institute of Information Theory and Automation, Czech Academy of Sciences)

Dynamical damage and phase-field fracture models

Abstract: Damage is a phenomenon/concept in continuum mechanics of solid materials undergoing various degradation processes with numerous applications in engineering and in computational mechanics and (geo)physics. Combination with inertial effects may be important modelling issue to prevent various undesired effects otherwise occuring in quasistatic models. Various damage models and their variants as a phase-field fracture will be overviewed. Also, several numerical approaches will be presented, amenable to compute vibrations or waves emitted during fast damage/fracture, together with various extensions of the basic scenario, combining mass or heat transfer, plasticity, or fluid interactions.

Anja Schlömerkemper (University of Würzburg)

Homogenization in the passage from discrete to continuous systems - fracture in composite materials

Abstract: We consider the limiting behaviour of a one-dimensional toy model of a chain of atoms interacting via potentials of Lennard-Jones type, which are periodically or randomly distributed. The limit of the corresponding energy as the number of atoms tends to infinity is calculated by means of Γ-convergence. We will discuss the homogenized limiting functionals and interpret them in view of failure.
This is joint work with L. Lauerbach (Würzburg), S. Neukamm (TU Dresden) and M. Schäffner (TU Dresden).

Viktor Shcherbakov (Lavrentyev Institute of Hydrodynamics, Novosibirsk State University)

Conservation laws and energy release rates for interfacial cracks in fiber-reinforced composites

Abstract: Recent developments in materials science have generated interest in mathematical models of composite materials. The aim of this talk is to present some rigorous results for a novel model arising from the study of fiber-reinforced composites. We focus on a model of elastostatics for a two-dimensional elastic body with a thin elastic inclusion and interfacial crack. We use of a semi-linear Berger–Kirchhoff type equation with nonlocal coefficients to describe the transverse deflection of the inclusion and nonlinear boundary conditions/Signorini conditions that do not allow the opposite crack faces to penetrate each other. As a result, the model is formulated as an elliptic variational inequality. We assume that the crack can propagate along the interface only, and thus the crack path is known a priori. We are interested in an energy criterion on the basis of which one may decide whether or not the crack will propagate for given external forces. Following the Griffith energy concept, the energy release rate associated with perturbation of the crack along the interface is introduced to describe crack propagation. We give a rigorous justification of a formula for the energy release rate. Further, we investigate a regularity question for the weak solution, adapting suitably different quotient approximations for weak derivatives. The regularity result allows us to deduce conservation laws (path-independent energy integrals) that are related to the energy release rates associated with local translation and expansion of the crack. These path-independent integrals are analogues of the Eshelby–Cherepanov–Rice J- integral and the Knowles–Sternberg M-integral from fracture mechanics. In comparison with the classical formulae, new terms appear depending on the displacements of the thin elastic inclusion. Finally, the theoretical results are illustrated with some numerical calculations. These results extend our previous studies $[1, 2]$. This is a joint work with E.M. Rudoy.

References:

[1] Khludnev A.M., Shcherbakov V.V., Singular path-independent energy integrals for elastic bodies with Euler–Bernoulli inclusions. Math. Mech. Solids. **22**(11) (2017) 2180–2195.

[2] Rudoy E.M., Domain decomposition method for crack problems with nonpenetration condition. ESAIM-Math. Model. Num. **50**(4) (2016) 995–1009.

Francesco Solombrino (Università di Napoli Federico II)

Multiscale analysis of singularly perturbed finite dimensional gradient flows: the minimizing movement approach

Abstract: We perform a convergence analysis of a discrete-in-time minimization scheme approximating a finite dimensional singularly perturbed gradient flow. We allow for different scalings between the viscosity parameter μ and the time scale τ. When the ratio μ/τ diverges, we rigorously prove the convergence of this scheme to a (discontinuous) Balanced Viscosity solution of the quasistatic evolution problem obtained as formal limit, when μ tends to 0, of the gradient flow. We also characterize the limit evolution corresponding to an asymptotically finite ratio between the scales, which is of a different kind. In this case, a discrete interfacial energy is optimized at jump times. This is a joint work with G. Scilla.

Nicolas Van Goethem (Universidade de Lisboa)

Around the incompatibility operator and a new model of elasto-plasticity

Abstract: In a first part of this talk, we introduce the incompatibility operator for symmetric tensor fields with some of its functional properties such as trace theorems, boundary lifitings, coerciveness and the Beltrami decomposition of any square-integrable symmetric tensor field in a symmetric gradient and a divergence-free tensor. On this basis we present a novel model of elasto-plasticity which is intrinsic, i.e., takes the deformation tensor as basic variable, and shows linearized elasticity as a limit case. The general model is able to capture some plasticity phenomena, underlying the specific role of the strain incompatibility as related to the density of dislocations. Intriguing issues and open problems will also be presented.

Caterina Zeppieri (University of Münster)

Stochastic homogenization of free-discontinuity problems

Abstract: In this talk I will present a stochastic homogenization result for free-discontinuity functionals. Assuming stationarity for the random volume and surface integrands, we prove the existence of a homogenized random free-discontinuity functional, which is deterministic in the ergodic case. Moreover, by establishing a connection between the deterministic convergence of the functionals at any fixed realization and the pointwise Subadditive Ergodic Theorem by Akcoglou and Krengel, we characterize the limit volume and surface integrands in terms of asymptotic cell formulas. Our qualitative homogenization result extends to the SBV-setting the classical qualitative results by Papanicolaou and Varadhan, Kozlov, and Dal Maso and Modica, which were formulated in the Sobolev setting.

Joint work with F. Cagnetti (Sussex) , G. Dal Maso (SISSA), and L. Scardia (Bath)

Barbara Zwicknagl (TU Berlin)

Variational models for stress-free martensitic nuclei

Abstract: Microstructures in shape-memory alloys are often modelled in the context of the calculus of variations by non-convex multiwell elasticity functionals. In this talk, I shall discuss some recent analytical progress on related partial differential inclusion problems modelling stress-free martensitic nuclei, with and without surface energy constraints. This talk is based on joint work with S. Conti and M. Klar, and with A. Räkland and C. Zillinger.
