DVR 0065528

ABSTRACTS

Workshop on "Higher topological quantum field theory and categorical quantum mechanics"
October 19 – October 23, 2015

Manuel Bärenz (U of Nottingham)
Dichromatic state sum models and four-dimensional topological quantum field theories from pivotal functors

There is a scarcity of four-dimensional topological state sum models. Apart from the Crane-Yetter model, defined in the 90s, little is known. In this talk, a family of invariants of four-dimensional manifolds is presented. Some of them are stronger than the signature and Euler invariant. The invariants are parameterised by a pivotal functor from a spherical fusion category into a ribbon fusion category. A state sum formula for the invariant can be constructed via a chain mail procedure, so a large class of topological state sum models (and therefore, a 4-dimensional topological quantum field theories) can be phrased in terms of it. Most prominently, the Crane-Yetter state sum over an arbitrary ribbon fusion, possibly nonmodular category, and the 4-dimensional Dijkgraaf-Witten model are recovered. Derivations of state space dimensions of some TQFTs as special cases agree with recent calculations of ground state degeneracies in Walker-Wang models. It is also shown that the Crane-Yetter invariant for nonmodular categories is stronger than signature and Euler invariant.

John Barrett (U of Nottingham)
The non-commutative geometry of defects

A diagrammatic calculus for non-commutative geometry is introduced. This calculus provides a quantum invariant of surfaces with spin structure in which the defect lines are labelled with the Hilbert space of the non-commutative geometry. The axioms of non-commutative geometry follow from the topological properties of defect lines. From this one can derive some categorical generalisations of non-commutative geometry (this part due to work of Manuel Bärenz).

Bruce Bartlett (U of Oxford)
The centre of a fusion category, the adjoint representation, and the tube algebroid

Motivated by the string-net description of 3-dimensional TQFT, I will describe two ways of understanding the vector space V assigned to the torus. Given a fusion category C I will introduce the "adjoint representation" of the fusion ring of C, and show that V is the space of invariants under this action. I will also describe an explicit contravariant equivalence between the Drinfeld centre $Z(C)$ and the category of representations $\text{Rep}(T)$ of the associated tube algebroid T of C.

Richard Blute (U of Ottawa)
Towards a Theory of Integral Linear Logic via Rota-Baxter algebras

Differential linear logic, as introduced by Ehrhard & Regnier, extends linear logic with an inference rule which is a syntactic version of differentiation. The corresponding categorical structures, called differential categories, were introduced by Blute, Cockett & Seely. Differential categories are monoidal categories equipped with a comonad which endows objects in its image with a cocommutative coalgebra structure. There is also a natural transformation, called the deriving transform, which models the differential inference rule. The large number of examples of differential categories demonstrate the utility of the idea. These include the convenient vector spaces of Frohlicher and Kriegl.

It is an ongoing project to develop similar notions of integral linear logic and integral categories. An appropriate place to draw inspiration for this is the theory of Rota-Baxter algebras. Rota-Baxter algebras are associative algebras with an endomorphism which satisfies an abstraction of the integration by parts formula. There are many examples of such algebras and multi-object versions of these examples should provide important examples of models of integral linear logic. In particular, there is a naturally occurring Rota-Baxter operator in the Connes-Kreimer Hopf algebra associated to renormalization in quantum field theory.

Bob Coecke (U of Oxford)

From quantum foundations to natural language meaning via string diagrams

We introduce the idea of a process theory, as developed in the textbook [1]. The mathematical underpinning is entirely diagrammatic, or if you want, it's category theory in disguise, although accessible pretty much to anyone who has half a brain. Conceptually, it involves a logical stance that focusses on the interactions rather than on the description of the individual. It's successes so far are a high-level conceptual underpinning for quantum theory, known as CQM, as well a framework to reason about meaning in natural language, solving the open problem on how to compute the meaning (not just true or false!) of a sentence given the meaning of its words [2, 3]. This talk will survey all of this, and focus in particular how structures from quantum theory are also fundamental for natural language meaning.

References: [1] B. Coecke & A. Kissinger (2015, 750 pp) Picturing Quantum Processes. Cambridge University Press.

[2] B. Coecke, M. Sadrzadeh & S. Clark (2010) Mathematical foundations for a compositional distributional model of meaning. Linguistic analysis - Lambek Festschrift. arXiv:1003.4394

[3] M. Sadrzadeh, S. Clark and B. Coecke, (2013) The Frobenius anatomy of word meanings I: subject and object relative pronouns. Journal of Logic and Computation. arXiv:1404.5278

Alexei Davydov (Ohio U, Athens)

Higher Witt categories of modular categories

Higher category structure of defects in 3d TFTs and 2d RCFTs indicate to the existence of a 3-category with modular fusion categories as objects. This 3-category provides a higher categorification of the Witt group of modular categories.

Ross Duncan (U of Strathclyde)

Interacting Frobenius Algebras Are Hopf

Commutative Frobenius algebras play an important role in both TQFT and CQM; in the first case they correspond to 2d TQFTs, while in the second they are non-degenerate observables. I will consider the case of "special" Frobenius algebras, and their associated group of phases. This gives rise to a free construction from the category of abelian groups to the PROP generated by this Frobenius algebra. Of course a theory with only one observable is not very interesting. I will consider how two such PROPs should be combined, and show that if the two algebras (i) jointly form a bialgebra and (ii) their units are "mutually real"; then they jointly form a Hopf algebra. This gives a free model of a pair of strongly complementary observables. I will also consider which unitary maps must exist in such models.

Domenico Fiorenza (U Roma 1)

Group actions on boundary structures in Dijkgraaf-Witten theory

It has been remarked by Douglas and Sharpe that Chan-Paton factors and D-branes for a 2-dimensional gauge theory with finite group G and discrete torsion carry a natural projective action of the gauge group G. This corresponds to a genuine group action of the $U(1)$-central extension of G defined by the discrete torsion of the theory as an element in $H^2(G, U(1))$. Similarly, boundary conditions for the 3-dimensional Dijkgraaf-Witten theory with gauge group G and $U(1)$-valued 3-cocycle α carry a natural action of the 2-group which is the central extension of G by α. In the talk we will see how, from the point of view of fully extended TQFTs, these "higher projective representations" of the group G arise in a completely natural way. The talk is based on Freed-Hopkins-Lurie-Teleman "TQFTs from compact Lie groups" as well as on several conversations with Urs Schreiber and Alessandro Valentino.

Sergei Gukov (Caltech, Pasadena)
LG interfaces and categorification of interesting algebras

The main goal of this talk is to motivate the study of a potentially rich dictionary between interfaces in Landau-Ginzburg model, on the one hand, and categorification of important algebraic structures in mathematics. In this dictionary, a simple drawing of interface lines and their fusion corresponds to diagrams in the "diagrammatic approach" to categorification a la Khovanov-Lauda and others. Even the simplest example of interfaces in minimal models leads to categorification used in the cutting edge knot homology, and one can only imagine what other LG models offer via this dictionary. This talk is based on recent work with Daniel Roggenkamp and Sungbong Chun.

Nick Gurski (U of Sheffield)
2-categorical methods in abstract homotopy theory

Quasicategories are becoming a standard tool in abstract homotopy theory, but recent work of Riehl and Verity shows that there are significant advantages in building up the theory of quasicategories using insights from the Australian school of 2-category theory. I will discuss a variety of projects which follow a similar philosophy, including the K-theory of symmetric monoidal (2-)categories (joint with Johnson and Osorno), homotopy coherent distributive laws and ring structures (joint with Cranch), and weak maps between homotopy coherent algebras (joint with Schaeppi).

Chris Heunen (U of Oxford)
Introduction to Categorical Quantum Mechanics

These two lectures cover the very basics of the use of monoidal categories to model quantum mechanical systems. Their graphical calculus will be an important tool. We will discuss dual objects (modelling entanglement), monoids and comonoids (leading to a no-cloning theorem), Frobenius algebras (embodying measurement), bialgebras (giving complementary observables), and complete positivity (adding mixed states). We focus on the mathematical structure but point out physical interpretations and computer science applications.

Paul-Andre Mellies (U Paris Denis Diderot)
Dialogue categories and Frobenius algebras

About ten years ago, Brian Day and Ross Street discovered a beautiful and unexpected connection between the notion of star-autonomous category in proof theory and the notion of Frobenius algebra in mathematical physics. In this talk, I will investigate the logical content of this connection by formulating a two-sided presentation of Frobenius algebras. The presentation is inspired by the idea that every logical dispute has two sides consisting of a Prover and of a Denier. This dialogical point of view leads us to a correspondence between tensorial logic, dialogue categories and a lax notion of Frobenius monoid. The correspondence refines Day and Streets original correspondence in the same way as tensorial logic (or equivalently dialogue games and innocent strategies) refines linear logic. I will explain at the end of the talk how to depict tensorial proofs in the graphical language of cobordism.

Catherine Meusburger (U Erlangen)
Gray categories with duals and their diagrammatical description

Gray categories can be viewed as maximally strict tricategories. We introduce a notion of duals for Gray categories and a description in terms of diagrams. These diagrams are three-dimensional stratifications of a cube, with regions, surfaces, lines and vertices labelled by Gray category data. They can be viewed as a generalisation of ribbon diagrams.

The duals are introduced in the diagrammatical framework. We show that they give rise to Gray category functors and natural isomorphisms of Gray category functors, which correspond to the symmetries of the cube. The Gray categories and these functors can be strictified such that the symmetries of the diagrams are realised exactly. We show that for a certain class of Gray category diagrams, the evaluation of a diagrams is invariant under homomorphisms of diagrams.

Simon Perdrix (U de Lorraine, CNRS)

Supplementary of Interacting Frobenius Algebras

In categorical quantum mechanics, the axiomatisation of interacting observables has led to the ZX calculus, a graphical language introduced by Coecke and Duncan. The ZX calculus is mainly based on interacting Frobenius algebras. The language is complete for the stabiliser fragment of quantum mechanics. The completeness for a larger and universal fragment of quantum mechanics, the so-called 'Clifford+T' fragment, has been conjectured and even proved when restricted to a single qubit. We show that supplementarity, a property of interacting observables pointed out by Coecke and Edwards, cannot be derived in the ZX-calculus. It implies in particular that the ZX-calculus is not complete for the 'Clifford+T' fragment.

Dorette Pronk (Dalhousie U)

Orbifold Atlases Revisited

Classical orbifolds (called V-manifolds at that time) were introduced by Satake in terms of an underlying space with an atlas of orbifold charts. Local charts were given in terms of an open subset of Euclidean space with an effective action of a finite group. Because of the effectiveness of the actions, embeddings between these charts could be taken to be all embeddings between the charts that commute with the group actions. However, the fact that not all structure present was really used became apparent in the definition of maps between orbifolds. Satake presented two different notions in his papers, and neither one of those was the right one for orbifold homotopy theory. This problem was remedied by considering orbifolds as a particular kind of (effective) Lie groupoids. In this description the natural notion of map would be that of a Hilsum Skandalis module or a so called generalized map. (Generalized maps are obtained as maps in the bicategory of fractions constructed to invert the Morita equivalences between Lie groupoids.)

In the study of orbifold homotopy theory and its applications to mathematical physics it became apparent that it would be desirable to study orbifolds for which the group actions on the charts are not necessarily effective. In terms of groupoids it was easy to drop the effective requirement and generally non-effective (also called, non-reduced) orbifolds have been studied in terms of their representing groupoids. Several authors do include a sketch of how they would consider these objects as an underlying space with an atlas of (possibly non-effective) charts with a collection of embeddings that need to satisfy certain properties. However, none of those definitions provide a notion of orbifold that actually corresponds to the non-effective Lie groupoids.

In order to obtain this correspondence one needs to require additional structure on the collection of all embeddings between two charts and the non-effectiveness needs to be built right into the embedding structure. In this talk I will present a way to do this and I will also discuss how we can then define maps between orbifolds in terms of these atlases. I hope that understanding this aspect in the construction of orbifolds from the category of open subsets of Euclidean space and embeddings between them will assist in obtaining an understanding of the relationship between orbifolds and the orbifold construction for bicategories.

Gregor Schaumann (U Vienna)

Introduction to Topological Quantum Field Theory

A TFT is often introduced as a systematic collection of manifold invariants. Here we focus instead on the physical origins of 3d TFTs and from there we consider defects and corresponding higher categories in TFT.

From 3d quantum gravity as well as from string-net models one arrives at the Turaev-Viro-Barrett-Westbury TFT. 1d defects in these models have the interpretation of particles and are used for topological quantum computing.

Then we focus on the structure of the (higher dimensional) defects and explain the appearance of higher categories. Finally a short overview of current research directions in TFT is given.

Claudia Scheimbauer (MPI Bonn)
(Op)lax natural transformations for higher categories and relative field theories

A relative (also called twisted) quantum field theory should be some transformation between functorial quantum field theories, which themselves are symmetric monoidal functors out of a space-time category. In examples, the notion of natural transformation turns out to be too strong, making it necessary to relax it. In joint work with Theo Johson-Freyd we provide a framework for both lax and oplax transformations and their higher analogs (called transfors) between strong (∞, n)-functors and propose a definition of relative (extended) field theories. A natural target for such should be a delooping of a (higher) category $\mathcal{C}$, e.g. the category of vector spaces or of categories. One such delooping is given by taking algebras and bimodules thereof, and bimodule morphisms. Its higher analog is the higher Morita category of E_d-algebras in the higher category $\mathcal{C}$. I will explain how to use our framework to construct this higher delooping.

Christoph Schweigert (U of Hamburg)
Traces for bimodule categories, generalized Wilson lines and generalized conformal blocks

Defects in (extended) three-dimensional topological field theories have important applications, ranging from solid state physics to representation categories.

In particular, such defects lead to generalized Wilson lines. To find categories in which labels for these Wilson lines take their values, we study a 2-functor that assigns to a bimodule category over a finite tensor category a k-linear category and has a coherent cyclic invariance of a categorified trace for the relative Deligne product. It turns out that this 2-functor also enters crucially into the construction of generalized conformal blocks.

Constantin Teleman (U Berkeley)
Matrix Factorizations and Lie group representations

We describe a categorical inverse to the Kirillov correspondence between co-adjoint orbits and unitary representations of compact Lie group and their Loop groups in terms of families of Dirac operators and Matrix Factorization categories. We will also discuss generalizations to real semi-simple Lie groups (work in progress) and, time permitting, touch upon the relation to 4D super-symmetric Yang-Mills theory. This is based on joint work with Dan Freed and (independently) Kiran Luecke.

Constantin Teleman (U Berkeley) Colloquium talk
Introduction to quadratic topology

A symmetric bilinear form $B : \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}$ has a unique *quadratic refinement*, a homogeneous quadratic function $q : \mathbb{Q} \to \mathbb{Q}$ such that $B(x, y) = q(x + y) - q(x) - q(y)$. It is given by the formula $q(x) = \frac{1}{2}B(x, x)$. When division by 2 is problematic, the relation between quadratic and symmetric bilinear forms becomes more complicated. We will review this relation on abelian groups, and several appearances of these notions in topology. In particular, we will see why the problem, though connected with the number 2, is not one of division by 2.

Alessandro Valentino (MPI Bonn)
Boundary conditions for 3d TQFTs and module categories

In this talk I will discuss some aspects of boundary conditions for a 3d TFT of Reshetikhin-Turaev type, and their description in terms of module categories.

Jamie Vicary (U of Oxford)
Higher categories and quantum computation

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