Discussion of Felix Otto's talk

The structure group in regularity structures: avoiding trees

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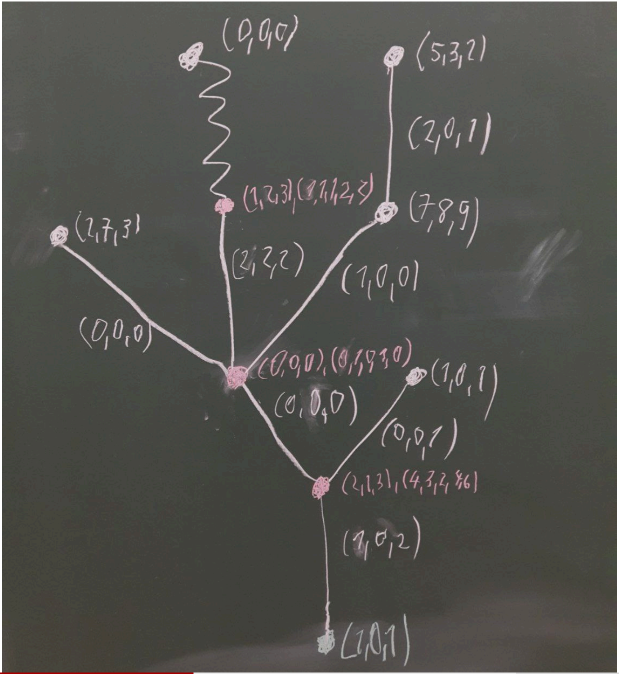
Higher Structures Emerging from Renormalisation, ESI Vienna, November 16, 2021


(0,0,0)
(5,3,2)
(2,0,1)
(7,8,9)
(2,7,3)
(2,2,2)
(1,0,0)
(0,0,0)
(1,0,1)
(0,0,1)
(1,0,2)
(1,0,1)

We look at the vector space T^+ generated by $(Z^{(J,\mathbf{m})})_{(J,\mathbf{m})}$ as the dual basis to the differential operators

$$D_{(J,\mathbf{m})} := \frac{1}{J!\mathbf{m}!} \prod_{(\gamma,\mathbf{n})} (\mathbf{z}^\gamma)^{J(\gamma,\mathbf{n})} \partial_1^{m_1} \partial_2^{m_2} \prod_{(\gamma,\mathbf{n})} (D^{(\mathbf{n})})^{J(\gamma,\mathbf{n})}$$

where $\mathbf{m} \in \mathbb{N}_0^2$ is a multindex and J is a multiindex on multiindices $(\gamma, \mathbf{n}) \in \mathbb{N}_0^{\mathbb{N}_0} \times \mathbb{N}_0^{\mathbb{N}_0^2 \setminus (0,0)} \times (\mathbb{N}_0^2 \setminus (0,0))$.

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$$\text{E.g.} \quad (J', \mathbf{m}') = (3e_{(\gamma_1,\mathbf{n}_1)} + 2e_{(\gamma_2,\mathbf{n}_2)}, (4,3))$$

with

$$\gamma_1 = 5\epsilon_3 + 7\epsilon_1 + 2\epsilon_{(2,0)} + 2\epsilon_{(0,1)}, \quad \mathbf{n}_1 = (0,3)$$

and

$$\gamma_2 = 8\epsilon_5 + 3\epsilon_{(1,0)}, \quad \mathbf{n}_2 = (1,1)$$

But only those $e_{(\gamma,\mathbf{n})}$ are allowed in J with

$$[\gamma] = \sum_{k\geq 0} k\gamma(k) - \sum_{\mathbf{m}\neq 0} \gamma(\mathbf{m}) > 0, \quad |\gamma| = \alpha([\gamma]+1) + \sum_{\mathbf{m}\neq 0} |\mathbf{m}|\gamma(\mathbf{m}) > |\mathbf{n}|$$

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We have a product

$$Z^{(J',\mathbf{m}')}Z^{(J'',\mathbf{m}'')} = Z^{(J'+J'',\mathbf{m}'+\mathbf{m}'')}$$

and a multiplicative coproduct generated by

$$\Delta^+ Z^{(e_{(\gamma,\mathbf{n})},0)} = \sum_{(J,\mathbf{m})} \sum_{\beta:[\beta]>0,|\beta|>|n|} \langle \mathbf{z}_\gamma, D_{(J,\mathbf{m})}\mathbf{z}^\beta \rangle Z^{(J,\mathbf{m})} \otimes Z^{(e_{(\beta,\mathbf{n})},0)} + \sum_{\mathbf{m}:|\mathbf{n}+\mathbf{m}|<|\gamma|} \binom{\mathbf{n}+\mathbf{m}}{\mathbf{m}} Z^{(e_{(\gamma,\mathbf{n}+\mathbf{m})},0)} Z^{(0,\mathbf{m})},$$

where $\langle \mathbf{z}_\alpha, \mathbf{z}^\beta \rangle = \delta_{\alpha,\beta}$,

$$\Delta^+ Z^{(0,(1,0))} = Z^{(0,(1,0))} \otimes \mathbf{1} + \mathbf{1} \otimes Z^{(0,(1,0))}$$

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We can also write the coproduct via the comodule $\Delta : T \to T^+ \otimes T$:

$$\Delta \mathbf{z}_\gamma = \sum_{\beta,(J,\mathbf{m})} \langle \mathbf{z}_\gamma, D_{(J,\mathbf{m})} \mathbf{z}^\beta \rangle Z^{(J,\mathbf{m})} \otimes \mathbf{z}_\beta$$

Then

$$\Delta^+ \mathcal{J}_\mathbf{n} \mathbf{z}_\gamma = (\mathrm{id} \otimes \mathcal{J}_\mathbf{n}) \Delta \mathbf{z}_\gamma + \sum_\mathbf{m} \mathcal{J}_{\mathbf{m}+\mathbf{n}} \mathbf{z}_\gamma \otimes \frac{Z^{(0,\mathbf{m})}}{\mathbf{m}!}$$

where

$$\mathcal{J}_\mathbf{n} \mathbf{z}_\gamma = \left\{ \begin{array}{ll} \mathbf{n}!\, Z^{(e_{(\gamma,\mathbf{n})},\mathbf{0})} & \text{if } [\gamma] \geq 0,\ |\gamma| > |\mathbf{n}| \\ 0 & \text{otherwise} \end{array} \right\}.$$

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2. What does the renormalisation group look like?
3. Is there a cointeraction?