Workshop on
"Heights in Diophantine geometry, group theory and additive combinatorics"

organized by
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Abstracts

Paul Fili (University of Oklahoma)
Title: *Energy integrals and height bounds*
Abstract: Bombieri and Zannier established lower bounds for the height of algebraic numbers which are totally p-adic. We will present an improvement on these bounds for non-archimedean places which also allows for archimedean splitting conditions. Our proof relies on solving an energy minimization problem over local fields. This is joint work with Clayton Petsche.

Robert Grizzard (University of Texas at Austin)
Title: *On the compositum of all degree d extensions of a number field*
Abstract: Bombieri and Zannier asked the question of whether the compositum $k^{(d)}$ of all extensions of degree at most d of a number field k has the Northcott property, but were only able to prove this property for the maximal abelian subextension, denoted $k_{ab}^{(d)}$. In their proof, it is important that all finite subextensions of $k_{ab}^{(d)}/k$ are generated by elements of degree at most d. Following work of Checcoli and Zannier, we explore this "boundedness" property, particularly for Galois subextensions of the compositum of all extensions of degree exactly d. This is joint work with Itamar Gal.

Francesco Amoroso (University of Caen)
Title: *Bounded height problems and Silverman Specialization Theorem (joint work with D. Masser and U. Zannier)*
Abstract: Let $\mathcal{C}$ be a curve defined over $\overline{\mathbb{Q}}$. Bombieri, Masser and Zannier proved a result which may be rephrased as a toric analogue of Silverman's Specialization Theorem: *Let $\Gamma \subset \mathbb{G}_{\mathrm{m}}(\mathcal{C})$ be a finitely generated subgroup of non zero rational functions on $\mathcal{C}$ which does not contain non trivial constant functions. Then the set of $P \in \mathcal{C}(\overline{\mathbb{Q}})$ such that the restriction of the specialization map $\sigma_P\colon \mathbb{G}_{\mathrm{m}}(\mathcal{C}) \to \mathbb{G}_{\mathrm{m}}(\overline{\mathbb{Q}})$, $x \mapsto x(P)$ to Γ is not injective is a set of bounded height.* It turns out that in fact a weaker assumption suffices to have bounded height: *Let V be an algebraic subvariety of $\mathbb{G}_{\mathrm{m}}^r(\mathcal{C})$ and let $\sigma_P\colon \mathbb{G}_{\mathrm{m}}^r(\mathcal{C}) \to \mathbb{G}_{\mathrm{m}}^r(\overline{\mathbb{Q}})$ be the specialization map. Then the set of $P \in \mathcal{C}(\overline{\mathbb{Q}})$ such that for some $\mathbf{x} \in \Gamma^r \backslash V$ we have $\sigma_P(\mathbf{x}) \in \sigma_P(V)$ is a set of bounded height.* As a corollary we obtain a bounded height result for some degenerate unlikely intersections.

Patrice Philippon (University of Paris VI)
Title: *Essential minima of toric varieties II*
Abstract: I will present a work in progress with J.Burgos Gil and M.Sombra, giving combinatorial descriptions of arithmetic quantities such as metrics, heights, algebraic successive minima, ... associated to toric divisors on toric varieties. I will also show how the obtained dictionary enable one to compute effectively these quantities on specific examples.

Laura Capuano (Scuola Normale Pisa)
Title: *An example of Unlikely Intersections in the multiplicative group*
Abstract: In this seminar we are going to give a different proof of the theorem of Bombieri, Masser and Zannier of 1999 about intersecting a curve with algebraic subgroups of the multiplicative group. To do that, we mainly use Pila-Zannier method and some estimates about rational points of bounded height of Pila-Wilkie type. This method was used for the first time in 2008 by Pila and Zannier to give a new proof of Manin-Mumford conjecture but is very general and has been used to prove other cases of unlikely intersections problems in many different contexts.

Francesco Veneziano (Graz University of Technology)
Title: *Torsion-anomalous intersections*
Abstract: Anomalous Intersections are a fairly recent framework introduced by Bombieri, Masser and Zannier, which comprises and generalises a vast body of problems and conjectures in Arithmetic Geometry. Let V be a variety contained in a group variety G, which is usually taken to be an abelian variety or a torus. When intersecting V with an algebraic subgroup B, if the intersection $V \cap B$ has a component of dimension strictly greater than "expected", then such a component is said to be torsion-anomalous. In analogy with many fundamental results in the field, there are conjectures giving geometrical conditions for the variety V to have only finitely many (maximal) torsion-anomalous subvarieties. The formulation of these conjectures generalises famous problems such as the Manin-Mumford Conjecture and is related to the Mordell-Lang problem.

Fabrizio Barroero (Graz University of Technology)
Title: *Counting S-integers of fixed degree and bounded height*
Abstract: Northcott's Theorem says that sets of algebraic numbers of fixed degree and uniformly bounded height are finite. It is therefore natural to ask for asymptotic formulas for the cardinality of such sets as the height bound tends to infinity. Let k be a number field and S a finite set of places of k containing the archimedean ones. Let e be a positive integer. We give an asymptotic formula for the number of $\overline{S}$-integers of degree e over k and bounded height, where $\overline{S}$ is the set of places of $\overline{k}$ that extend the ones in S.