ABSTRACTS

Workshop on "The interrelation between mathematical physics, number theory and noncommutative geometry", Master Class, March 2 - 6, 2015:

Stefan Weinzierl (3 lectures)
"Feynman integrals and the functions associated to them"

The computation of Feynman integrals is a prerequisite for precision calculations in particle physics. On the other hand, Feynman integrals are also very interesting from the mathematical point of view. In these lectures I will discuss Feynman integrals and some of the functions to which they evaluate. Whereever possible, I will try to make the connection to algebraic geometry clear. In this spirit I will introduce Feynman integrals through two graph polynomials and discuss first the properties of these polynomials. I will then consider Feynman integrals which evaluate to multiple polylogarithms and discuss the algebraic structure of the latter. Finally I will consider Feynman integrals which are beyond the class of multiple polylogarithms.

Karen Yeats (3 lectures)
"Introduction to a combinatorial perspective on QFT"

Lecture 1: Introduction to a combinatorial perspective on QFT: An introduction to my perspective on Feynman graphs, Feynman integrals, and their sums, with definitions and examples which will be useful in the next two lectures. Lecture 2: Graph theory and QFT periods: What can we learn about periods in QFT from the graphs themselves? What graph symmetries preserve the periods? What graph invariants preserve these symmetries? What are some tools and results? Lecture 3: Recursive and algebraic structure via DSEs: We see new structure when we sum graphs together the way physics tells us to. What are some different forms the result can take? What can we learn from them?

Raimar Wulkenhaar (4 lectures)
"Quantum field theory on noncommutative spacetime"

It is an old idea to use noncommutative spaces as a regulator for quantum field theory. Such a regularisation can 1) be regarded as a toy model for quantum gravity, or 2) as a technical tool to overcome problems in traditional quantum field theories. I am going to review recent progress in the second approach. We consider a regularisation of the $\lambda\phi^4_4$-model on a noncommutative geometry which turns that model into a matrix model. The huge remnant symmetry allows to exactly solve this 4D model in a certain scaling limit. The limit describes Schwinger functions on standard $\mathbb{R}^4$ for which the easy Osterwalder-Schrader axioms are proved and for which the decisive reflection positivity axiom looks promising.

Jacob Bourjaily (5 lectures)
"Scattering Amplitudes and Grassmannian Geometry"

The past several years have been witness to an ongoing revolution in our understanding of (perturbative) quantum field theory. In particular, a concrete proposal now exists for how to reformulate any theory recursivelywithout any reference to virtual particles, gauge redundancies, or any of the other unphysical baggage that so complicates computations in the traditional formalism. In addition to greatly simplifying computations, the recursive reformulation provides an important connection between field theory and the geometry (and combinatorics) of certain subspaces of the Grassmannian—a connection that has proven extremely fruitful for both physics and mathematics in recent years. Because both sides of this connection are greatly simplified (and best understood) in the case of planar, maximally supersymmetric Yang-Mills, this will be the primary example discussed. I will provide a broad introduction to these ideas, starting from basic principles of quantum mechanics alone. No prior familiarity with supersymmetric theories or Grassmannian geometry will be assumed.

Pierre Vanhove (Erwin Schrödinger Lecture, Wednesday, March 4 at 5 p. m.)
"Modular invariance and duality symmetries in Quantum Field Theory and String Theory"

An apocryphal quote (sometime attributed to Martin Eichler) says that "There are five elementary arithmetical operations: addition, subtraction, multiplication, division, and . . . modular forms." Modular and automorphic forms play prominent role in the understanding of the fundamental properties of quantum field theory and string theory. They are consequences of fundamental duality symmetries connecting weakly coupled and strongly coupled regimes of the theory. In this talk we will describe the appearance of modular and automorphic forms in string theory in connection to fundamental properties of black hole. We will explain how physically motivated questions connect to deep properties of automorphic representation. We will as well report on the recent progress in perturbative computations in QCD and quantum gravity, and the role of modular forms in the evaluation of Feynman integrals.

Alain Connes ("The Arithmetic Site" Lecture Series, March 5 at 11 a.m. part 1 of 3 at the ESI)
"Noncommutative Geometry"