Ward Schwinger Dyson equations

Enrico I. Russo & Marc P. Bellon

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ESI: Higher Structures Emerging from Renormalisation

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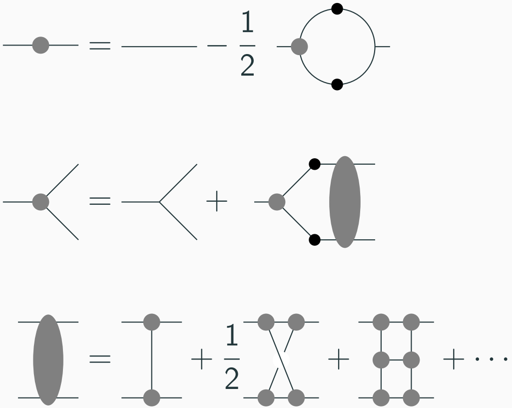
Why WSD

Introduction

- pQFT gives divergent series
- N-P physics might be hidden in these infinities
- Resurgence needs a differential equation to work well

Idea: SD and RG equations play this role.

The ϕ_6^3 model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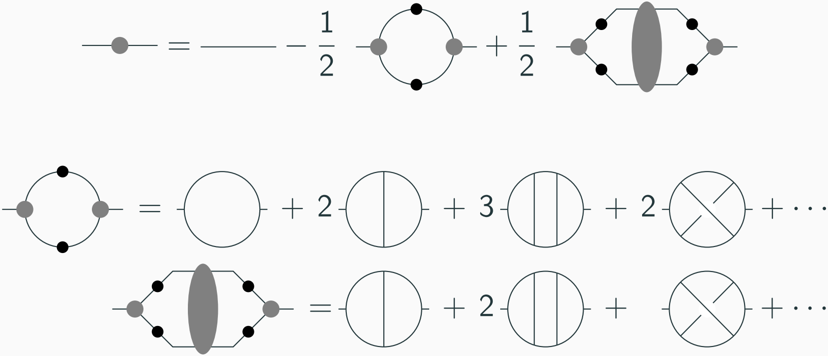
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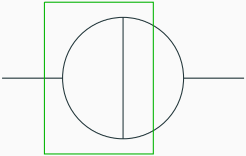

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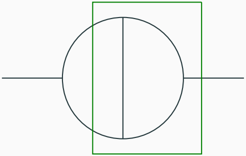
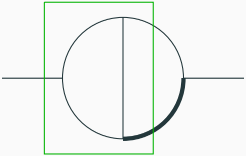
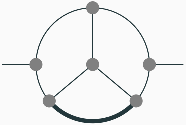
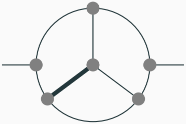
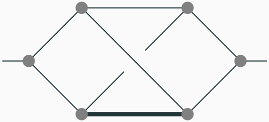


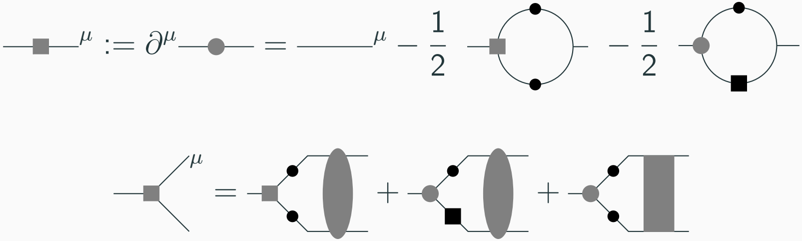


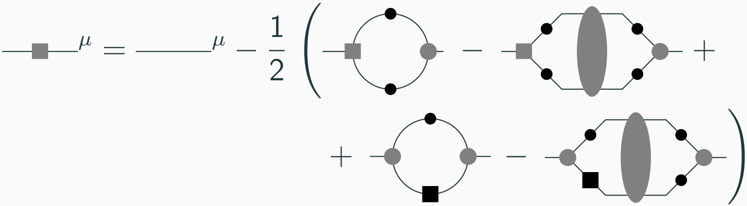


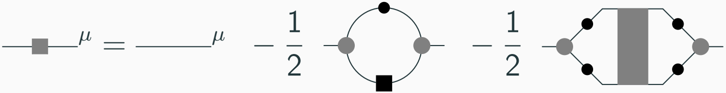




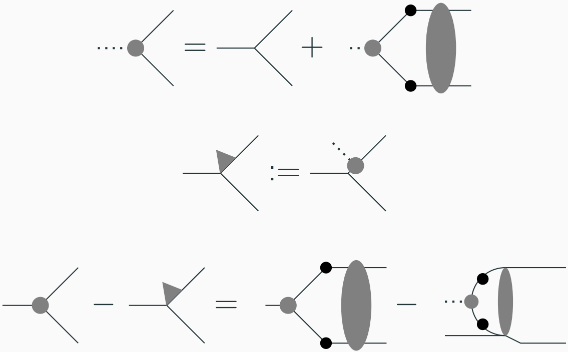
Ward's trick

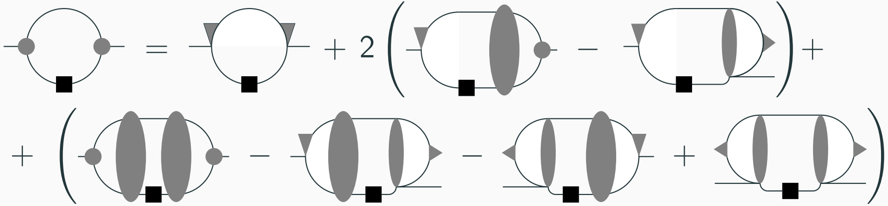




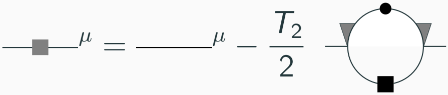


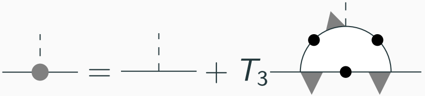
IR rearrangement





Renormalised ingredients





$$\begin{cases} G(a,L) := \sum_{n\geq 0} \gamma_n(a)\frac{L^n}{n!} & \text{for} \\ Y(a,L) := \sum_{n\geq 0} \upsilon_n(a)\frac{L^n}{n!} & \text{for} \\ K^\nu(a,L) := \frac{2p^\nu}{(p^2)^2}\sum_{n\geq 0}(\gamma_{n+1}-\gamma_n)\frac{L^n}{n!} & \text{for} \end{cases}$$



$$\begin{cases} \mp(a,L) := YGY(a,L) = \sum_{n\geq 0} s_n(a)\frac{L^n}{n!} & \text{for} \\ \Phi(a,L) := GYG(a,L) = \sum_{n\geq 0} q_n(a)\frac{L^n}{n!} & \text{for} \end{cases}$$

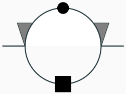

$$\partial_L G = (\gamma + \beta a \partial_a)\, G \qquad \partial_L \mp = (\gamma + 2\upsilon + \beta a \partial_a)\, \mp$$

$$\partial_L Y = (\upsilon + \beta a \partial_a)\, Y \qquad \partial_L \Phi = (2\gamma + \upsilon + \beta a \partial_a)\, \Phi$$

What about K^ν? *WSD* impose a *RG* equation on it and provide a recipe for $\beta = 3\gamma + 2\upsilon$!

$$L \leftrightarrow \partial \qquad L^n = \partial_x^n e^{xL} \sim \partial_x^n (p^2)^x$$

The Cat

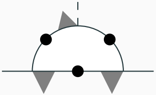


$$:= a \int_{\mathbb{R}^6} \frac{du}{(2\pi)^6} \mp(u) K^\nu(u+p)$$

$$= a G_x \mp_y \partial^\nu \left((p^2)^{1+x+y} \frac{\Gamma(-1-x-y)\Gamma(2+x)\Gamma(2+y)}{\Gamma(4+x+y)\Gamma(1-x)\Gamma(1-y)} \right)$$

$$H^\gamma(x,y) = \frac{\Gamma(1-x-y)\Gamma(2+x)\Gamma(2+y)}{\Gamma(4+x+y)\Gamma(1-x)\Gamma(1-y)}$$

The Walrus



$$:= a \int_{\mathbb{R}^6} \frac{du}{(2\pi)^6}\, \Phi(u)\,\text{Ŧ}(u+p)$$

$$= a\Phi_x \text{Ŧ}_y e^{(x+y)L} \frac{\Gamma(-x-y)\Gamma(1+x)\Gamma(2+y)}{\Gamma(3+x+y)\Gamma(2-x)\Gamma(1-y)}$$

$$H^{\upsilon}(x,y) = \frac{\Gamma(1-x-y)\Gamma(1+x)\Gamma(2+y)}{\Gamma(3+x+y)\Gamma(2-x)\Gamma(1-y)}$$

General procedure

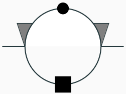
$\mathcal{W} \in \{$  $\}$

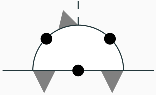
$$\partial_L \mathcal{W} = (w + \beta a \partial_a)\, \mathcal{W}$$

$$w := \# G\, \gamma + \# Y\, \upsilon$$

$$\mathcal{O} = a\mathcal{W}_1\mathcal{W}_2$$

$$\mathcal{O}_\bullet = \begin{cases} aG_\bullet Ŧ_\bullet \quad \text{for} \\ a\Phi_\bullet Ŧ_\bullet \quad \text{for} \end{cases}$$

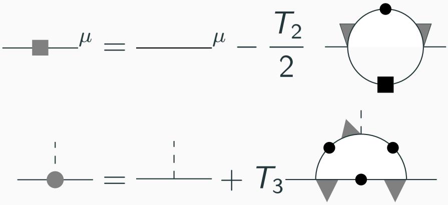




$$\gamma_\mathcal{O} := \#G\,\gamma + \#Y\,\upsilon - \beta$$

$$\partial_L\mathcal{O} = (\gamma_\mathcal{O} + \beta a\partial_a)\,\mathcal{O}$$

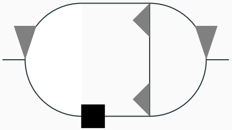
The final form of *WSD*



$$\gamma(a) = (\gamma - \beta a \partial_a)\,\gamma + \frac{T_2}{2}\mathcal{O}_\gamma H^\gamma(x, y)$$

$$\upsilon(a) = -a T_3 \mathcal{O}_\upsilon H^\upsilon(x, y)$$

Corrections



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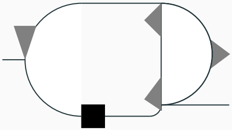
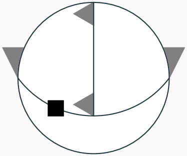
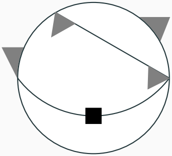


$\sim a^2$



$$\psi_{\tilde{1}} = t_6 \psi_1 + \phi_1$$



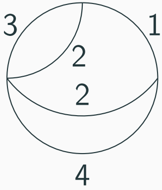
$$\psi_{\tilde{2}} = t_6 \psi_2 + \phi_2$$

$$\psi_{\tilde{1}} - \psi_{\tilde{2}} = \phi_1 - \phi_2$$

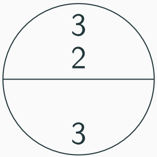
$$\int dt_1 \dots dt_6 \, t_5 t_6^3 \left(\frac{C_{\widetilde{1}}}{\psi_{\widetilde{1}}^4} - \frac{C_{\widetilde{2}}}{\psi_{\widetilde{2}}^4} \right) \delta_H .$$

$$C_{\widetilde{1}} = t_4(t_1 + t_2 + t_3) + t_3 t_2$$
$$C_{\widetilde{2}} = t_4(t_1 + t_2 + t_3) + (t_1 + t_2) t_3$$

$$\frac{C_{\widetilde{1}}}{\psi_{\widetilde{1}}^4} - \frac{C_{\widetilde{2}}}{\psi_{\widetilde{2}}^4} = C_{\widetilde{1}} \left(\frac{1}{\psi_{\widetilde{1}}^4} - \frac{1}{\psi_{\widetilde{2}}^4} \right) - \frac{t_1 t_3}{\psi_{\widetilde{2}}^4}$$
$$= C_{\widetilde{1}} \left(\psi_{\widetilde{2}} - \psi_{\widetilde{1}} \right) \left(\frac{1}{\psi_{\widetilde{1}}^4 \psi_{\widetilde{2}}} + \frac{1}{\psi_{\widetilde{1}} \psi_{\widetilde{2}}^4} + \frac{1}{\psi_{\widetilde{1}}^2 \psi_{\widetilde{2}}^3} + \frac{1}{\psi_{\widetilde{1}}^3 \psi_{\widetilde{2}}^2} \right) +$$
$$- \frac{t_1 t_3}{\psi_{\widetilde{2}}^4} .$$



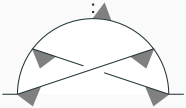
$$\approx \frac{\Gamma(0)}{\Gamma(4)}$$



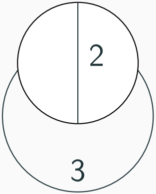
$$\approx \frac{\Gamma(0)}{24}$$

So this contributes to γ with a factor $-\frac{a^2}{24}$.

$\frac{1}{2}$



$\frac{1}{2}$



$$\frac{1}{2}\int_{\mathbb{R}_+^6} dt_1 \ldots dt_6 \frac{t_5 t_6^2}{\tilde{\psi}^3}\delta_H$$

This contributes to υ with a factor of $\frac{a^2}{4}$.

Result

Results

$$\gamma = \frac{T_2}{12} a + (-11\, T_2 + 48\, T_3)\, \frac{T_2}{432} a^2$$

$$\upsilon = -\frac{T_3}{2} a + \left(-\frac{T_5}{4} + \frac{T_3}{16} \left(T_2 - 6\, T_3 \right) \right) a^2$$

$$\beta = \left(T_2 - 4\, T_3 \right) \frac{a}{4} + \left(-11\, T_2^2 + 66\, T_2\, T_3 - 108\, T_3^2 - 72\, T_5 \right) \frac{a^2}{144}$$

but also trans-series exponents!