# BV and BFV for the H-twisted Poisson sigma model

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## §1. Introduction

### To do

Construct the BV and BFV formalism of the H-twisted Poisson sigma model

### Purpose

Geometry and quantization of the twisted Poisson and the Dirac structure (and higher structures)

Generalization of the AKSZ sigma models

# Plan of Talk

(H-)twisted Poisson sigma model

BV Lagrangian formalism

Generalization of AKSZ formalism

(BFV Hamiltonian formalism)

# §2. (H-)twisted Poisson sigma model (HPSM)

**Twisted Poisson structure** Klimcik-Strobl, Park, Ševera-Weinstein

**Definition 1.** *Let $M$ be a smooth manifold. $\pi \in \Gamma(\wedge^2 TM)$ and $H \in \Omega^3(M)$ is a closed 3-form. $(M, \pi, H)$ is a twisted Poisson structure if*

$$\frac{1}{2}[\pi, \pi] = \langle \pi \otimes \pi \otimes \pi, H \rangle \,.$$

***Note*** : *It is a Dirac structure on $TM \oplus T^*M$.*

**Theorem 1.** *Let $(M, \pi, H)$ be a twisted Poisson structure. Then, a Lie algebroid is defined on $T^*M$.*

Define

$$\begin{aligned} \rho &= -\pi^\sharp, \\ [\alpha, \beta]_\pi &= L_{\pi^\sharp(\alpha)}\beta - L_{\pi^\sharp(\beta)}\alpha - d(\pi(\alpha, \beta)) + H(\pi^\sharp(\alpha), \pi^\sharp(\beta), -). \end{aligned}$$

Here $\pi^\sharp : T^*M \to TM$ and $\alpha, \beta \in \Omega^1(M)$. Then, $(\rho, [-,-]_\pi)$ is a Lie algebroid on $T^*M$,

**Definition 2.** ***A Lie algebroid*** $(E, [\cdot,\cdot], \rho)$ *is a vector bundle* $E \to M$ *together with a bundle morphism* $\rho \colon E \to TM$ *as well as a Lie algebra* $(\Gamma(E), [\cdot,\cdot])$*, satisfying the Leibniz rule* $[s, fs'] = f[s, s'] + \rho(s)f\, s'$ *for all* $s, s' \in \Gamma(E)$ *and* $f \in C^\infty(M)$.

## Twisted Poisson sigma model

Klimcik-Strobl,Park

$(M, \pi, H)$: twisted Poisson manifold.

$N$: 3D manifold with a 2D boundary $\Sigma = \partial N$.

Fields: $(X, A)$.

$X : N \to M$

$A = A_\mu \mathrm{d}\sigma^\mu$: 1-form on $\Sigma$ taking a value on $X^* T^* M$. $(\sigma^\mu) \equiv (\sigma^0, \sigma^1)$ are coordinates on $\Sigma$.

$(X, A)$ is regarded as a local coordinate of maps, $a : T\Sigma \to T^* M$.

The classical action functional is

$$S = \int_{\Sigma = \partial N} A_i \wedge \mathrm{d}X^i + \tfrac{1}{2} X^* \pi^{ij} A_i \wedge A_j + \int_N X^* H \,.$$

**Note** : If $H = 0$, it reduces to the Poisson sigma model (PSM).

Equations of motion are ($\pi^{ij}{}_{,k} \equiv \partial \pi^{ij} / \partial x^k$)

$$F^i := \mathrm{d}X^i + \pi^{ij} A_j = 0 \,,$$

$$G_i := \mathrm{d}A_i + \tfrac{1}{2} \pi^{jk}{}_{,i} A_j \wedge A_k + \tfrac{1}{2} H_{ijk} \mathrm{d}X^j \wedge \mathrm{d}X^k = 0 \,.$$

# Hamiltonian formalism

Let $\Sigma = \boldsymbol{R} \times S^1$ or $T^2$. Let $p_i = A_{1i}$, which is the spatial component of $A_i$. The symplectic form is

$$\omega = \oint_{S^1} \mathrm{d}\sigma \left( \delta X^i \wedge \delta p_i + \tfrac{1}{2} H_{ijk}(X)\, \partial X^i \delta X^j \wedge \delta X^k \right) ,$$

where $\sigma \equiv \sigma^1$ is the spatial coordinate and $\partial \equiv \partial / \partial \sigma$. This gives the following fundamental classical Poisson brackets

$$\begin{aligned} \{X^i(\sigma), X^j(\sigma')\} &= 0, \qquad \{X^i(\sigma), p_j(\sigma')\} = \delta^i{}_j \delta(\sigma - \sigma'), \\ \{p_i(\sigma), p_j(\sigma')\} &= -H_{ijk}(X) \partial X^k \delta(\sigma - \sigma'). \end{aligned}$$

The Hamiltonian is

$$\mathcal{H} = \oint_{S^1} \mathrm{d}\sigma A_{0i} J^i,$$

where $J^i \equiv F_1^i = \partial_1 X^i + \pi^{ij}(X) p_j$ is a constraint.

**Theorem 2.** *If $(\pi, H)$ is a twisted Poisson structure, $J^i$ consists of a closed Lie algebra,*

$$\{J^i(\sigma), J^j(\sigma')\} = -f_k^{ij}(X(\sigma))\, J^k(\sigma)\delta(\sigma - \sigma')\,,$$

*where $f_k^{ij}$ are the structure functions of the Lie algebroid induced* *from the twisted Poisson structure,*

$$f_k^{ij} \equiv \pi^{ij},_k + \pi^{il}\pi^{jm}H_{klm} \,.$$

The mechanics is consistent with the twisted Poisson structure on $M$. $J^i$ are called first class constraints.

## Gauge transformation

**Theorem 3.** *If $(\pi, H)$ is a twisted Poisson structure, the action functional $S$ is invariant under the following gauge transformation,*
*Kotov-Salnikov-Strobl*

$$
\begin{aligned}
\delta X^i &= -\pi^{ij}\epsilon_j, \\
\delta A_i &= \mathrm{d}\epsilon_i + \pi^{jk}{}_{,i}\, A_j\epsilon_k + \tfrac{1}{2}\pi^{jk}H_{ijl}(\mathrm{d}X^l - \pi^{lm}A_m)\epsilon_k. \\
&= \mathrm{d}\epsilon_i + f_i^{jk}A_j\epsilon_k + \tfrac{1}{2}\pi^{jk}H_{ijl}\, F^l\, \epsilon_k,
\end{aligned}
$$

*where* $F^i = \mathrm{d}X^i + \pi^{ij}A_j$ *and* $\epsilon_i \in C^\infty(N, X^*T^*M)$.

## §3. Geometry of BV formalism

### Gauge transformation

$$\begin{aligned} \delta X^i &= -\pi^{ij}\epsilon_j, \\ \delta A_i &= \mathrm{d}\epsilon_i + f_i^{jk} A_j \epsilon_k + \frac{1}{2}\pi^{jk} H_{ijl}\, F^l\, \epsilon_k. \end{aligned}$$

Meaning of the third term in $\delta A_i$ becomes clear by introducing a target space connection.

## Gauge transformation with connection

The the action $S$ of the HPSM is also invariant under the target space covariant gauge transformation with a connection,

$$\begin{aligned} \delta^\nabla X^i &= -\pi^{ij}\epsilon_j, \\ \delta^\nabla A_i &= \mathrm{d}\epsilon_i + f_i^{jk} A_j \epsilon_k - \Gamma_{ij}^k F^j \epsilon_k. \end{aligned}$$

Here the Christoffel symbol of an affine connection $\nabla$ on the target space $M$ is

$$\Gamma_{ij}^k = \overset{\circ}{\Gamma}{}_{ij}^k - \frac{1}{2}\pi^{km} H_{mij} \,,$$

with a torsion $\Theta = \langle \pi, H \rangle$ and $\overset{\circ}{\Gamma}{}_{ij}^k = \overset{\circ}{\Gamma}{}_{ji}^k$. Nonzero $H$ introduces the torsion.

## BRST transformation

We consider on-shell closed BRST transformations. Replace gauge parameters $\epsilon$ by odd and anti-commuting ghost fields $c \in C^\infty(\Sigma, X^*T^*[1]M)$.

$$\begin{aligned} sX^i &= -\pi^{ij} c_j, \\ sA_i &= \mathrm{d}c_i + f_i^{jk} A_j c_k - \Gamma_{il}^k F^l c_k. \end{aligned}$$

We put the BRST transformation of $c$,

$$sc_i := -\frac{1}{2}[c, c]_i = -\frac{1}{2} f_i^{jk} c_j c_k \, .$$

Then, $s^2$ are

$$s^2 X^i \;=\; s^2 c_i = 0\,.$$

However $s^2 A_i \neq 0$,

**Lemma 1. [I-Strobl]** $s^2 A_i = -\frac{1}{2} S_{ij}{}^{kl} F^j c_k c_l$,

where Mayer-Strobl, Abad-Crainic, Blaom, Kotov-Strobl

$$S = \nabla T + 2\mathrm{Alt}(\iota_\rho R_\nabla) \in \Gamma(\Lambda^2 TM \otimes S^2 T^* M).$$

is a *basic curvature*.

Here $R_\nabla \in \Omega^2(M, \mathrm{End}\, TM)$ is the curvature of the affine connection $\nabla$.

$\rho \in \Gamma(T^*M \otimes TM)$ is the anchor map, and $\mathrm{Alt}$ denotes an antisymmetrization over $T^*M \otimes T^*M$.

$T \in \Gamma(\Lambda^2 TM \otimes T^*M)$ is the $E$-*torsion* of the Lie algebroid covariant derivative,

$$T(e, e') = -\nabla_{\rho(e)} e' + \nabla_{\rho(e')} e + [e, e'].$$

$e, e' \in \Gamma(TM)$.

## Space of fields in BV

The space with classical fields and ghosts (fields in BRST) is

$$\mathcal{M}_{BRST} := \left\{ \left( X \colon \Sigma \to M,\ A \in \Omega^1(\Sigma, X^*T^*M),\ c \in C^\infty(\Sigma, X^*T^*[1]M) \right) \right\}.$$

The space of fields of the BV formulation is

$$\mathcal{M}_{BV} := T^*[-1]\mathcal{M}_{BRST}.$$

$\mathcal{M}_{BV}$ is equipped with a degree $-1$ BV symplectic form,

$$\omega_{BV} \;=\; \int_\Sigma \left( \delta X^i \wedge \delta X^+_i + \delta A_i \wedge \delta A^{+i} + \delta c_i \wedge \delta c^{+i} \right).$$

**Bigrading** fdeg is the form degree. gh is the ghost number.

$$\mathrm{fdeg}(\Phi^+) = 2 - \mathrm{fdeg}(\Phi), \qquad \mathrm{gh}(\Phi^+) = -1 - \mathrm{gh}(\Phi).$$

$$\begin{aligned}
\mathrm{fdeg}(X^i) &= 0\,, & \mathrm{gh}(X^i) &= 0,\\
\mathrm{fdeg}(A_i) &= 1\,, & \mathrm{gh}(A_i) &= 0,\\
\mathrm{fdeg}(c_i) &= 0\,, & \mathrm{gh}(c_i) &= 1,\\
\mathrm{fdeg}(X_i^+) &= 2\,, & \mathrm{gh}(X_i^+) &= -1,\\
\mathrm{fdeg}(A^{+i}) &= 1\,, & \mathrm{gh}(A^{+i}) &= -1,\\
\mathrm{fdeg}(c^{+i}) &= 2\,, & \mathrm{gh}(c^{+i}) &= -2.
\end{aligned}$$

## BV functional

We construct a BV action functional which satisfies the classical master equation,

$$(S_{BV}, S_{BV}) = 0\,,$$

where $(-,-)$ is a BV bracket defined from $\omega_{BV}$. The BV functional is determined by the addition of further contributions to the minimal BRST extension of the classical action:

$$S_{BV} = S_{BV}^{(0)} + S_{BV}^{(1)} + S_{BV}^{(2)} + \cdots\,,$$

where $(k)$ is the order of antifields $\Phi^+$.

$$
\begin{aligned}
S_{BV}^{(0)} &= S_{cl}, \\
S_{BV}^{(1)} &= \int_\Sigma (-1)^{\mathrm{gh}(\Phi)} \Phi^+ s\Phi = \int_\Sigma \left( X_i^+ \, sX^i + A^{+i} \, sA_i - c^{+i} \, sc_i \right),
\end{aligned}
$$

where $\Phi$ denotes all fundamental fields in $\mathcal{M}_{BRST}$. In general, the expansion does not terminate.

**Proposition 1. [I-Strobl]** *In the HPSM, it does at level two,*

$$
S_{BV} = S_{BV}^{(0)} + S_{BV}^{(1)} + S_{BV}^{(2)},
$$

*where*

$$S_{BV}^{(2)} = \int_\Sigma \tfrac{1}{4} S_{nk}{}^{ij}(X) A^{+n} A^{+k} c_i c_j \,.$$

**Proof.** $(S_{BV}, S_{BV}) = 0$ is proven using the expansion $S_{BV} = S_{BV}^{(0)} + S_{BV}^{(1)} + S_{BV}^{(2)}$ and the BV brackets of fields.

A nontrivial identity is $(S_{BV}^{(1)}, S_{BV}^{(2)}) = 0$, which comes from the Bianchi identity of the basic curvature:

$$\pi^{m[l} \nabla_m S_{nk}{}^{ij]} + T_m^{[jl} S_{nk}{}^{i]m} + T_n^{m[i} S_{mk}{}^{jl]} + T_k^{m[l} S_{nm}{}^{ij]} = 0.$$

□

The resulting BV action functional is

$$S_{BV} = \int_\Sigma \left(A_i \wedge \mathrm{d}X^i + \tfrac{1}{2}\pi^{ij} A_i \wedge A_j\right) + \int_N H$$
$$+ \int_\Sigma \left[-\pi^{ij} X_i^+ c_j + A^{+i} \wedge \left(\mathrm{d}c_i + f_i^{jk} A_j c_k - \Gamma_{ij}^k F^j c_k\right) + \tfrac{1}{2} f_k^{ij} c^{+k} c_i c_j\right]$$
$$+ \int_\Sigma \tfrac{1}{4} S_{nk}{}^{ij} A^{+n} \wedge A^{+k} c_i c_j.$$

# Manifestly target space covariant form

We define the covariantized antifield of $X^i$:
$X_i^{+\nabla} := X_i^{+} + \Gamma_{ji}^{k}(A^{+j} \wedge A_k + c^{+j} c_k)$.

The manifestly covariant BV action is

$$
\begin{aligned}
S_{BV}^{\nabla} &= \int_{\Sigma} \left[ \langle A, \mathrm{d}X \rangle + \tfrac{1}{2}(\pi \circ X)(A, A) \right] + \int_{N} X^{*} H \\
&+ \int_{\Sigma} \left[ \langle A^{+}, \mathrm{D}c - (T \circ X)(A, c) \rangle - (\pi \circ X)(X^{+\nabla}, c) \right. \\
&\left. - \tfrac{1}{2} \langle c^{+}, (T \circ X)(c, c) \rangle \right] + \int_{\Sigma} \tfrac{1}{4} \langle A^{+}, (S \circ X)(A^{+}, c, c) \rangle .
\end{aligned}
$$

## §4. Superfield formalism

We consider an **AKSZ**-like formulation on a graded manifold. Here we put $\overset{\circ}{\Gamma}{}^k_{ij} = 0$. (Even in the PSM, the AKSZ construction works in this case.) Setting $\Gamma^k_{ij} \mapsto -\frac{1}{2}\pi^{kl}H_{ijl}$,

$$S_{BV} = \int_\Sigma \left(A_i \wedge \mathrm{d}X^i + \frac{1}{2}\pi^{ij} A_i \wedge A_j\right) + \int_N H$$
$$+ \int_\Sigma \Big[-\pi^{ij} X^+_i c_j + A^{+i} \wedge \Big(\mathrm{d}c_i + f_i^{jk} A_j c_k + \frac{1}{2}\pi^{kl} H_{ijl} F^j c_k\Big)$$
$$+\frac{1}{2} f_k^{ij} c^{+k} c_i c_j - \frac{1}{4}\left(f_n^{ij},_k + \frac{1}{2}\pi^{ci}\pi^{ja}\pi^{bd} H_{nab} H_{kcd}\right) A^{+n} \wedge A^{+k} c_i c_j\Big].$$

# Mapping space of graded manifolds

All the fields in the BV phase space can be combined into elements of the space of (not necessarily degree-preserving) maps:

$$\mathcal{M}_{BV} \cong \underline{\mathrm{Hom}}(T[1]\Sigma, T^*[1]M).$$

Let $(\sigma^\mu, \theta^\mu)$ be coordinates on $T[1]\Sigma$ of degree $(0,1)$. Superfields are

$$\begin{aligned}
\boldsymbol{X}^i(\sigma,\theta) &\equiv X^i(\sigma) - \underline{A}^{+i}(\sigma,\theta) + \underline{c}^{+i}(\sigma,\theta) \\
&:= X^i(\sigma) - \theta^\mu A_\mu^{+i}(\sigma) + \tfrac{1}{2}\theta^\mu\theta^\nu c_{\mu\nu}^{+i}(\sigma), \\
\boldsymbol{A}_i(\sigma,\theta) &\equiv -\underline{c}_i(\sigma) + \underline{A}_i(\sigma,\theta) + \underline{X}_i^+(\sigma,\theta) \\
&:= -c_i(\sigma) + \theta^\mu A_{\mu i}(\sigma) + \tfrac{1}{2}\theta^\mu\theta^\nu X_{\mu\nu i}^+(\sigma).
\end{aligned}$$

$$\deg(\phi) := \mathrm{fdeg}(\phi) + \mathrm{gh}(\phi).$$

$\deg(\boldsymbol{X}^i) = 0$ and $\deg(\boldsymbol{A}_i) = 1$,

The BV symplectic form is combined into the natural symplectic form:

$$\omega \;=\; \int_{T[1]\Sigma} d^2\sigma d^2\theta \, \delta\boldsymbol{X}^i \delta\boldsymbol{A}_i.$$

**Note** : $\mathcal{M}_{BV}$ and $\omega$ are the same as the AKSZ construction of the PSM.

## AKSZ procedure does not work

In the case of the PSM ($H = 0$), the BV functional $S$ is simply obtained from the classical action, $S_{cl}$, by the replacements $X \mapsto \boldsymbol{X}$, $A \mapsto \boldsymbol{A}$, the derivatives to the superdrerivatives, and the integration on $\Sigma$ to $T[1]\Sigma$: $S_{cl} \mapsto S_{BV}$. Alexandrov-Kontsevich-Schwartz-Zaboronsky

This procedure does not work in the case $H \neq 0$. In fact,

$$S_{BV} = \int_{T[1]\Sigma} d^2\sigma d^2\theta \left[\boldsymbol{A}_i \, \mathrm{d}\boldsymbol{X}^i + \tfrac{1}{2}\pi^{ij}(\boldsymbol{X}) \, \boldsymbol{A}_i \boldsymbol{A}_j\right] + \int_{T[1]N} d^3\sigma d^3\theta \, H(\boldsymbol{X})$$

does not satisfy the classical master equation, $(S_{BV}, S_{BV}) \neq 0$.

Park, I-X.Xu

**Theorem 4. [I-Strobl]** *The BV action functional in terms of the superfields* $\boldsymbol{X}$ *and* $\boldsymbol{A}$ *is (* $\varepsilon$ *is the Euler vector field,* $\varepsilon = \theta^\mu \frac{\partial}{\partial \theta^\mu}$. *)*

$$
\begin{aligned}
S_{BV} \;=\; & \int_{T[1]\Sigma} d^2\sigma d^2\theta \left[\boldsymbol{A}_i \, \mathrm{d}\boldsymbol{X}^i + \tfrac{1}{2}\pi^{ij}(\boldsymbol{X}) \, \boldsymbol{A}_i \boldsymbol{A}_j\right] + \int_{T[1]N} d^3\sigma d^3\theta \, H(\boldsymbol{X}) \\
& + \int_{T[1]\Sigma} d^2\sigma d^2\theta \left[\tfrac{1}{4}(\pi^{il}\pi^{jm}H_{lmk})(\boldsymbol{X}) \, \boldsymbol{A}_i \boldsymbol{A}_j \boldsymbol{\varepsilon} \boldsymbol{X}^k \right. \\
& \qquad \left. + \tfrac{1}{2}(\pi^{il}H_{jkl})(\boldsymbol{X}) \, \boldsymbol{A}_i (\mathrm{d}\boldsymbol{X}^j) \boldsymbol{\varepsilon} \boldsymbol{X}^k \right] \\
& + \int_{T[1]\Sigma} d^2\sigma d^2\theta \left[\tfrac{1}{8}(\pi^{im}\pi^{jn}\pi^{pq}H_{mql}H_{npk})(\boldsymbol{X}) \, \boldsymbol{A}_i \boldsymbol{A}_j (\boldsymbol{\varepsilon}\boldsymbol{X}^k) \boldsymbol{\varepsilon}\boldsymbol{X}^l\right].
\end{aligned}
$$

**Proof.** The expansion of $S_{BV}$ coincides with the BV action functional $S_{BV}$ in Section 3. □

# Generalized AKSZ sigma model

Let $\Sigma$ be a $d$ dimensional manifold and $\mathcal{M}$ is a graded manifold. Assume the BV symplectic form $\omega$ of degree $-1$ on the mapping space,

$$\mathcal{M}_{BV} \cong \underline{\mathrm{Hom}}(T[1]\Sigma, \mathcal{M}).$$

The BV action functional $S_{BV}$, which is a homological function $(S_{BV}, S_{BV}) = 0$ is constructed by

$$S_{BV} = \sum_{k=0}^{d} S_{BV(k)},$$

where $S_{BV(k)}$ is the $k$-th order part of the Euler vector field $\varepsilon$.

Here $S_{BV(0)}$ is simply the replacement of fields to superfields in the classical action $S_{cl}$.

1. $S_{BV(k)} = 0$ for $k \geq 1$ is the normal AKSZ formulation,

2. In the HPSM, $S_{BV(k)} = 0$ for $k \geq 3$.

# §5. BFV formalism

The BFV formalism is a pair of odd and even functions $(S_{BFV}, H_{BFV})$ with a BFV symplectic form $\omega_{BFV}$ such that

$$\{S_{BFV}, S_{BFV}\} = \{S_{BFV}, H_{BFV}\} = \{H_{BFV}, H_{BFV}\} = 0,$$

where $\{-,-\}$ is a Poisson bracket induced from $\omega_{BFV}$.

$H_{BFV}$ is an extension of the Hamiltonian.

$S_{BFV}$ is an extension of the 'charge' of the symmetry.

# BFV formulation of twisted Poisson sigma models

We introduce two odd fields $c_i(\sigma) \in C^\infty(S^1, X^*T^*[1]M)$ and $b^i(\sigma) \in C^\infty(S^1, X^*T[-1]M)$, such that the fundamental Poisson brackets are $\{c_i(\sigma), b^j(\sigma')\} = \delta_i{}^j \delta(\sigma - \sigma')$.

The BFV symplectic form is

$$\omega_{BFV} = \oint_{S^1} \mathrm{d}\sigma \left(\delta X^i \wedge \delta p_i + \delta c_i \wedge \delta b^i + \tfrac{1}{2} H_{ijk}(X) \partial X^i \delta X^j \wedge \delta X^k\right).$$

$H_{BFV} \approx 0$ in the HPSM.

## $S_{BFV}$

$$
\begin{aligned}
S_{BFV} &= \oint_{S^1} \mathrm{d}\sigma \left( c_i\, J^i + \tfrac{1}{2} f_k^{ij}\, c_i c_j b^k \right) \\
&= \oint_{S^1} \mathrm{d}\sigma \left[ c_i(\partial X^i + \pi^{ij} p_j) + \tfrac{1}{2}(\pi^{ij}{}_{,k} + \pi^{il}\pi^{jm} H_{klm}) b^k c_i c_j \right].
\end{aligned}
$$

From the Poisson bracket

$$
\{J^i(\sigma), J^j(\sigma')\} = -f_k^{ij}(X(\sigma))\, J^k(\sigma)\delta(\sigma - \sigma'),
$$

we obtain $\{S_{BFV}, S_{BFV}\} = 0$.

## Superfield formalism

We reformulate $S_{BFV}$ using superfields.

$S^1$ is extended to a super-circle $T[1]S^1$ parametrized by the coordinates $(\sigma, \theta)$ of degree zero and one, respectively. We consider the super BFV phase space,

$$\mathcal{M}_{BFV} = \underline{\mathrm{Hom}}(T[1]S^1, T^*[1]M)\,.$$

Coordinates are the following superfields of degree $0$ and $1$, respectively:

$$\widetilde{X}^i(\sigma, \theta) := X^i(\sigma) + \theta\, b^i(\sigma), \qquad \widetilde{A}_i(\sigma, \theta) := -c_i(\sigma) + \theta\, p_i(\sigma).$$

Now the BFV symplectic form and the BFV-BRST charge become

$$\omega_{BFV} = \int_{T[1]S^1} \mathrm{d}\sigma \mathrm{d}\theta \left( \delta \widetilde{X}^i \wedge \delta \widetilde{A}_i + \frac{1}{2} H_{ijk}(\widetilde{X}) \tilde{\mathrm{d}} \widetilde{X}^i \delta \widetilde{X}^j \wedge \delta \widetilde{X}^k \right),$$

$$\begin{aligned} S_{BFV} &= \int_{T[1]S^1} \mathrm{d}\sigma \mathrm{d}\theta \left( \widetilde{A}_i \tilde{\mathrm{d}} \widetilde{X}^i + \frac{1}{2} \pi^{ij}(\widetilde{X}) \widetilde{A}_i \widetilde{A}_j \right. \\ &\quad \left. + \frac{1}{2} \pi^{il} \pi^{jm} H_{klm}(\widetilde{X}) \, \widetilde{A}_i \widetilde{A}_j \, \tilde{\varepsilon} \widetilde{X}^k \right), \end{aligned}$$

where $\tilde{\mathrm{d}} = \theta \partial$ is a super-derivative on $T[1]S^1$, and $\tilde{\varepsilon}$ denotes the Euler vector field, $\tilde{\varepsilon} = \theta \frac{\partial}{\partial \theta}$.

## §9. Conclusions

- We have constructed the BV formalism of the twisted Poisson sigma model (HPSM).

- We analyzed geometric structures of the BFV and BV formalisms by introducing a target space connection. They are described by geometry of a Lie algebroid.

- The superfield formulation is a generalization of the AKSZ sigma models.

- We constructed the BFV formalism of the twisted PSM.

## Further work

[I-Strobl arXiv:2007.15912] We applied the formula of Grigoriev-Damgaard to construct the BV action from the Hamiltonian-BFV formalism to the HPSM. A generalization of GD formalism is needed.

## Outlook

• Quantization (a generalization of the deformation quantization)

Kontsevich, Cattaneo-Felder

• A generalization to the Dirac structure or higher structures. (a Dirac sigma model, Kotov-Schaller-Strobl etc.)

• Generalized AKSZ sigma models?

**Thank you for your attention!**