Remarks on A. Chandra's talk

Path integrals and SPDEs

Aim: Understand (rigorously) Euclidean Path integrals

$$\int F(\phi)\, e^{-S(\phi)}\, d\phi = \mathbb{E}_\mu(F(\phi))$$

μ: measure on space of distributions

Langevin dynamics (auxiliary time t)

$$d\phi = -A\nabla S(\phi)\, dt + \sqrt{A}\,\sqrt{2}\, dW_t$$

Motivation:
- Monte Carlo Simulations
- New information on μ through dynamics
- Similar to "Glauber/Kawasaki" dynamics in spin models.

Examples: ϕ^4 - theory $\quad S(\phi) = \int \frac{1}{2}|\nabla\phi|^2 + \frac{\phi^4}{4}\,dx$

$$\rightsquigarrow \quad d\phi = (\Delta\phi - \phi^3)\,dt + dW$$

$\uparrow$ "$\nabla S(\phi)$" $\qquad$ $\uparrow$ S-T whitenoise

e.g. Hairer '14.

- Yang-Mills : $S(A) = \int |F_A|^2\,dx$

vector valued one forms $\downarrow$ $\qquad$ $\uparrow$ curvature

$$dA_i = \left(\Delta A_i + \sum [A_j, 2\partial_j A_i - \partial_i A_j + [A_j, A_i]]\right)dt + dW^i$$

Chandra, Chevyrev, Hairer, Shen '20. $\quad d=2$

$d=3$ forthcoming

Regularity structures (Hairer '14)

Scope: Local-in-time solution theory for Super-renormalisable/ subcritical theories

(also non-reversible dynamics; e.g. KPZ).

Ansatz:
- Perturbative analysis localised in space. To finite order!
- Use pathwise PDE / harmonic analysis arguments (regularity estimates, reconstruction theorem) to bound remainder.

Challenge:
- Describe renormalisation for "localised" expansions
  e.g. Bruned, Hairer Zambotti '19, Linares, Otto, Tempelmayr '21.
- Perturbative calculations, Chandra, Hairer '16

Example: gPAM

$$(\partial_t - \Delta)\, u = \sigma(u)\,\xi$$

cf. "freezing of coefficients"

$$\approx \sigma(u(z_0)) + \sigma'(u(z_0))(u(\cdot) - u(z_0)) + \dots$$

Suggests that to leading order $\langle \Pi_{z_0}, P \rangle$

$$u(z) - u(z_0) = \sigma(u(z_0))\big(Z(z) - Z(z_0)\big)$$

$$(\partial_t - \Delta) Z = \xi$$

$$\approx \sigma'(u(z_0))\,\sigma(u(z_0))\big(Z(\,) - Z(z_0)\big)\,\xi\,.$$

Leads to local description of u near z_0

$$u \approx \sum_{\tau} \curlyvee[\tau](z_0) \prod_{z_0} \tau$$

closely related to elementary differentials in Butcher series

τ: decorated trees

The specific talk: Discusses tool to lift "scalar valued" trees to "vector valued" trees. Used in YM.

Questions:

- What is the role of "finite dimensionality" condition (c.f. quasilinear equations).
- Applications beyond YM.
- What's next? Links to QFT community?

- Some smallness required in theory (just like in RG ... time small enough),
- There are however eqn.... coupled Felix, Tsatsoulis, Ignal micromagnetism
- Specific talk ... technical tool for Yang-Mills

(connect conjectures BDH2...