Discussion on the talk: "Revisiting the Okubo–Marshak Argument" by José Gracia-Bondía

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Higher Structures Emerging from Renormalisation
Vienna, November 16, 2021

Quick summary

- A new approach to construct QFT models describing higher spin particles.
- String localized fields. Principle of string independence.
- Application to Standard Model (QCD and Electroweak sector).
- No CP violating terms in QCD: solution of the strong CP problem.

Relativistic perturbative QFT

Relativistic QFT $\equiv$ QFT in Minkowski space

- Hilbert space $\mathcal{H}$,
- Poincaré covariance,
- pointlike localization $[\hat{\phi}(x), \hat{\phi}(y)] = 0$ if x and y spatially separated,
- ...

Perturbative approach

Basic object $\equiv$ $\mathbb{S}$-matrix (formal power series in the coupling constant g):

$$\mathbb{S} = \mathrm{Texp}\left(\mathrm{i}\, g \int \mathcal{L}_{\mathrm{int}}(x)\, \mathrm{d}x\right)$$
$$= 1 + \mathrm{i}g \int \mathcal{L}_{\mathrm{int}}(x)\, \mathrm{d}x - \frac{g^2}{2} \int \mathrm{T}(\mathcal{L}_{\mathrm{int}}(x_1), \mathcal{L}_{\mathrm{int}}(x_2))\, \mathrm{d}x_1 \mathrm{d}x_2 + \ldots .$$

Spin $\geqslant 1$ particles problematic!

Top-down approach (*gauge principle*)

Key steps:

1. classical action functional (renormalizability, gauge invariance),
2. quantization procedure (BRST technique, unphysical fields, auxiliary space),
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Example: QCD, gauge group $SU(3)$.

Classical action functional: $S[A_\mu] := \int \mathrm{tr}(F_{\mu\nu}(x)F^{\mu\nu}(x))\,\mathrm{d}x$
(extrema $\equiv$ classical solutions of equations of motion).

- $A_\mu(x)$ – gauge potential valued in $\mathfrak{su}(3)$ (Lie algebra).
- $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + g[A_\mu(x), A_\nu(x)]$ – interacting field tensor,

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- Gauge invariance $\Rightarrow S[A_\mu] = S[A'_\mu]$, where
  $A'_\mu(x) = U^{-1}(x) A_\mu(x) U(x) + U(x)^{-1} \partial_\mu U(x)$ and $U : \mathbb{R}^4 \to SU(3)$.

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Quantization (problematic because of gauge symmetry):

- $S_{\mathrm{mod}}[A_\mu, B_\mu, C, \bar{C}] = S[A_\mu] + S_{\mathrm{gf}}[A_\mu, B_\mu] + S_{\mathrm{gh}}[C, \bar{C}, A_\mu]$.
- Observables: $[\mathcal{O}, Q_{\mathrm{BRST}}] = 0$, physical Hilbert space: $\frac{\mathrm{ker} Q_{\mathrm{BRST}}}{\mathrm{Ran} Q_{\mathrm{BRST}}}$.

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Massless free field describing spin 1 particle:

- $F_{\mu\nu}(x)$ - anti-symmetric tensor satisfying the Maxwell equation.
- Bad UV properties.
- $A_\mu(x,e) = \int_0^\infty F_{\mu\nu}(x+\tau e)e^\nu \, \mathrm{d}\tau$, e – spatial four-vector (direction of string).
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Interaction terms: polynomials in $A_{a\mu}(x,e)$, $a \in \{1,\ldots,8\}$.

- $\mathrm{d}_e\mathcal{L}_{\mathrm{int}}(x,e) =$(total derivative) $\Rightarrow \mathcal{L}_{\mathrm{int}}(x,e)$ at first order in g coincides with $\sum_{a,b,c} f_{abc} A_{a\mu} A_{b\nu} \partial^\mu A_c^\nu$ and f_{abc} completely anti-symmetric,

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- $\mathrm{d}_{e_1} \mathrm{T}(\mathcal{L}_{\mathrm{int}}(x_1,e_1), \mathcal{L}_{\mathrm{int}}(x_2,e_2)) =$(total derivative) $\Rightarrow f_{abc}$ satisfies the Jacobi identity [$\Rightarrow f_{abc}$ structure constant of compact Lie algebra] and $\mathcal{L}_{\mathrm{int}}(x,e)$ contains second order term.

Comparison

Top-down approach (standard, gauge principle):

- unphysical degrees of freedom (obscure physical interpretation),
- very general, proof of perturbative renormalizability to all orders.

Bottom-up approach (new, string independence):

- only physical degrees of freedom,
- simplification of computations at tree level,
- no complete proof of renormalizability to all orders.

Questions

- Can one derive all couplings of the SM using the bottom-up approach?
- New physics: spin 2 particles, quantum gravity?