Abstracts

Behrend, **Roger** - *Multiply-refined enumeration of alternating sign matrices*

I will review a range of results for the exact enumeration of alternating sign matrices of fixed size with prescribed values of some or all of the following statistics: the numbers of generalized inversions and -1's, and the positions of the 1's in the first and last rows and columns. Many of these results can be obtained by using connections with the statistical mechanical six-vertex model with domain-wall boundary conditions.

Bousquet-Mélou, **Mireille** - *The Potts model on planar maps*

Bouttier, **Jérémie** - *Distances in random planar maps and discrete integrability*

Metric properties of random maps (graphs embedded in surfaces) have been subject to a lot of recent interest. In this talk, I will review a combinatorial approach to these questions, which exploits bijections between maps and some labeled trees. Thanks to an unexpected phenomenon of "discrete integrability", it is possible to enumerate exactly maps with two or three points at prescribed distances, and more. I will then discuss probabilistic applications to the study of the Brownian map (obtained as the scaling limit of random planar maps) and of uniform infinite planar maps (obtained as local limits). If time allows, I will also explain the combinatorial origin of discrete integrability, related to the continued fraction expansion of the so-called resolvent of the one-matrix model. Based on joint works with E. Guitter and P. Di Francesco.

David, **François** - *Planar maps, circle patterns, conformal point processes and 2D gravity*

A model of random planar triangulations is presented. It exemplifies the relations between discrete geometries in the plane (circle packings and circle patterns), conformally invariant point processes and two dimensional quantum gravity (topological gravity and Liouville theory).

de Tilière, **Béatrice** - *From the critical Ising model to spanning trees*

We will show an explicit relation between the square of the critical Ising model partition function and the partition function of critical spanning trees, thus proving a relation on the level of configurations between two classical models of statistical mechanics at criticality.

Duplantier, **Bertrand** - *Liouville quantum gravity, KPZ and Schramm-Loewner evolution*

Ellis-Monaghan, **Joanna** - *Graph theoretical expressions for statistical mechanics models*

The interactions between graph theory and statistical mechanics have been exceptionally fruitful, beginning with the fundamental identification of the Tutte polynomial of graph theory and the Potts model of statistical mechanics. We continue this byplay with new ramifications of the 2011 V-polynomial which extended the classical Tutte-Potts connection to settings with external fields, considering in particular zero-temperature antiferromagnetic models and n-vertex models.

Fischer, **Ilse** - *Extreme diagonally and antidiagonally symmetric alternating sign matrices*

DADASMs is the only symmetry class of alternating sign matrics for which David Robbins provided a conjectural product formula that has not been proven so far. We study DADASMs with a certain extreme behavior along the diagonal and antidiagonal. In particular, we will present operator formulas, constant term identities and linear equation systems for related enumerative quantities.

Guttmann, **Tony** - *Unusual critical behaviour of some lattice and combinatorial models*

Many lattice models, such as self-avoiding walks, self-avoiding polygons, self-avoiding bridges, and simple combinatorial models like Dyck paths, have generating functions $A(x) = \sum a_n x^n$, (where the coefficient a_n gives the number of such objects of size n) with algebraic singularities. The coefficients thus behave as $a_n \sim B\mu^n n^g$. This remains true when such models are considered in a half-space, with their origin anchored in the surface. In this geometry, each such object achieves a maximum height h. If we associate with each such object a fugacity y conjugate to the height h of the object, this can model the behaviour of such objects subject to a stretching force ($y > 1$), no force ($y = 1$) or a compressive force ($y < 1$).

We find that, for all the models considered, the critical behaviour is different in the three regimes. When there is no force ($y = 1$), the usual algebraic singularity is observed. For $y > 1$, – the stretched regime – we find the generating function has a simple pole, but the radius of convergence $1/\mu(y)$ decreases as y increases.

For the compressive case however, we find, in all cases, that $a_n \sim C\mu^n \mu_1^{n^\sigma} n^g$, where $\mu(y) = \mu(1)$, and typically $\sigma = 1/2$ or $1/3$. In the case of Dyck paths this result has been proved by McKay (here $\sigma = 1/3$). In the other cases mentioned, we obtain estimates of μ, μ_1, σ and g by careful numerical work. We also find similar behaviour (with $\sigma = 1/2$) for certain pattern-avoiding permutations. We discuss these results, and the numerical methods whereby they are obtained.

Kazakov, **Vladimir** - *Summing planar graphs in zero, one, two, three and four dimensions*

Moffatt, **Iain** - *Ribbon graphs, their polynomials, and delta-matroids*

A matroid is a mathematical structure that generalises the notion of linear independence in vector spaces. There is a natural way to associate a matroid with a graph that results in a very close connections between graph theory and matroid theory. In fact, many results and objects in graph theory are properly understood in terms of matroids (for example, the Tutte polynomial of a graph is actually a matroid polynomial).

Here we are interested in ribbon graphs. The (graphic) matroid associated with a ribbon graph does not record any of its topological information. Thus matroids do not appear to provide a 'correct' generalisation of ribbon graphs. This leads us to ask if matroids don't, what do? In this talk I will propose an answer to this question. I will describe how delta-matroids arise as the natural extension of matroids to the setting of ribbon graphs, showing that there is a fundamental compatibility between ribbon graph and delta-matroid theory, just as there is for graph and matroid theory. In particular, I will show that several ribbon graph polynomial, such as the Bollobas-Riordan polynomial, are delta-matroidal, just as the Tutte polynomial is matroidal.

This is joint work with Carolyn Chun, Steve Noble, and Ralf Rueckriemen.

Procacci, **Aldo** - *Witness trees in the Moser-Tardos algorithmic Lovász Local Lemma and Penrose trees in the hard-core lattice gas*

We point out a close connection between the Moser-Tardos algorithmic version of the Lovász Local Lemma, a central tool in probabilistic combinatorics, and the cluster expansion of the hard-core lattice gas in statistical mechanics. We show that the notion of witness trees given by Moser and Tardos is essentially coincident with that of Penrose trees in the Cluster expansion scheme of the hard-core gas. Such an identification implies that the Moser-Tardos algorithm is successful in a polynomial time if the Cluster expansion converges.

Romik, **Dan** - *Algebraic and bijective combinatorics in the $O(1)$ loop model: constant term identities and commutation of the transfer matrices*

The $O(1)$ loop model is a random configuration of planar loops, that gives rise to a random noncrossing matching, known as the link pattern or connectivity pattern of the configuration. This random combinatorial object has fascinating properties tying it to the study of alternating sign matrices via results such as the Razumov-Stroganov-Cantini-Sportiello theorem from 2010, and the less detailed "sum rule" proved a few years earlier by Di Francesco and Zinn-Justin.

In this talk I will describe two recent results on the $O(1)$ loop model. The first result concerns an explicit formula representing the probabilities of a general class of "connectivity events" as constant terms in certain multivariate Laurent polynomials. This is especially interesting since empirically the probabilities in question seem to be rational functions of a size parameter N; the desire to prove this empirical observation therefore reduces to proving a conjectural family of constant term identities, which generalize a beautiful identity proved in 2007 by Di Francesco, Zinn-Justin and Zeilberger.

The second result (a joint work by Ron Peled and myself) is a direct combinatorial proof of the commutation of transfer matrices in the $O(1)$ loop model; this important property of the model was previously known to follow using mysterious (to me, at least) algebraic reasoning based on the Yang-Baxter equation.

Scott, **Jeanne** - *Laurent expansions for twisted Plücker coordinates via dimer partition functions*

I'll discuss the Grassmannian's cluster algebra struture and its BFZ 'twist' automorphism. Following that I'll explain how to compute Laurent expansions for twisted Plücker coordinates (predicted by cluster theory) using dimer partition functions for bipartite graphs associated with a special class of planar networks called Postnikov diagrams.

Steinacker, **Harold** - *From matrix models to the theory of fundamental interactions*

We explain why certain multi-matrix models of "Yang-Mills type" - notably the so-called IKKT or IIB model - are considered as candidates for a theory of fundamental interactions, capturing non-perturbative aspects of string theory. A large class of solutions of these models can be understood in a semi-classical picture, and interpreted as D-branes. These branes carry (nonabelian) gauge theory, and might play the role of physical space-time. Essential steps and ingredients of this picture are discussed, and some non-trivial geometries and solutions corresponding to intersecting and/or linked manifolds are obtained.

Stoltzfus, **Neal** - *Ribbon graph constructions, generating functions and topological rank polynomial applications*

Many geometric constructions in the theory of classical links can be mirrored in combinatorial setting of the associated Kauffman state ribbon graphs (e.g. all-A or Seifert). In particular, connected sum, mutations, closed braid powers can be treated by these techniques. Analogous constructions will be described for the associated state graphs, together with associated formulae for the rank polynomial of (Whitney, Tutte, Bollobas & Riordan) and the associated link polynomial invariants.

Viennot, **Xavier** - *Heaps of pieces and 2D Lorentzian quantum gravity*

Lorentzian quantum gravity is a discrete approach to $2D$ quantum gravity where triangulations of the space-time admit a causal structure. The model can be solved exactly performing the path integral over these causal dynamical triangulations (Ambjørn, Loll, ...). Further studies have been done introducing a higher curvature weight and correspondence with directed random walks (Di Francesco, Guitter, Kristjansen). This correspondence is strongly related to the theory of heaps of pieces introduced by the author in 1985 as a geometric interpretation of the so-called commutation monoids of Cartier and Foata.

In a first part, I will recall some fundamental lemma about this theory and survey some works which have been done relating Lorentzian triangulations and heaps of pieces (in fact heaps of dimers). The power of the connection with heaps of pieces is shown by giving two possible extensions: a $(2+1)D$ model (Benedetti, Loll, Zamponi), and by removing the usual border condition making the model solvable (Viennot with the introduction of the "nordic decomposition" of a heap of dimers).

The topics is strongly related to the directed animals model and hard gas model in statistical mechanics (Bousquet-Mélou, Rechnitzer, Bacher). I will propose some new results relating the curvature parameter with heaps of segments and further possible researches for a pure combinatorial explanation of the appearance of Bessel functions in the continuum limit.