(Higher-spin) gravity as a quantum effect on quantum space-time

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(higher-spin) gravity emerges on suitable background within

IKKT = IIB Matrix Model Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S = Tr([Y^a, Y^b][Y_a, Y_b] + \bar{\Psi}\Gamma_a[Y^a, \Psi])$$

closely related to string theory, maximal SUSY
(crucial for quantization)

- leads to $\mathfrak{hs}$ gauge theory on suitable quantum space-time,
  class. action differs from GR
- quantum effects (1-loop)
  $\rightarrow$ induced Einstein-Hilbert term, gravity * new! *
- weak coupling, no holography, no target space compactification

summary & outline:

- Yang-Mills matrix models & emergent geometry
- covariant quantum spaces: fuzzy H_n^4
  higher-spin modes — M. Sperling, HS 1806.05907
- covar. space-time solution $\mathcal{M}_n^{3,1}$ — M. Sperling, HS 1901.03522
  linearized fluctuations, no ghosts — HS 1910.00839
- nonlinear regime:
  - volume-preserving diffeos
  - eom for frame, spherically symmetric solutions
    HS 2002.02742, S. Fredenhagen HS 2101.07297, Y. Asano, HS 21xx.xxxx
- **quantization**: * new, unpublished *
  1-loop effective action
  $\rightarrow$ Einstein-Hilbert action (+ extras)
  no cosm. const. problem

introductory review: arXiv:1911.03162

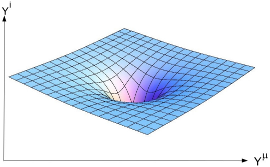
geometric interpretation of Yang-Mills matrix models:

$$S = Tr([Y^a, Y^b][Y_a, Y_b] \; + ...)$$

solution Y^a (matrix configuration) interpreted as

$$Y^a \sim y^a : \quad \mathcal{M} \to \mathbb{R}^D$$

$\mathcal{M}$ ... symplectic manifold ("brane")



Y^a generates algebra of functions $End(\mathcal{H}) \sim \mathcal{C}(\mathcal{M})$

semi-classical regime:

$$[\Phi, \Psi] \sim i\{\phi, \psi\}$$

$$S \sim \int_{\mathcal{M}} \rho_M(-\{y^a, y^b\}\{y_a, y_b\} \; + ...), \qquad \rho_M = \sqrt{|\theta^{-1}_{\mu\nu}|}$$

$\sim$ higher-dim. Poisson-sigma model !

The effective metric in M.M.

general rule: write kinetic term in semi-class. form, extract eff. metric

eff. action for fluctuations $Y^a + \mathcal{A}^a$ around background (solution) Y^a:

$$S[Y+\mathcal{A}] = S[Y] - Tr\Big([Y^b,\mathcal{A}^a][Y^b,\mathcal{A}^a] + (\text{gauge}-\text{fix etc.})\Big)$$

simplest: transversal fluctuations = scalar fields $\phi \in End(\mathcal{H})$

$$\begin{aligned} S[\phi] &= -Tr\eta_{ab}[Y^a,\phi][Y^b,\phi] \\ &\sim \int \rho_M \eta_{ab} E^{a\mu}\partial_\mu\phi E^{b\nu}\partial_\nu\phi \sim \int \sqrt{|G|}\, G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \end{aligned}$$

semi-classical:

$$-i[Y^a,\phi] \sim \{Y^a,\phi\} = E^{a\mu}\partial_\mu\phi$$

eff. vielbein (frame)

$$E^a := \{Y^a, .\}, \qquad E^{a\mu} := \{Y^a, x^\mu\}$$

governs all fluctuations in M.M.

eff. metric

$$G^{\mu\nu} = \rho^{-2}\eta_{ab}E^{a\mu}E^{b\nu} = \rho^{-2}\gamma^{\mu\nu}$$

$$\rho^2 = \rho_M\sqrt{|\gamma^{\mu\nu}|} \qquad \text{....dilaton}$$

governs all fluctuations in M.M.
universal metric $\Rightarrow$ gravity !?

HS 1003.4134 ff

($D \neq 2$)

can show: $\Box = -\{Y^a, \{Y_a, .\}\} = \rho^2\Box_G$ ... Matrix Laplacian

issues:

- frame $E^a = \{Y^a, .\}$ ... not enough dof on 4D $\mathcal{M}$ (... ?)
- $\theta^{\mu\nu}$ explicitly breaks Lorentz invariance
- class. M.M.: no Einstein equations ("pre-gravity")
  but: quantum effects $\rightarrow$ induced gravity (later, cf. Sakharov)

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4D covariant quantum spaces & $\mathfrak{hs}$

key step:

consider bundle of $\{\theta^{\mu\nu}\}$ over space(time) $\mathcal{M}$

- no Lorentz-violating VEV $\langle\theta^{\mu\nu}\rangle$!
- price to pay: higher-spin theory
- vol.-preserving diffeos on $\mathcal{M} \subset$ higher-dim symplectomorphisms

main examples:

- prototype: fuzzy S^4_N

  Grosse-Klimcik-Presnajder; Castelino-Lee-Taylor; Medina-o'Connor; Ramgoolam; Kimura; Abe; Karabail-Nair; Zhang-Hu; HS
- noncompact H^4_n Hasebe 1207.1968 , M. Sperling, HS 1806.05907
- projection $\rightarrow$ cosmological space-time $\mathcal{M}^{3,1}_n$

  HS, 1710.11495, M. Sperling, HS 1901.03522, ff.

Euclidean fuzzy hyperboloid H_n^4 $(=EAdS_n^4)$

$\mathcal{M}^{ab}$ ... hermitian generators of $\mathfrak{so}(4,2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}) \ .$$

$$\eta^{ab} = \mathrm{diag}(-1,1,1,1,1,-1)$$

choose "short" discrete unitary irreps $\mathcal{H}_n$ ("minireps", doubletons)

special properties:

- irreps under $\mathfrak{so}(4,1)$, multiplicities one
- positive discrete spectrum

$$\mathrm{spec}(\mathcal{M}^{05}) = \{E_0, E_0+1, \ldots\}, \qquad E_0 = 1 + \frac{n}{2}$$

lowest eigenspace is $n+1$-dim.

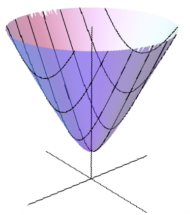
fuzzy hyperboloid H_n^4

5 hermitian generators

$$X^a := r\mathcal{M}^{a5}, \qquad a = 0, ..., 4$$

satisfy

$$\eta_{ab}X^aX^b = X^iX^i - X^0X^0 = -R^2\mathbf{1}, \qquad R^2 = r^2(n^2-4)$$

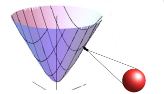


hyperboloid $H^4 \subset \mathbb{R}^{1,4}$, covariant under $SO(4,1)$

noncommutative structure $[X^a, X^b] = ir^2\mathcal{M}^{ab} =: i\Theta^{ab}$

claim:

$H_n^4 \;=\;$ quantized $\mathbb{C}P^{1,2} = S^2$ -bundle $\{\theta^{\mu\nu}$ selfdual$\}$ over H^4



best seen from oscillator construction:
4 bosonic oscillators $[\psi_\alpha, \bar\psi^\beta] = \delta_\alpha^\beta$

$$\mathcal{M}^{ab} = \bar\psi\Sigma^{ab}\psi, \qquad \gamma_0 = diag(1,1,-1,-1)$$
$$X^a = r\bar\psi\gamma^a\psi \qquad \text{cf. Hopf map}$$

$End(\mathcal{H}_n) \cong$ functions on $\mathbb{C}P^{1,2} \cong$ functions on $H^4 \otimes$ harmonics on S_n^2

local stabilizer acts on S^2 $\Rightarrow$ harmonics =higher spin modes

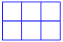
fuzzy "functions" on H_n^4:

$$End(\mathcal{H}_n) \cong \bigoplus_{s=0}^{n} \mathcal{C}^s$$

$\mathcal{C}^0 \ni \quad \phi(X)$

$\mathcal{C}^1 \ni \quad \phi_{ab}(X)\theta^{ab} \quad =$ 

$\vdots$

$End(\mathcal{H}_n) \cong$ fields on H^4 taking values in $\mathfrak{hs} = \oplus$  $\ni \theta^{a_1 b_1} \ldots \theta^{a_s b_s}$

cf. Vasiliev

higher spin modes = would-be KK modes on S^2

matrix model defines higher spin gauge theory, truncated at n

M. Sperling, HS 1806.05907

Relation with tensor fields:

one-to-one map with symm. div.-free traceless tensor fields:

$$
\begin{aligned}
\Gamma^{(s)}H^4 &\to \mathcal{C}^s \\
\phi_{a_1\ldots a_s}(x) &\mapsto \{x^{a_1},\ldots\{x^{a_s},\phi_{a_1\ldots a_s}\}\ldots\} \\
&= \theta^{a_1b_1}\ldots\theta^{a_sb_s}\eth_{b_1}\ldots\eth_{b_s}\phi_{a_1\ldots a_s} \\
&=: \mathcal{R}_{a_1\ldots a_s;b_1\ldots b_s}(x)\,\theta^{a_1b_1}\ldots\theta^{a_sb_s} =: \phi^{(s)}\ .
\end{aligned}
$$

conversely:

$$
\begin{aligned}
\mathcal{C}^s &\to \Gamma^{(s)}H^4 \\
\phi^{(s)} &\mapsto \{x^{a_1},\ldots\{x^{a_s},\phi^{(s)}\}\ldots\}_0
\end{aligned}
$$

$\mathcal{M}^{3,1}$ FLRW quantum space-time

IKKT model with mass term:

$$S[Y] = Tr([Y^\mu, Y^\nu][Y_\mu, Y_\nu] + \frac{6}{R^2} Y^\mu Y_\mu)$$

solution $\mathcal{M}_n^{3,1}$

$$\bar{Y}^\mu := \frac{1}{R}\mathcal{M}^{\mu 4}, \qquad \Box \bar{Y}^\mu = \frac{3}{R^2}\bar{Y}^\mu, \quad \mu = 0, ..., 3$$

generates algebra of "functions"

$$\mathrm{End}(\mathcal{H}_n) \cong \mathcal{C}(H^4 \times S^2)_n \quad \ldots \quad \text{fuzzy } H_n^4$$

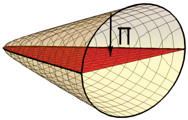
same underlying bundle, different metric $\Box = [\bar{Y}^\mu, [\bar{Y}_\mu, .]] \cong \rho^2 \Box_G$

$\rightarrow$ Lorentzian effective metric:

$ds_G^2 = -dt^2 + a(t)^2 d\Sigma^2$ ... FLRW space-time $\mathcal{M}^{3,1}$

relation with H_n^4:

$\mathcal{M}_n^{3,1}$ = projection of $H_n^4 \subset \mathbb{R}^{1,4} \xrightarrow{\Pi} \mathbb{R}^{1,3}$.



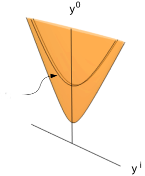
algebraically: $\mathcal{M}_n^{3,1}$ generated by

$$X^\mu := \mathcal{M}^{\mu 5}, \quad \text{for } \mu = 0,1,2,3 \qquad \text{(drop } X^4 \text{)}$$

metric acquires Minkowski signature!

- $\Box = [\bar{Y}^\mu, [\bar{Y}_\mu, .]]$ encodes FLRW metric $ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2$
- manifest $SO(3,1)$ $\Rightarrow$ foliation into space-like 3-hyperboloids H_τ^3
- Big Bounce $a(t) \sim (t - t_0)^{1/5}$

  HS arXiv:1710.11495



functions on $\mathcal{M}^{3,1}$:

generators: bundle space $\to$

$X^\mu = r\mathcal{M}^{\mu 5} \sim x^\mu$ ... base space

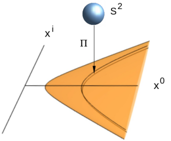
$T^\mu = \frac{1}{R}\mathcal{M}^{\mu 4} \sim t^\mu$ ... fiber / momenta

commutation relations / Poisson brackets

$$\begin{aligned} \{t^\mu, x^\nu\} &= \sinh(\eta)\eta^{\mu\nu} \\ \{x^\mu, x^\nu\} &= \theta^{\mu\nu} \\ \{t^\mu, t^\nu\} &= -\frac{1}{r^2 R^2}\theta^{\mu\nu} \end{aligned}$$

constraints:

$$\begin{aligned} x_\mu x^\mu &= -R^2\cosh^2(\eta), \qquad x^4 = R\sinh(\eta) \\ t_\mu t^\mu &= r^{-2}\cosh^2(\eta) \\ t_\mu x^\mu &= 0, \\ \theta^{\mu\nu} &= c(x^\mu t^\nu - x^\nu t^\mu) + b\epsilon^{\mu\nu\alpha\beta}x_\alpha t_\beta \end{aligned}$$



t^μ ... generates space-like S^2 fiber

fluctuations & $\mathfrak{hs}$ gauge fields on $\mathcal{M}^{3,1}$

add fluctuations to $\mathcal{M}^{3,1}$ background $\quad Y^\mu = \overline{Y}^\mu + \mathcal{A}^\mu$

expand action to second order in $\mathcal{A}_\mu(x,t)$ ...$\mathfrak{hs}$-valued 1-form on $\mathcal{M}^{3,1}$

$$S[Y] = S[\overline{Y}] + \frac{2}{g^2}\mathrm{Tr}\mathcal{A}_\mu\Big(\underbrace{(\Box - \frac{3}{R^2})\delta^\mu_\nu + 2[[\overline{Y}^\mu,\overline{Y}^\nu],.\,]}_{\mathcal{D}^2} - \underbrace{[\overline{Y}^\mu,[\overline{Y}^\nu,.]]}_{g.f.}\Big)\mathcal{A}_\nu$$

eigenmodes of $\mathcal{D}^2$: M. Sperling, HS: 1901.03522

- 4 towers of off-shell modes for each $s>0$:

$$\mathcal{D}^2\mathcal{A}^{(i)}_\mu[\phi] = \lambda\,\mathcal{A}^{(i)}_\mu[\phi], \qquad i\in\{+,-,n,g\} \qquad \Box\phi = \lambda\phi$$

- 4 towers of on-shell modes for each $s>0$, massless

$$\mathcal{D}^2\mathcal{A}^{(i)}[\phi] = 0 \qquad \text{for} \qquad \Box\phi = 0, \qquad i\in\{+,-,n,g\}$$

  universal propagation $\Box \sim \Box_G$

- 2 spin 0 modes (+ tower of exceptional spin 0 modes) gauge-fixing $\{t^\mu, \mathcal{A}_\mu\} = 0$

physical Hilbert space

$$\mathcal{H}_{\rm phys} = \{\mathcal{D}^2\mathcal{A} = 0, \ \ \mathcal{A} \ \text{ gauge fixed}\}/_{\{\text{pure gauge}\}}$$

results:

- generically 2 physical modes $\Box\phi^{(s)} = 0$ for each $s \geq 1$
  would-be massive, $m^2 = 0$ (+ exceptional spin 0 modes)
- no ghosts (t^μ is space-like!) HS 1910.00839
  no tachyons
- same propagation for all modes

linearized geometric fluctuation modes

off-shell:

12 geometric d.o.f. of frame captured by $\mathcal{A}_\mu^{(i)} \in \mathcal{C}^1$

$\rightarrow$ all 10 dof for lin. metric $h_{\mu\nu}[\mathcal{A}]$, & axion & dilaton

physical (vacuum) modes:

$$\mathcal{A}^{(-)}[\phi^{(2)}]$$

... 5 modes of (would-be massive) spin 2 irrep

+ 1 exceptional scalar mode (axion ?)

Ricci-flat $\boxed{\mathcal{R}_{\mu\nu}^{(\mathrm{lin})}[h_{\mu\nu}] \approx 0}$ up to cosm. scales

(basically $\Box h_{\mu\nu} = 0$)

nonlinear regime: frame

any background $Y^{\dot\alpha}$ defines $\mathfrak{hs}$ - valued frame

$$E_{\dot\alpha}[\phi] = \{Y_{\dot\alpha}, \phi\} = E_{\dot\alpha}{}^{\mu}\partial_\mu\phi, \qquad \dot\alpha = 0, ..., 3$$

fundamental object: frame

$$E^{\dot\alpha\mu} = \{Y^{\dot\alpha}, x^\mu\}$$

divergence constraint $\nabla_\nu(\rho^{-2}E_{\dot\alpha}{}^{\nu}) = 0$ (Jacobi)

no local Lorentz transformation of the frame! (diffeo ✓)

eff. metric:

$$G^{\mu\nu} = \rho^{-2}\eta^{\dot\alpha\dot\beta}E_{\dot\alpha}{}^{\mu}E_{\dot\beta}{}^{\nu} \qquad (\mathfrak{hs}-\text{valued})$$

frame contains more physical info than the metric!

conjecture:

for any metric, there is a frame $E^{\dot\alpha}$ such that the divergence-constraint is satisfied, thereby fixing $\rho, \tilde\rho$

gauge transformations and $\mathfrak{hs}$-valued diffeos

scalar fields:

$$\delta_\Lambda \phi = \{\Lambda, \phi\} = \xi^\mu \partial_\mu \phi = \mathcal{L}_\xi \phi, \qquad \xi^\mu = \{\Lambda, x^\mu\}$$

... push-forward of Hamiltonian VF (symplectomorphisms) on the bundle to $\mathcal{M}$ by bundle projection

frame:

$$\begin{aligned} \delta_\Lambda Y_{\dot\alpha} &= \{\Lambda, Y_{\dot\alpha}\} \\ (\delta_\Lambda E_{\dot\alpha})\phi &= \{\Lambda, \{Y_{\dot\alpha}, \phi\}\} - \{Y_{\dot\alpha}, \{\Lambda, \phi\}\} = (\mathcal{L}_\xi E_{\dot\alpha})\phi \qquad \text{(Jacobi)} \end{aligned}$$

hence

$$\boxed{\delta_\Lambda E_{\dot\alpha}{}^\mu = \mathcal{L}_\xi E_{\dot\alpha}{}^\mu, \qquad \delta_\Lambda G^{\mu\nu} = \mathcal{L}_\xi G^{\mu\nu}}$$

diffeos from NC gauge trafos

($\mathfrak{hs}$-valued) Weitzenböck connection:

$$\nabla^{(W)} E_{\dot\alpha} = 0 \qquad \text{(Weitzenböck)} \quad \Rightarrow \quad \nabla^{(W)} G^{\mu\nu} = 0$$

flat but ($\mathfrak{hs}$ - valued) torsion:

$$T_{\dot\alpha\dot\beta} \equiv T[E_{\dot\alpha}, E_{\dot\beta}] = \nabla_{\dot\alpha} E_{\dot\beta} - \nabla_{\dot\beta} E_{\dot\alpha} - [E_{\dot\alpha}, E_{\dot\beta}]$$

can show:

$$\boxed{T_{\dot\alpha\dot\beta}{}^{\mu} = \{\hat\Theta_{\dot\alpha\dot\beta}, x^\mu\},} \qquad \hat\Theta_{\dot\alpha\dot\beta} := -\{Y_{\dot\alpha}, Y_{\dot\beta}\}$$

$$T_{\dot\alpha} = dE_{\dot\alpha}, \qquad E_{\dot\alpha} = E_{\mu\dot\alpha} dx^\mu \qquad \text{...coframe}$$

torsion tensor encodes field strength of the NC gauge theory

HS arXiv:2002.02742 , cf. Langmann Szabo hep-th/0105094

classical dynamics of geometry: pre-gravity

matrix model eom: $\{Y^{\dot\alpha}, \hat\Theta_{\dot\alpha\dot\beta}\} = m^2 Y_{\dot\beta}$

can recast as $\nabla^{(W)}_\nu T^\nu{}_{\rho\mu} + T_\nu{}^\sigma{}_\mu T_{\sigma\rho}{}^\nu = m^2 \rho^{-2} G_{\rho\mu}$

HS arXiv:2002.02742 cf.

$\rightarrow$ eom for frame:

$$d(\rho^2 \star T^{\dot\alpha}) = d\tilde\rho \wedge T^{\dot\alpha} + m^2 \star E^{\dot\alpha} \, .$$

tot. AS part: $\epsilon^{\mu\nu\rho\sigma} T_{\mu\nu\rho} \stackrel{eom}{\sim} \partial^\sigma \tilde\rho$ ... axion

divergence: $T_{\mu\nu}{}^\mu \sim \rho^{-1}\partial_\nu \rho$ ... dilaton

fully covariant form of matrix model eom for frame, axion & dilaton

S. Fredenhagen, HS arXiv: 2101.07297

non-linear solution of class. equations

solve $\quad d(\rho^2 \star T^{\dot\alpha}) = d\tilde\rho \wedge T^{\dot\alpha} + m^2 \star E^{\dot\alpha} \quad \Leftrightarrow$

$$\nabla^{\nu}_{(G)}(\rho^2 T_{\nu\rho}{}^{\dot\alpha}) = \frac{1}{2}\sqrt{|G|}^{-1}\varepsilon^{\nu\rho'\sigma\mu}G_{\rho\rho'}\partial_\mu\tilde\rho\, T_{\nu\sigma'}{}^{\dot\alpha} + m^2 E^{\dot\alpha}{}_{\rho} \ .$$

spherically symmetric static solutions for frame:

simplest solution: S. Fredenhagen, HS arXiv: 2101.07297

$$\begin{aligned} ds^2_G &= -\frac{c_1 b_0^{-2}}{(1+\frac{M}{r})}dt^2 + c_1\left(1+\frac{M}{r}\right)(dr^2 + r^2 d\Omega^2) \\ \rho^2 &= c_1 b_0^{-2}\left(1+\frac{M}{r}\right) . \end{aligned}$$

linearized Schwarzschild, deviates at nonlin. level

most general spherical solution: Y. Asano, HS arXiv: 21xx.xxxxx vacuum equations: (using eom for torsion, no matter)

$$\mathcal{G}_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2} G_{\mu\nu}\mathcal{R} = 8\pi \mathbf{T}_{\mu\nu}$$

where

$$8\pi\mathbf{T}_{\mu\nu} = \mathbf{T}_{\mu\nu}[E^{\dot\alpha}] + \mathbf{T}_{\mu\nu}[\rho] + \mathbf{T}_{\mu\nu}[\tilde\rho] - \rho^{-2} m^2 G_{\mu\nu} = O(TT)$$

... effective e-m tensor due to frame, axion & dilaton

quadratic in $\rho, \tilde\rho, E^{\dot\alpha}$ $\Rightarrow$ $\boxed{\mathcal{R}_{\mu\nu} = 0}$ in linearized regime (in vacuum)

however: $\mathcal{R}_{\mu\nu} \neq 0$ in non-linear regime

pre-gravity from classical matrix model:

($\mathfrak{hs}$-extedend) theory of dynamical geometry, similar to gravity

differs from GR at non-lin level

action has 2 derivatives less than E-H action (good for quantization!!)

summary: classical emergent (pre-) gravity:

- univ. metric, gravitons, lin. Schwarzschild etc. recovered
- extra dof: dilaton, axion, $\mathfrak{hs}$; massive graviton modes (?)
- bare action: $S \sim \int \{Y, Y\}\{Y, Y\}$

  ... 2 derivatives less than

  $$S_{EH} \sim \int T_{\nu\lambda}{}^{\mu} T_{\nu\lambda}{}^{\mu} + ... , \qquad T \sim \{\{Y, Y\}, x\}$$

  $\Rightarrow$ different from GR, expected to dominate on large scales
- reasonable cosmology without any fine-tuning

1-loop effective action and physical perspectives

SUSY IKKT model $\rightarrow$ quantum effects suppressed:

claim:

Einstein-Hilbert action (+ extra) arises in the 1-loop effective action on $\mathcal{M}^{3,1}$ space-time (cf. Sakharov '67)

$$\Gamma_{1-\mathrm{loop}} \ni \int\limits_{\mathcal{M}} T_{\nu\lambda}{}^{\mu} T_{\nu\lambda}{}^{\mu} + \ldots \sim \int\limits_{\mathcal{M}} d^4x\sqrt{G}\, m_{\mathcal{K}}^2 \mathcal{R}[G] + \ldots$$

requires presence of fuzzy extra dimensions $\mathcal{K}$

different from bulk IIB sugra (also 1-loop)
= short-range $\sim r^{-8}$ correction

$\approx$ GR, matter will induce $\sim$ standard gravity!

M.M. provides consistent basis: quantizable "pre-gravity" nonperturbative quantization:

$$Z = \int dY d\Psi e^{iS[Y,\Psi]}, \qquad S = S_{\rm IKKT} + i\varepsilon Y^a Y^b \delta_{ab}$$

1-loop effective action

$$e^{i\Gamma_{\rm 1\text{-}loop}[Y]} = \int\limits_{\rm 1\ loop} d\mathcal{A} d\Psi e^{iS[Y+\mathcal{A},\Psi]}$$

Faddeev-Popov gauge fixing: $[Y^a, \mathcal{A}_a] = 0$,

$$\begin{aligned}\Gamma_{\rm 1loop}[Y] &= \tfrac{1}{2}{\rm Tr}\Big(\log(\Box - M_{ab}[\Theta^{ab},.]) - \tfrac{1}{2}\log(\Box - M^{(\psi)}_{ab}[\Theta^{ab},.]) - 2\log(\Box)\Big)\\ &= \tfrac{1}{2}{\rm Tr}\Big(\sum_{n=4}^{\infty}\tfrac{1}{n}\Big((\Box^{-1}M_{ab}[\Theta^{ab},.])^n - \tfrac{1}{2}(\Box^{-1}M^{(\psi)}_{ab}[\Theta^{ab},.])^n\Big)\Big)\end{aligned}$$

(max. SUSY !) where

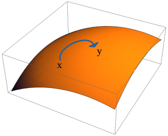
$$\begin{aligned} i\Theta^{ab} &= -[Y^a, Y^b] \\ M^{(\psi)}_{ab} &\quad \text{...spinorial generators of } so(5) \\ M_{ab} &\quad \text{...vector generators} \end{aligned}$$

UV-finite on 4D backgrounds due to max. SUSY evaluate trace use string state formalism

$$\mathrm{Tr}_{End(\mathcal{H})}\mathcal{O} = \frac{1}{(2\pi)^m}\int\limits_{\mathcal{M}\times\mathcal{M}} dxdy \left(\begin{smallmatrix}x\\y\end{smallmatrix}\right|\mathcal{O}\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)$$

string states:

$$\boxed{\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right) := |x\rangle\langle y|} \quad \in End(\mathcal{H})$$



$|x\rangle$ ... coherent state on $\mathcal{M} = \mathbb{C}P^{2,1}$

$\approx$ diagonalize kinetic operators:

$$\Box^{-1}\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right) \approx \frac{1}{|x-y|^2+2\Delta^2}\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)$$

$$\Box^{-1}[\Theta^{ab},.]\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right) \approx \frac{1}{|x-y|^2+2\Delta^2}\underbrace{(\Theta^{ab}(y)-\Theta^{ab}(x))}_{\delta\Theta^{ab}}\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)$$

H.S. arXiv:1606.00646

can evaluate

$$\Gamma_{1\text{loop};4}[Y] = \frac{1}{8}\mathrm{Tr}\Big((\Box^{-1}(M_{ab}[\Theta^{ab},.])^4 - \frac{1}{2}(\Box^{-1}M^{(\psi)}_{ab}[\Theta^{ab},.])^4\Big)$$
$$= \frac{1}{4}\frac{1}{(2\pi)^m}\int\limits_{\mathcal{M}\times\mathcal{M}} dxdy \frac{3S_4[\delta\Theta(x,y)]}{(|x-y|^2+2\Delta^2)^4}$$

where

$$-S_4[\delta\Theta] = 4tr(\delta\Theta\delta\Theta\delta\Theta\delta\Theta) - (tr\delta\Theta\delta\Theta)^2$$

note:

- UV regime = long string regime
  ($\rightarrow$ UV/IR mixing, non-local action)
- UV-finite only in maximally SUSY model $\rightarrow$ almost-local action: IIB supergravity in $\mathbb{R}^{9,1}$, $\sim r^{-8}$ interaction
- short-distance regime requires refined analysis:

string states as localized Gaussian wave-packets:

local isometric mapping $\mathrm{End}(\mathcal{H})_{loc} \to \mathcal{C}_{IR}(\mathcal{M})$:

$$e^{\frac{i}{2}k^a\theta^{-1}_{ab}y^b}\Big|y+\frac{k}{2}\Big\rangle\Big\langle y-\frac{k}{2}\Big| =: \Psi_{\tilde{k};y} \;\cong\; \psi_{\tilde{k};y}(x) = \frac{2}{\pi L_{\mathrm{NC}}^2}\, e^{i\tilde{k}x} e^{-|x-y|_g^2} .$$

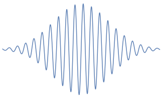
(for almost-Kähler manifolds $(\mathcal{M}, \theta^{-1}, g)$, locally) $\tilde{k}_\mu = \theta^{-1}_{\mu\nu}k^\nu$

size $\sim L_{NC}$

H.S., J. Tekel arxiv:21xx.xxxxx

semi-classical wavepackets:

$$\Psi^{(L)}_{k;y} := \int d^4z\, e^{-|y-z|^2/L^2}\Psi_{k;z} \cong e^{ikx} e^{-|x-y|^2/L^2} =: \psi^{(L)}_{k;y}$$



locally diagonalizes kinetic operators in IR:

$$\begin{aligned} \Box\Psi^{(L)}_{k;y} &\approx \gamma^{\mu\nu}(x)k_\mu k_\nu \Psi^{(L)}_{k;y} \\ [\theta^{ab}, \Psi^{(L)}_{k;y}] &\approx i\{\theta^{ab}, \psi^{(L)}_{k;y}\} \approx -\{\theta^{ab}, x^\mu\}k_\mu \psi^{(L)}_{k;y} \end{aligned}$$

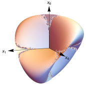
Trace formula for UV-finite traces on NC spaces:

$$\mathrm{Tr}\mathcal{O} = \frac{1}{(2\pi)^m}\int\limits_{\mathcal{M}\times\mathcal{M}} \Omega_x\Omega_y \left(\begin{smallmatrix}x\\y\end{smallmatrix}\right|\mathcal{O}\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right) \approx \frac{1}{(2\pi)^m}\int\limits_{\mathcal{M}}\sqrt{G}dx\int\frac{1}{\sqrt{G}}dk\langle\Psi^{(L)}_{k,x},\mathcal{O}\Psi^{(L)}_{k,x}\rangle$$

apply this to $\mathcal{O} = (\Box^{-1}[\Theta^{ab},.])^4$ in 1-loop eff. action

a priori: 4-derivative action ☹

however: assume fuzzy extra dimensions $\mathcal{M}\times\mathcal{K}$
from 6 transversal directions $\langle\phi^i\rangle = \lambda^i \neq 0$



($\rightarrow$ interesting gauge theory, cf. H.S., J. Zahn arxiv:1409.1440)

mixed term $(\delta\Theta^{\alpha\beta}\delta\Theta^{\alpha\beta})(\delta\Theta^{ij}\delta\Theta^{ij})$ leads to induced E-H action ☺

induced E-H action:

$$\begin{aligned}\{\theta^{\alpha\beta},\{\theta^{\alpha\beta},\psi_{k;y}\}\} &\approx -\{\theta^{\alpha\beta},x^\mu\}\{\theta^{\alpha\beta},x^\nu\}k_\mu k_\nu \psi_{k;y} \\ &= -T^{\alpha\beta\mu}k_\mu T^{\alpha\beta\nu}k_\nu\psi_{k;y}\ , \qquad k > \frac{1}{L}\ .\end{aligned}$$

recall torsion $T^{\alpha\beta\mu} = \{\theta^{\alpha\beta},x^\mu\}$

$$\begin{aligned}\Gamma_{1\text{loop}} = i\mathrm{Tr}\Big(\frac{V_{4,\text{mix}}}{(\Box - i\varepsilon)^4}\Big) &\sim \int_{\mathcal{M}} d^4x \sum_{lm} C^2_{lm}\int d^4k \frac{T^{\alpha\beta\mu}k_\mu T^{\alpha\beta\nu}k_\nu}{(k\cdot k + m^2_{lm} - i\varepsilon)^4} \\ &\sim -\int_{\mathcal{M}} d^4x\sqrt{G}\, m^2_{\mathcal{K}}\, T^\rho{}_{\sigma\mu}\, T_{\rho'}{}^\sigma{}_\mu\, G^{\mu\mu'} \\ &\sim \int d^4x\sqrt{G}\, m^2_{\mathcal{K}}\Big(8\mathcal{R}[G] + 6T_\nu T_\mu G^{\mu\nu}\Big)\end{aligned}$$

$m^2_{\mathcal{K}}$ ... Kaluza-Klein mass scale on $\mathcal{K}$

using identity

$$\int d^4x\sqrt{|G|}\mathcal{R} = \int d^4x\sqrt{|G|}\Big(-\frac{3}{4}T_\nu T_\mu G^{\mu\nu} - \frac{7}{8}T^\mu{}_{\sigma\rho}\, T_{\mu\sigma'}{}^\rho G^{\sigma\sigma'}\Big)$$

S. Fredenhagen, H.S. arxiv:2101.07297)

($T_\mu \sim \partial_\mu\tilde\rho$ ... totally antisymmetric torsion, grav. axion) bottom line: (to be published)

- $\Gamma_{\rm 1loop}$ includes Einstein-Hilbert action

$$\frac{1}{16\pi G_N} \sim c_{\mathcal{K}}^2 m_{\mathcal{K}}^2$$ ... eff. Newton constant

  set by Kaluza-Klein mass scale on $\mathcal{K}$

- no induced cosm. const (!!), rather

$$\Gamma_{\rm 1loop} \ni C \int \rho_M$$ ... symplectic volume, non-dynamical

- these are on top of matrix model action $S \sim \int [Y,Y][Y,Y]$, should dominate extreme IR (cosm.)
- attractive potential between $\mathcal{K}$ and cosm. $\mathcal{M}$
- + lots of other stuff (axion, dilaton, ... $\mathfrak{hs}$), to be understood

summary

gravity arises as quantum effect on 3+1-dim. quantum space-time in the maximally SUSY IKKT matrix model

- geometric structures arise automatically on given BG "pre-gravity", suitable for quantization
- covariant quantum spaces = twisted S^2 bundles over $\mathcal{M}^{3,1}$
  $\rightarrow$ higher spin gauge theory
  Lorentz invariance partially manifest
- expect cross-over GR $\leftrightarrow$ cosm. background (class.)
- quantization $\rightarrow$ induced Einstein-Hilbert action, no c.c. problem
- new physics (axion, dilaton, $\mathfrak{hs}$ ...)
  lots of things to be clarified ! (help is welcome!)

linearized Schwarzschild solution

HS 1905.07255

Ricci-flat "scalar" metric perturbation

from $\mathcal{A}^{(-)}[D^+ D^+ \phi]$

$$ds^2 = (G_{\mu\nu} - h_{\mu\nu}) dy^\mu dy^\nu = -dt^2 + a(t)^2 d\Sigma^2 + \phi'(dt^2 + a(t)^2 d\Sigma^2)$$

$$\phi' \sim \frac{M(t)}{2r} \frac{1}{a(t)}$$

$\approx$ lin. Schwarzschild (Vittie) solution on FRW, eff. mass $M(t) \sim \frac{M_0}{a(t)}$

more generally for any quasi-static lin. solution in GR geometric eom arise from effective action

$$S_{\rm eff}[E, G, \rho, \tilde\rho] = 2S_{\mathcal{R}} + S_E + S_{\tilde\rho} - \frac{1}{2} S_{\tilde E} - 4S_\rho + 2S_m - 8S_{\tilde m}$$

where

$$\begin{aligned}
S_{\mathcal{R}} &= \int d^4x \sqrt{|G|}\mathcal{R}[G], \\
S_E &= \int d^4x \sqrt{|G|}\rho^2 G^{\nu\nu'} G^{\sigma\sigma'} T_{\nu\sigma}{}^{\dot\alpha} T_{\nu'\sigma'\dot\alpha} \\
S_{\tilde E} &= \int d^4x \tilde\rho\, \varepsilon^{\nu\sigma\mu\kappa} T_{\nu\sigma}{}^{\dot\alpha} T_{\mu\kappa\dot\alpha} \\
S_{\tilde\rho} &= \int d^4x \sqrt{|G|} G^{\nu\nu} \rho^{-4} \partial_\mu\tilde\rho \partial_\nu\tilde\rho \\
S_\rho &= \int d^4x \sqrt{|G|} G^{\mu\nu} \rho^{-2} \partial_\mu\rho \partial_\nu\rho \\
S_m &= \int d^4x \sqrt{|G|} m^2 E^{\dot\alpha}{}_\kappa E_{\dot\alpha\kappa'} G^{\kappa\kappa'}
\end{aligned}$$

for independent variations of $G, E, \rho, \tilde\rho$

... geometric action, however vanishes upon imposing $G = \eta EE$