Instituto de Física Teórica UAM-CSIC

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On the uses of EFT strings in 4d $\mathcal{N}=1$

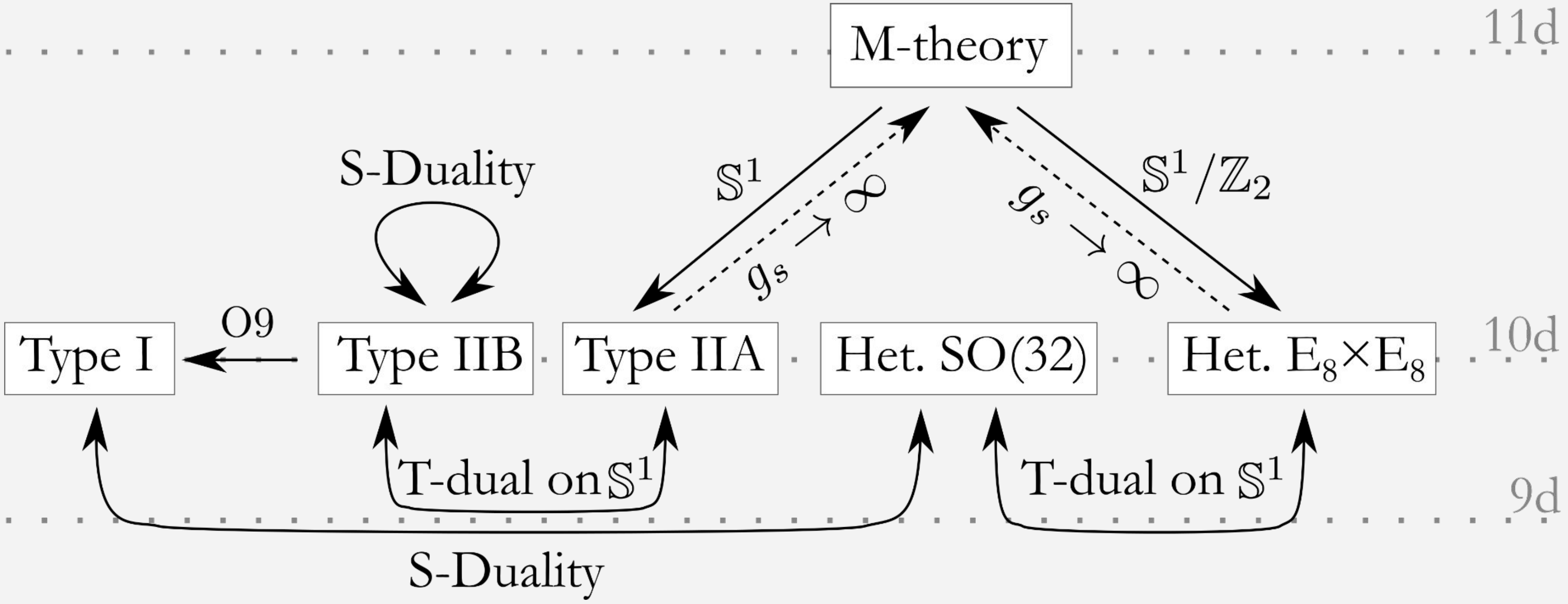
Based on [W.I.P] *with* Alessandra Grieco *and* Irene Valenzuela
and [2406.07614] *with* Gonzalo F. Casas *and* Miguel Montero

Ignacio Ruiz ,

The Landscape vs. the Swampland, ESI, Vienna, July 16th, 2024

Motivation:
Why 4d $\mathcal{N}=1$?

String Theory and dualities



Swampland Distance or **Duality Conjecture**

[Ooguri, Vafa, '07]

As we move towards infinite distance limits of moduli space, there is an **infinite tower of states** becoming **exponentially light**:

$$M(\Delta) \sim M(0)e^{-\alpha\Delta} \text{ as } \Delta \to \infty \text{ with } \alpha = \mathcal{O}(1).$$

[Álvarez-García, Basile, Baume, Blumenhagen, Buratti, Calderón-Infante, Castellano, Cecotti, Corvilain, Cribiori, Debusschere, Etheredge, Erkinger, Font, Gendler, Grimm, Heidenreich,van de Heisteeg, Herráez, Ibáñez, Joshi, Kaya, Klemm, Kläwer, Knapp, Lanza, Lee, Lerche, Li, Lockhart, McNamara, Marchesano, Montella, Martucci, Mohseni, Montero, Ooguri, Palti, Petri, Perlmutter, Qiu, Rastelli, Reece van Riet, Rudelius, **IR**, Schlechter, Stout, Uranga, Valenzuela, Vafa, Weigand, Wiesner, Wolf...]

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Emergent String Conjecture

[Lee, Lerche, Weigand, '19]

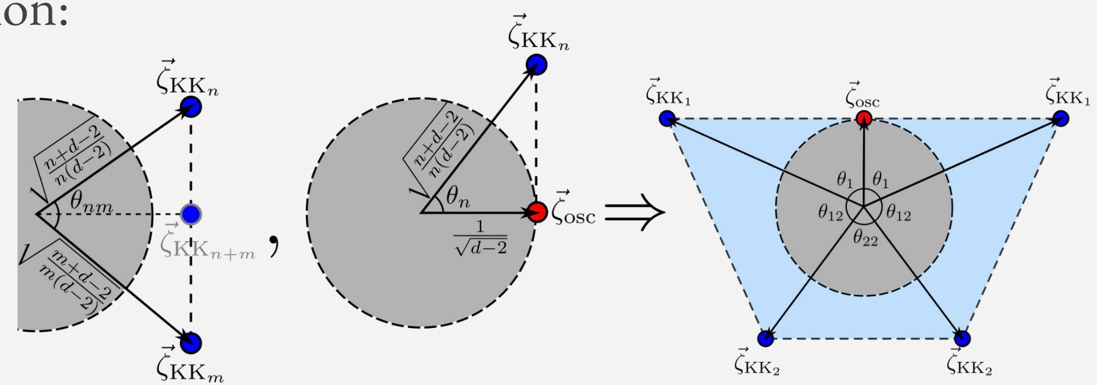
Any infinite distance limit is either a (1) **decompactification** limit or has a (2) **weakly coupled** (critical) **string** becoming **tensionless**.

[Álvarez-García, Aoufia, Basile, Baume, Calderón-Infante, Kläwer, Lanza, Lee, Leone, Lerche, Marchesano, Martucci, Perlmutter, Rastelli, Rudelius, Vafa, Valenzuela, Weigand, Wiesner, Xu...]

Taxonomy of Towers

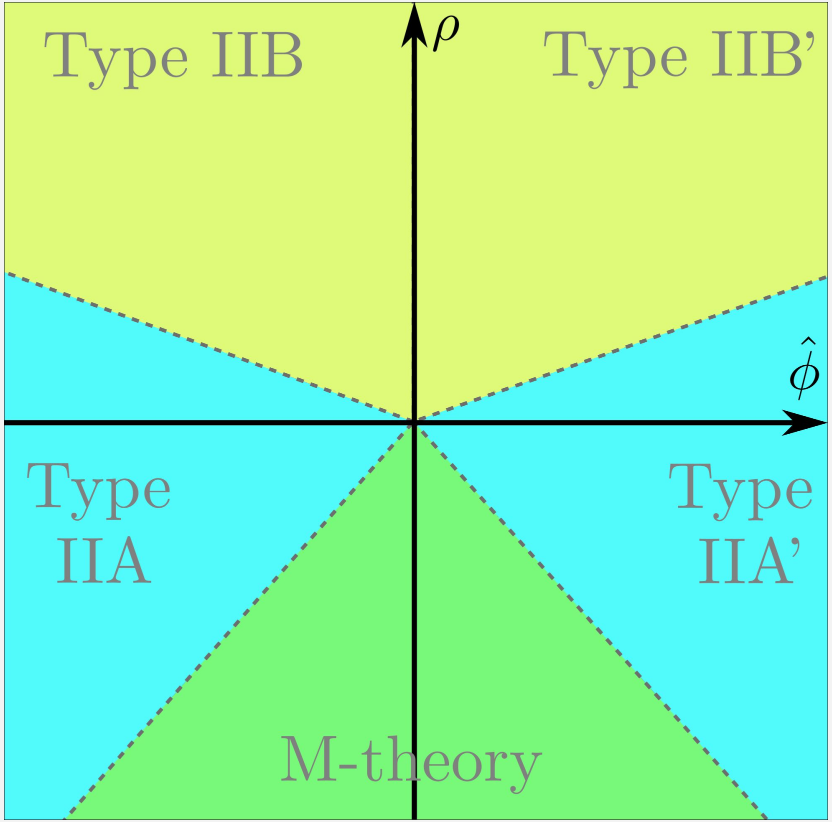
[Etheredge,Heindenreich,Rudelius,Ruiz,Valenzuela,2405.20332], c.f. Tom's talk!

Under relatively general assumptions we can obtain the rules relating the moduli dependence of the different towers of states, as well as that of the QG cut-off in the asymptotic region:



A classification of the light towers allows us to obtain information about the **global asymptotic structure** of **duality frames** and their **range of validity.**

A simple example: Type IIB on $\mathbb{S}^1$



where

$$\hat{\phi} = \frac{1}{\sqrt{2}}\phi \quad \rho = \sqrt{\frac{8}{7}}\log R_{\text{IIB}}$$

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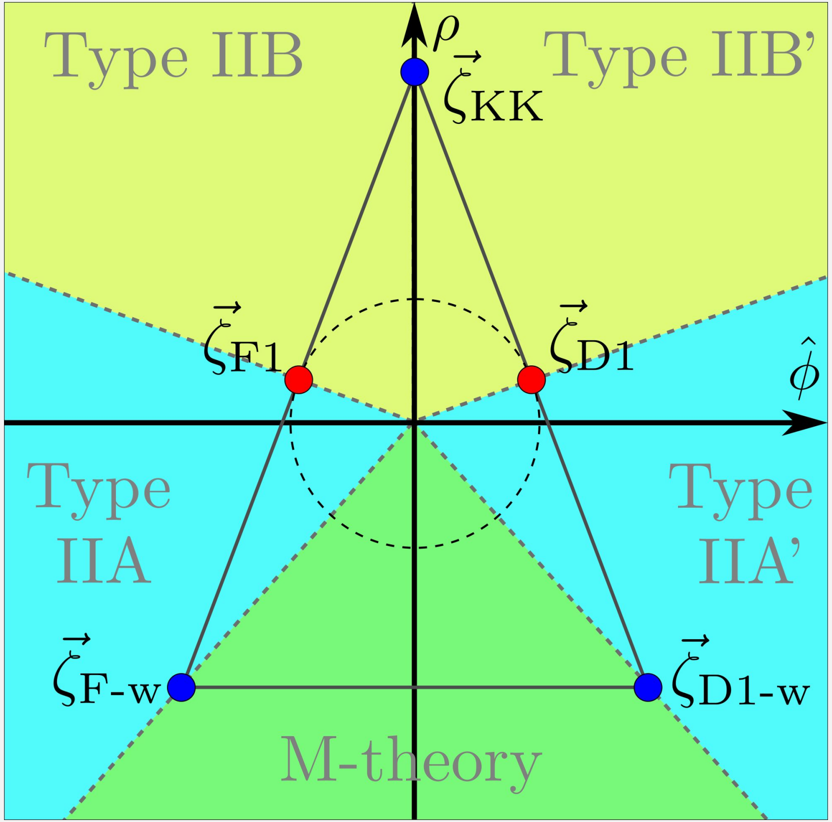

where

$$\hat{\phi} = \frac{1}{\sqrt{2}}\phi \quad \rho = \sqrt{\frac{8}{7}} \log R_{\text{IIB}}$$

and we define

$$\vec{\zeta}_I = -\vec{\nabla} \log m_I(\vec{\varphi})$$

[Calderón-Infante,Uranga,Valenzuela,'20]

Can we go down?

Motivation for 4d $\mathcal{N}=1$

- **Phenomenologically interesting:** 4d, low supersymmetry, positive potentials, instanton corrections, EFT string/membranes, etc.
- **Different constructions**: M-th on G2 manifold, heterotic on CY_3, type II on CY_3 orientifolds...
- Rich web of **duality relations.**
- Moduli space does not factorize and might receive corrections.

c.f. Max's talk!

Do we have additional tools?

In 5 minutes

4d $N=1$ for DUMMIES

4d N=1 action

[Cremmer,Ferrara,Girardello,Van Proeyen,'82]

The bosonic action for chiral multiplets $\{\phi^\alpha\}_\alpha$ is given by

$$S = \int \left\{ M_{\rm Pl}^2 \left[\frac{1}{2} \star R - K_{\alpha\bar{\beta}} \mathrm{d}\phi^\alpha \wedge \star \mathrm{d}\bar{\phi}^{\bar{\beta}} \right] - \star V(\vec{\phi}) \right\}$$

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For F-term potentials we have

$$V(\vec{\phi}) = M_{\rm Pl}^4 f_A f_B e^K \left[K^{\alpha\bar{\beta}} D_\alpha \Pi^A \bar{D}_\beta \bar{\Pi}^B - 3 \Pi^A \bar{\Pi}^B \right]$$

with flux quanta $f_A \in \mathbb{Z}$ and periods $\vec{\Pi}(\vec{\phi})$, holomorphic functions of $\vec{f}$ and $\vec{\phi}$.

4d *N*=1 action

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with flux quanta $f_A \in \mathbb{Z}$ and periods $\vec{\Pi}(\vec{\phi})$, holomorphic functions of $\vec{f}$ and $\vec{\phi}$.

The 4d *N*=1 theory is fully determined from Kähler potential K, superpotential $W(\vec{\phi}) = f_A \Pi^A + \ldots$ and gauge kinetic matrix.

Axionic shift symmetry and EFT strings

[Bandos,Isidro,'03; Bandos,(Lanza,Sorokin),'19]

Moduli $\{t^j = a^j + is^j\}_j \subseteq \{\phi^\alpha\}_\alpha$ with periodic directions $a^j \simeq a^j + e^j$ (with $e^j \in \mathbb{Z}$) might enjoy an **approximate axion-like shift symmetry** in some **asymptotic limits:**

$$a^j \to a^j + \lambda e^j \text{ with } \lambda \in \mathbb{R}$$

Kähler potential here has form $K = -\log P(s^j) + \ldots$ with homogeneous $P(s)$.

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Charges under these "axions" we can have **EFT strings** with charges $\vec{e}$:

$$S^{\vec{e}}_{\text{string}} = -M^2_{\text{Pl}} \int_{\mathcal{S}} |e^i l_i| \sqrt{-h} + e^i \int_{\mathcal{S}} \mathcal{B}_{2i} \begin{cases} \mathcal{T}_{\vec{e}} = M^2_{\text{Pl}} |e^i l_i| \\ \mathcal{Q}_{\vec{e}} = M_{\text{Pl}} \sqrt{\mathcal{G}_{ij} e^i e^j} \end{cases}$$

with $l_i = -\frac{1}{2}\frac{\partial K}{\partial s^i}$ and $\mathcal{G}_{ij} = \frac{1}{2}\frac{\partial^2 K}{\partial s^i \partial s^j}$.

Axionic shift symmetry and EFT strings

For ½-BPS strings $\mathcal{T}_{\vec{e}} = M_{\rm Pl}^2 e^i l_i > 0$ preserving 2D Poincaré invariance scalars backreact as

$$s^i \approx s_0^i - \frac{1}{2\pi} e^i \log\left(\frac{r}{r_0}\right) \qquad a^i \approx a_0^i + \frac{\theta}{2\pi} e^i$$

which allows us to prove infinite distance limits in moduli space.

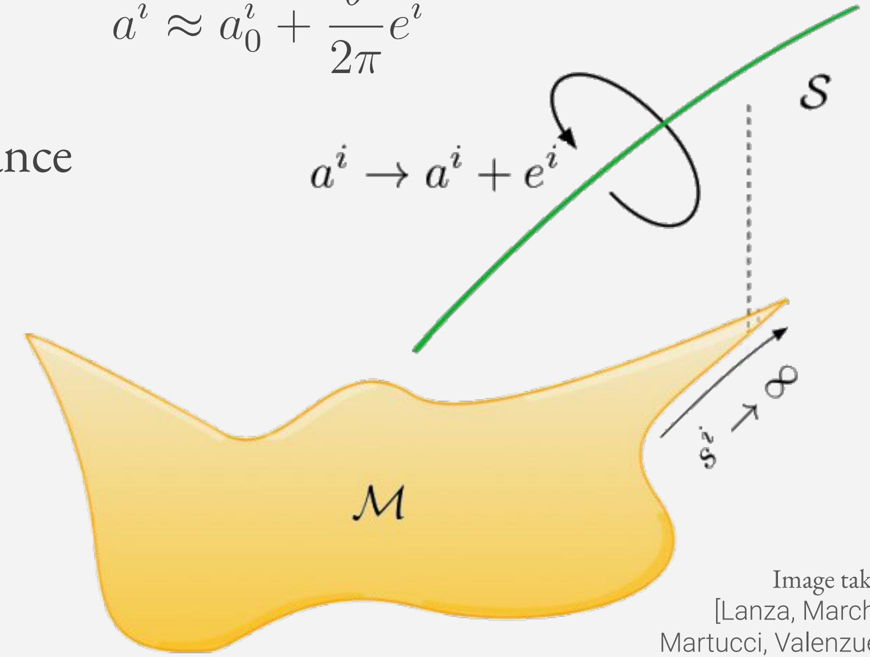

Image taken from [Lanza, Marchesano, Martucci, Valenzuela, '20]

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We also have towers at infinite distances!

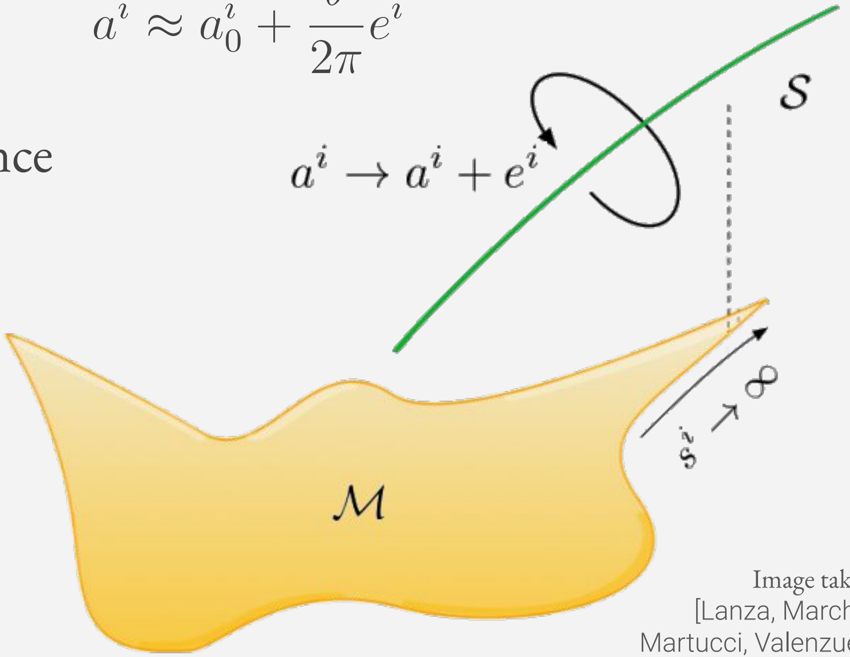

Image taken from [Lanza, Marchesano, Martucci, Valenzuela, '20]

Integral Scaling Conjecture

[Lanza, Marchesano, Martucci, Valenzuela, '21]

Along the asymptotic flow associated with an EFT string, its tension $\mathcal{T} \to 0$. Compared with the scaling of the leading light tower, we have

$$\left(\frac{m_\star}{M_{\rm Pl}}\right)^2 \sim \left(\frac{\mathcal{T}}{M_{\rm Pl}^2}\right)^w \to 0 \quad \text{with} \qquad w = 1, 2, 3$$

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This holds in all studied examples and actually **not only** for the leading towers but for **all** towers that are **principal**[1] in some limit (allowing also $w = 0$).

1.Leading tower which is not degenerate (there are no other towers becoming light at the same rate), see [2405.20332].

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The microscopic origin of the behavior is not yet fully-understood, but examples point

$w = 1$	$w = 2$	$w = 3$
Emerg. string limit	Decomp. to string theory	Decomp. to M-theory

[Lanza, Marchesano, Martucci, Valenzuela, '21; Marchesano, Melotti,'22]

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Recovering towers and duality frames from EFT strings

Can we say more?

Infinite distance limits become more involved when several moduli are considered:

Global moduli dependence rather than scaling along limits!

Different **top-down** constructions studied:

- Heterotic on CY_3
- Type IIA/Bon CY_3 orientifolds
- M-theory on G2 manifolds

The **Integral Scaling Conjecture** imposes **bottom-up** constraints on the different towers and duality frames we can have!

Example: Heterotic on CY $_3$

Consider Heterotic string theory on CY_3, with Kähler potential $K = -\log s^0 - \log \mathcal{V}_X$, where $\mathcal{V}_X$ is the CY_3 volume in string units and $s^0 = e^{-2\phi} \log \mathcal{V}_X$.
The moduli space metric of this slice is asymptotically flat, with canonical coordinates

$$\hat{s}^0 = \frac{1}{\sqrt{2}} \log \frac{s^0}{s^0_0} > 0 \qquad \hat{\mathcal{V}} = \frac{1}{\sqrt{6}} \log \frac{\mathcal{V}_X}{\mathcal{V}_{X,0}} > 0$$

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We have the following families of EFT strings:

$$\frac{\mathcal{T}_{(a,0)}}{M^2_{\text{Pl}}} \sim e^{-\frac{1}{\sqrt{2}}\hat{s}^0}$$

Fundamental Het. String

$$\frac{\mathcal{T}_{(0,b)}}{M^2_{\text{Pl}}} \sim e^{-\frac{1}{\sqrt{6}}\hat{\mathcal{V}}}$$

NS5 wrapped on eff. NEF divisor

$$\frac{\mathcal{T}_{(a,b)}}{M^2_{\text{Pl}}} \sim e^{-\frac{1}{4\sqrt{2}}\hat{s}^0 - \frac{1}{4}\sqrt{\frac{3}{2}}\hat{\mathcal{V}}}$$

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Fundamental Het. String

NS5 wrapped on eff. NEF divisor

and the following leading towers:

$$\frac{m_{\mathrm{osc}}}{M_{\mathrm{Pl}}} \sim e^{-\frac{1}{\sqrt{2}}\hat{s}^0} \qquad \frac{m_{\mathrm{KK}}}{M_{\mathrm{Pl}}} \sim e^{-\frac{1}{\sqrt{2}}\hat{s}^0 - \frac{1}{\sqrt{6}}\hat{\mathcal{V}}} \qquad \frac{m_{\mathrm{D0}}}{M_{\mathrm{Pl}}} \sim e^{-\sqrt{\frac{3}{2}}\hat{\mathcal{V}}}$$

Example: Heterotic on CY $_3$

The integer scaling can be checked pictorially!

$$\vec{\zeta}_I = -\vec{\nabla} \log m_I(\vec{\varphi}) \qquad \vec{\zeta}_{\mathcal{T}} = -\frac{1}{2}\vec{\nabla} \log \mathcal{T}$$

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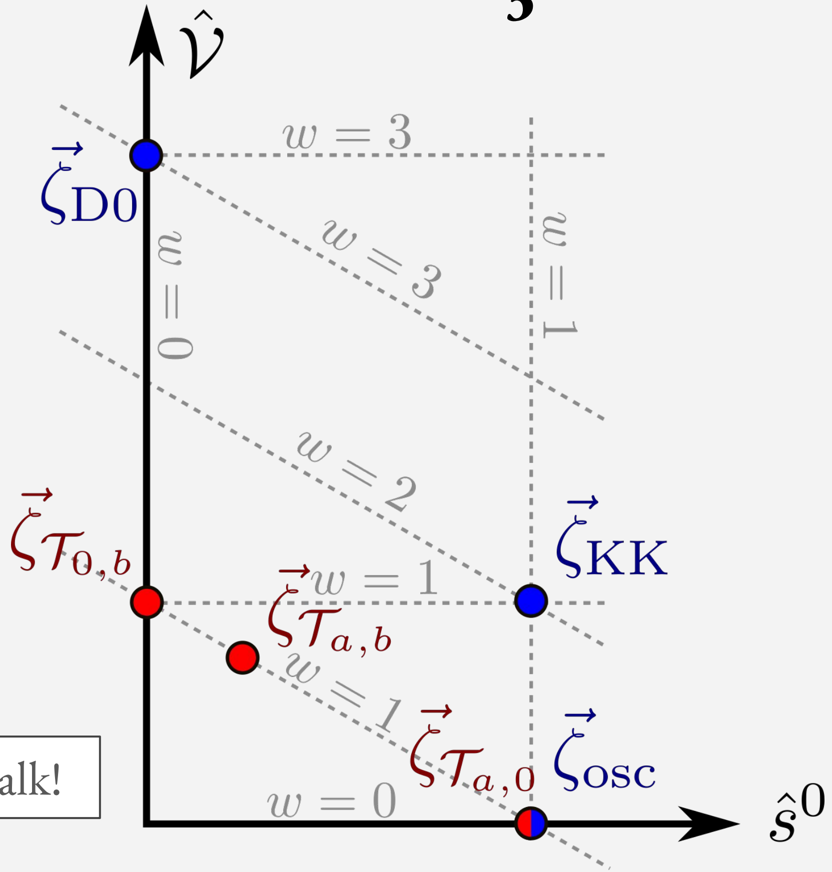
Integer scaling along EFT flow:

$$\left(\frac{m_\star}{M_{\rm Pl}}\right)^2 \sim \left(\frac{\mathcal{T}}{M_{\rm Pl}^2}\right)^w \to 0$$

$$\boxed{\vec{\zeta}_I \cdot \vec{\zeta}_{\mathcal{T}} = w_I |\vec{\zeta}_{\mathcal{T}}|^2}$$

with $w_I = 0,\ 1,\ 2,\ 3$

c.f. Muldrow's talk!



Towers from EFT strings?

The EFT strings can be read directly from the 4d $\mathcal{N}$=1 EFT data (i.e. the Kähler potential). However, naïvely we would need UV information to recover the light towers!

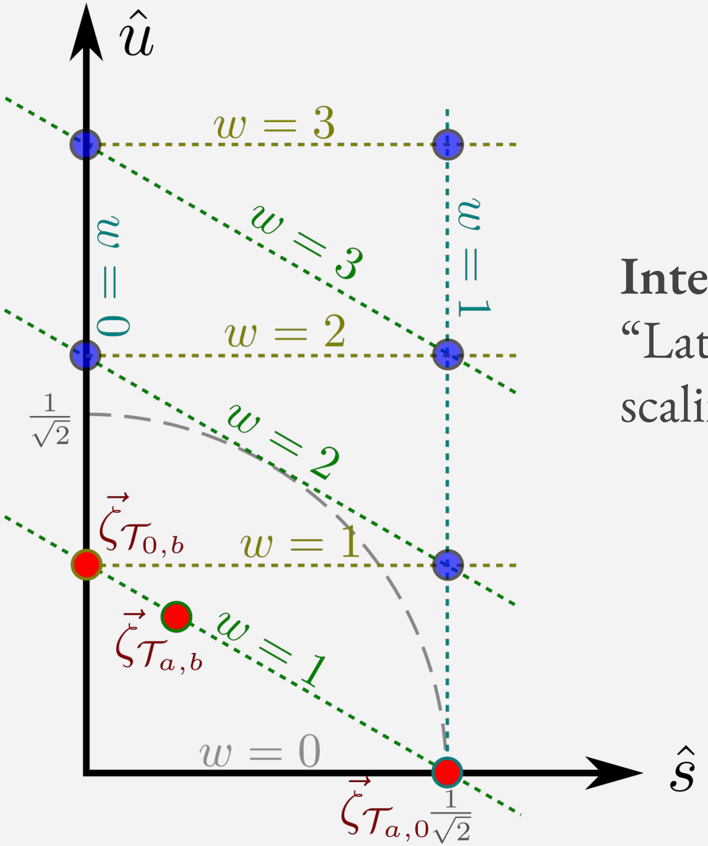
EFT Data:
Kähler potential→EFT Strings

\+

Swampland Constraints:
Integral Scaling Conjecture
Emergent String Conjecture

UV Information: Light towers and duality frames

Example 1: $K = -\log\left(su^3\right)$



Integer Scaling Condition:
"Lattice" of possible tower scalings.

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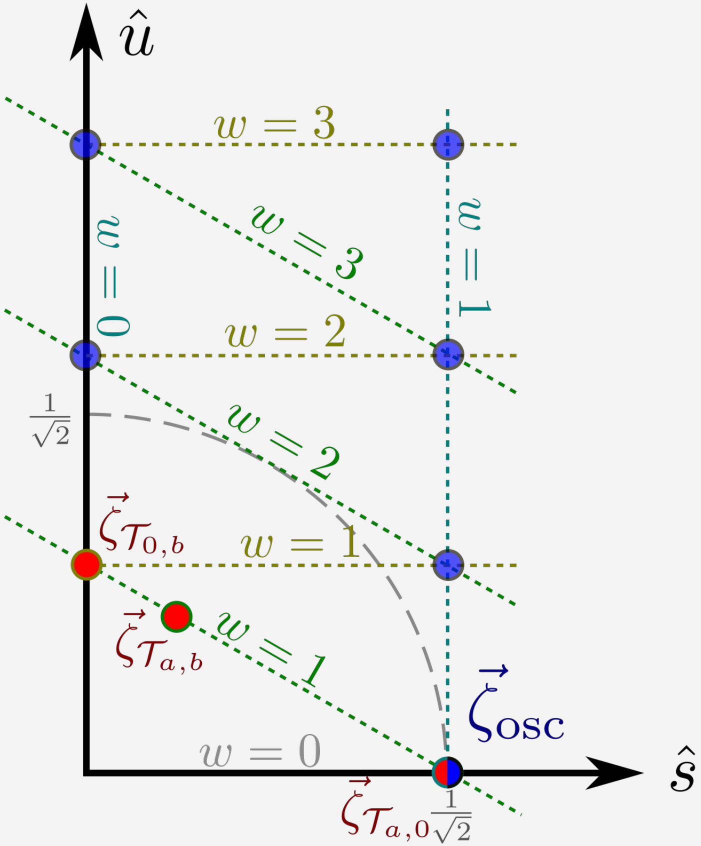

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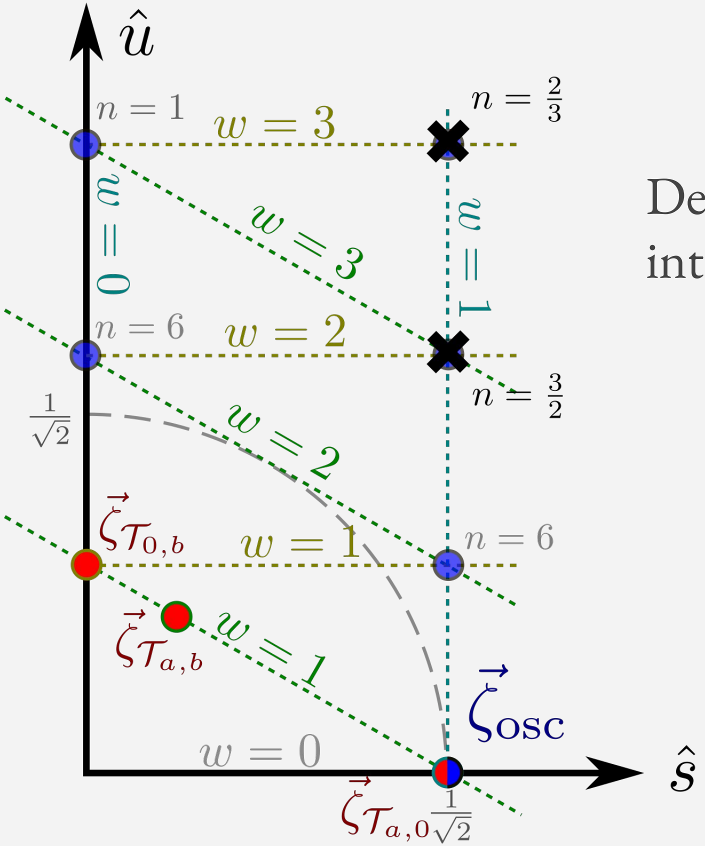

Decompactification of n integer dimensions:

$$|\vec{\zeta}_{\text{KK},n}| = \sqrt{\frac{2+n}{2n}}$$

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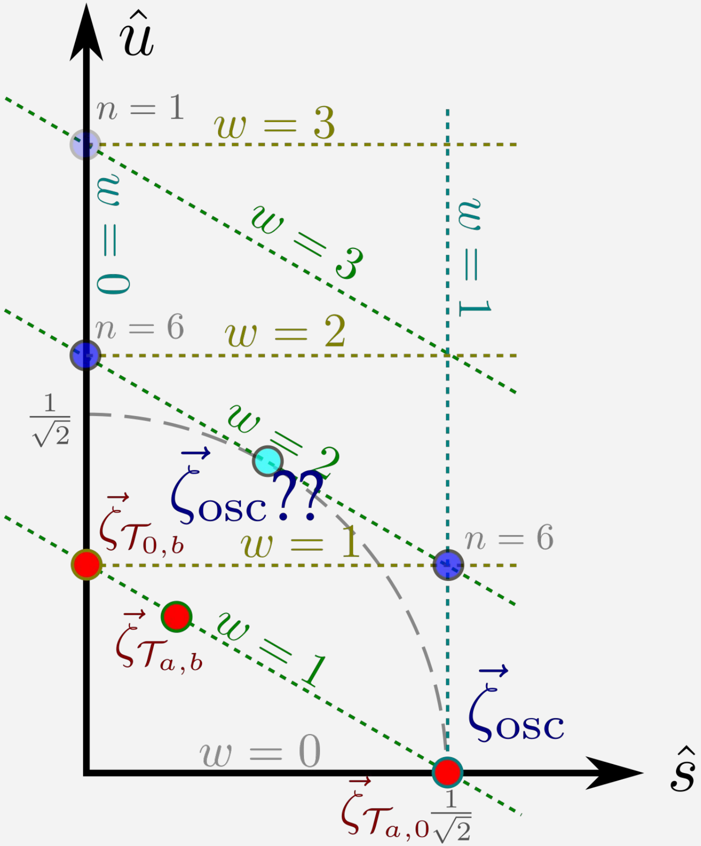

Emergent Strings are always EFT strings

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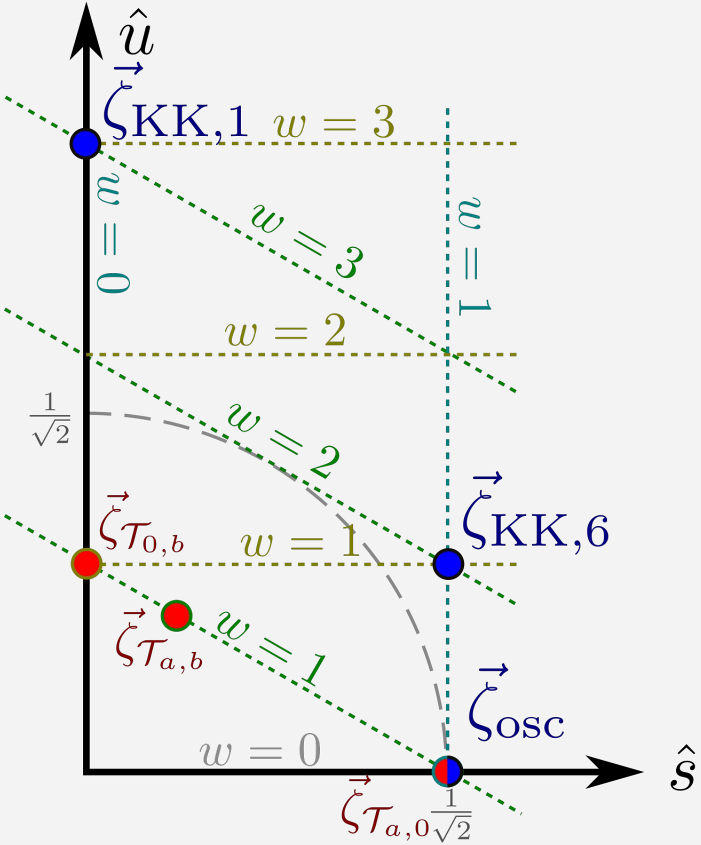

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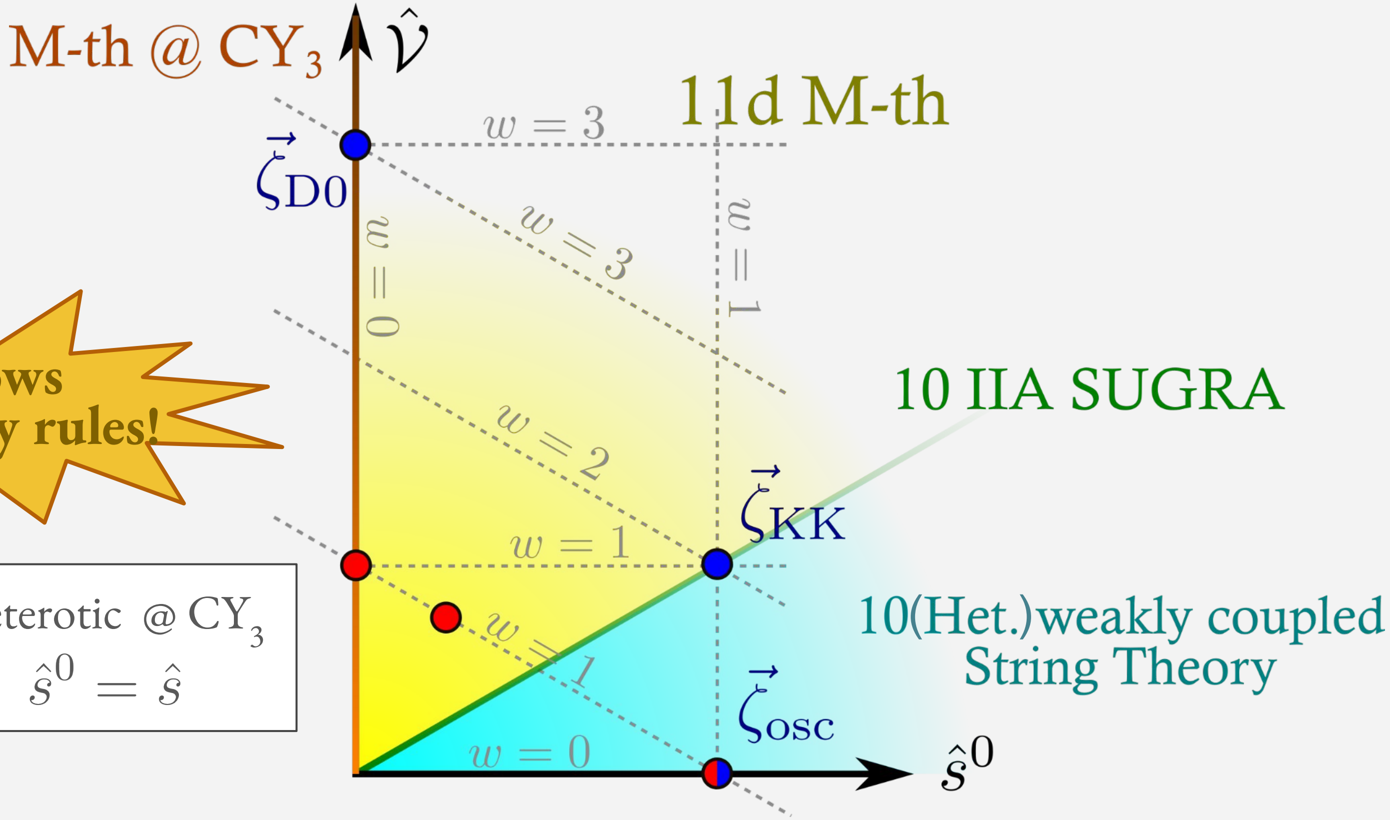

Follows taxonomy rules!

Realized on Heterotic @ CY$_3$

$\hat{\mathcal{V}} = \hat{u} \quad \hat{s}^0 = \hat{s}$

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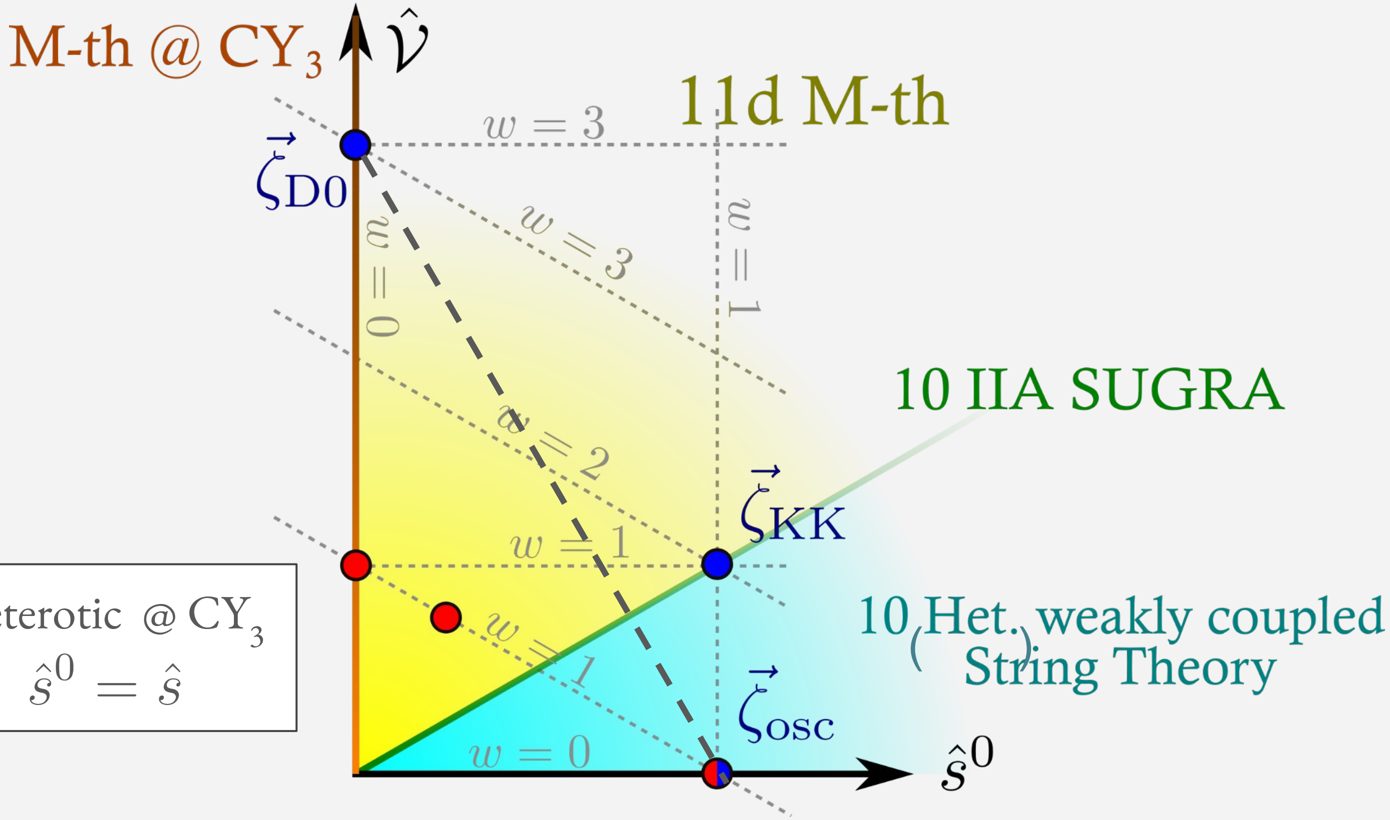

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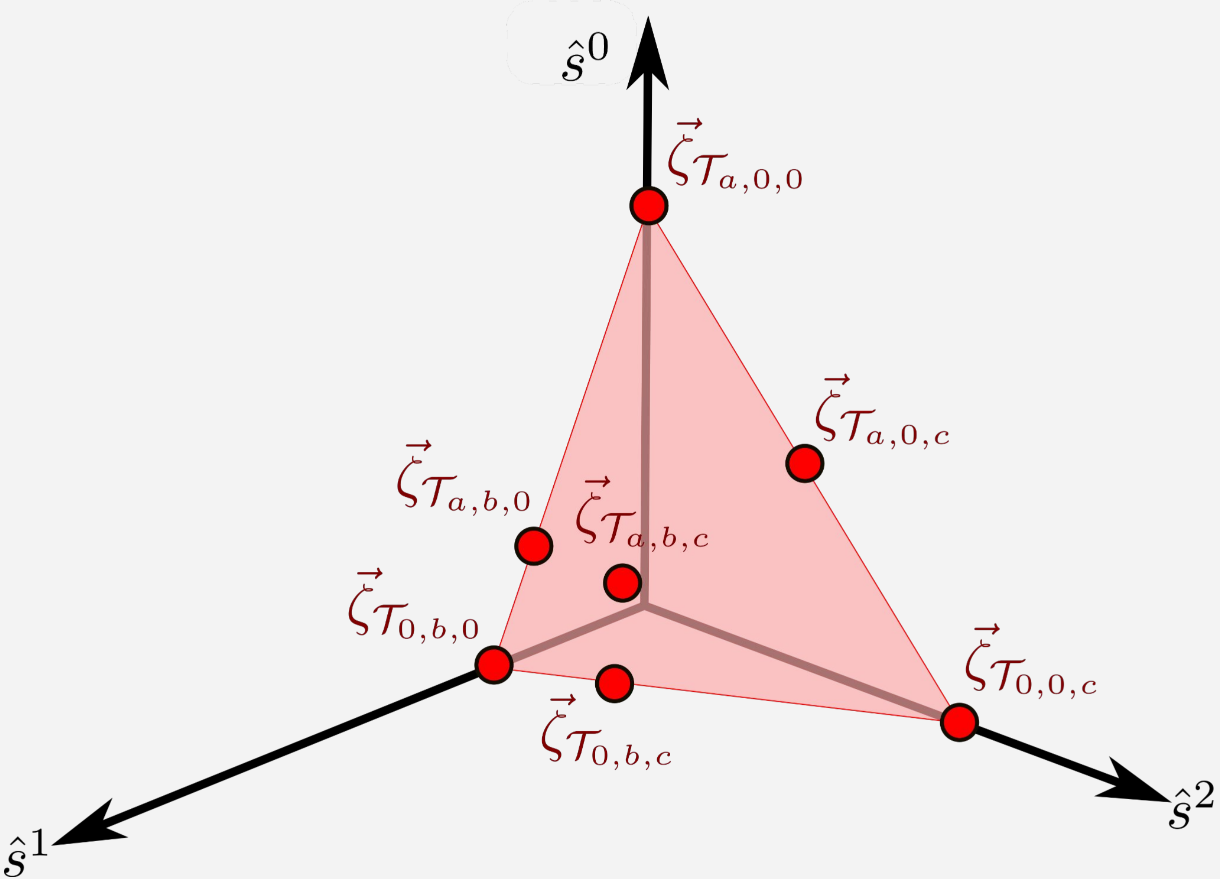
Example 2: $K = -\log\left(s^0 (s^1)^2 s^2\right)$



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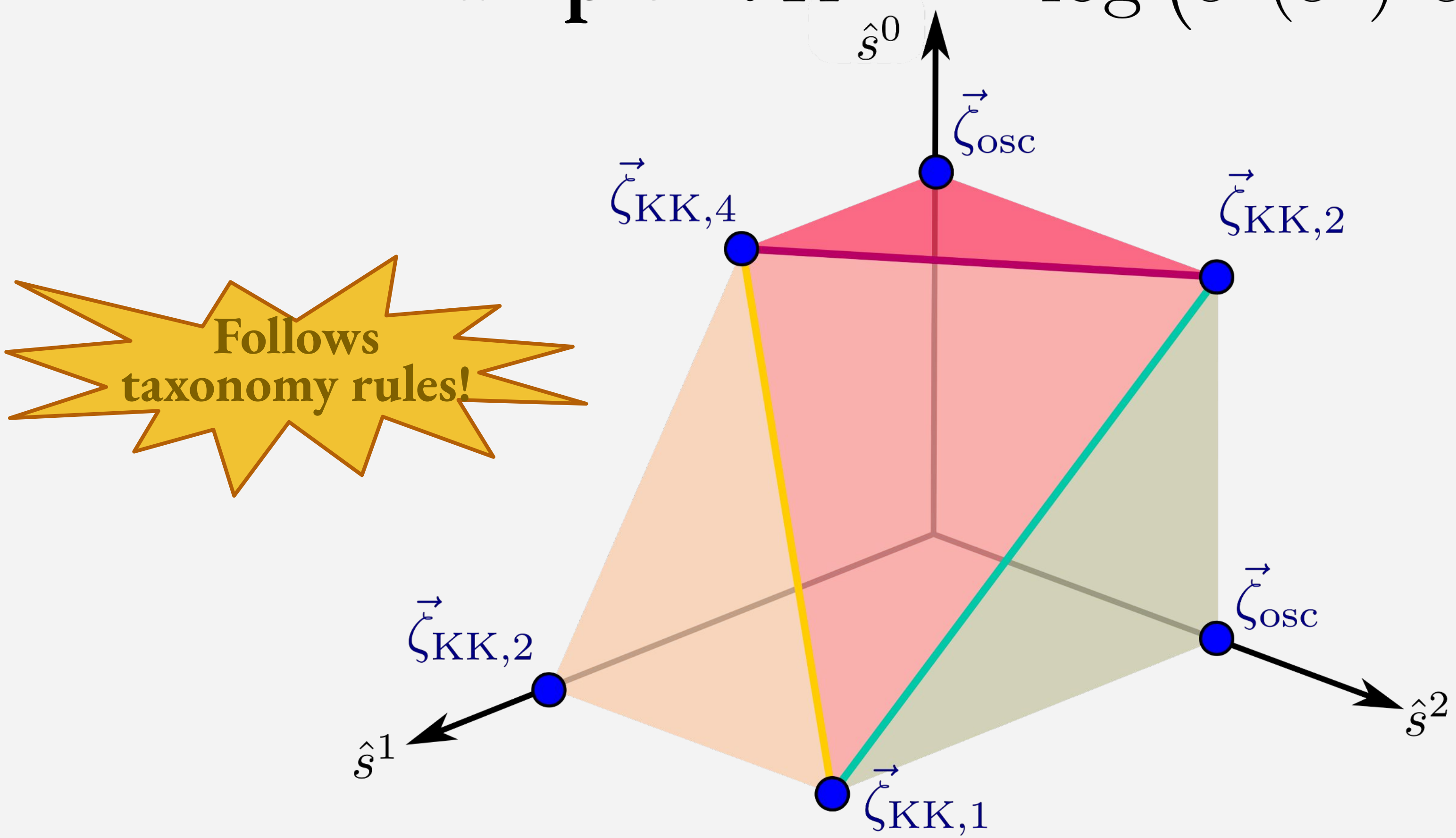

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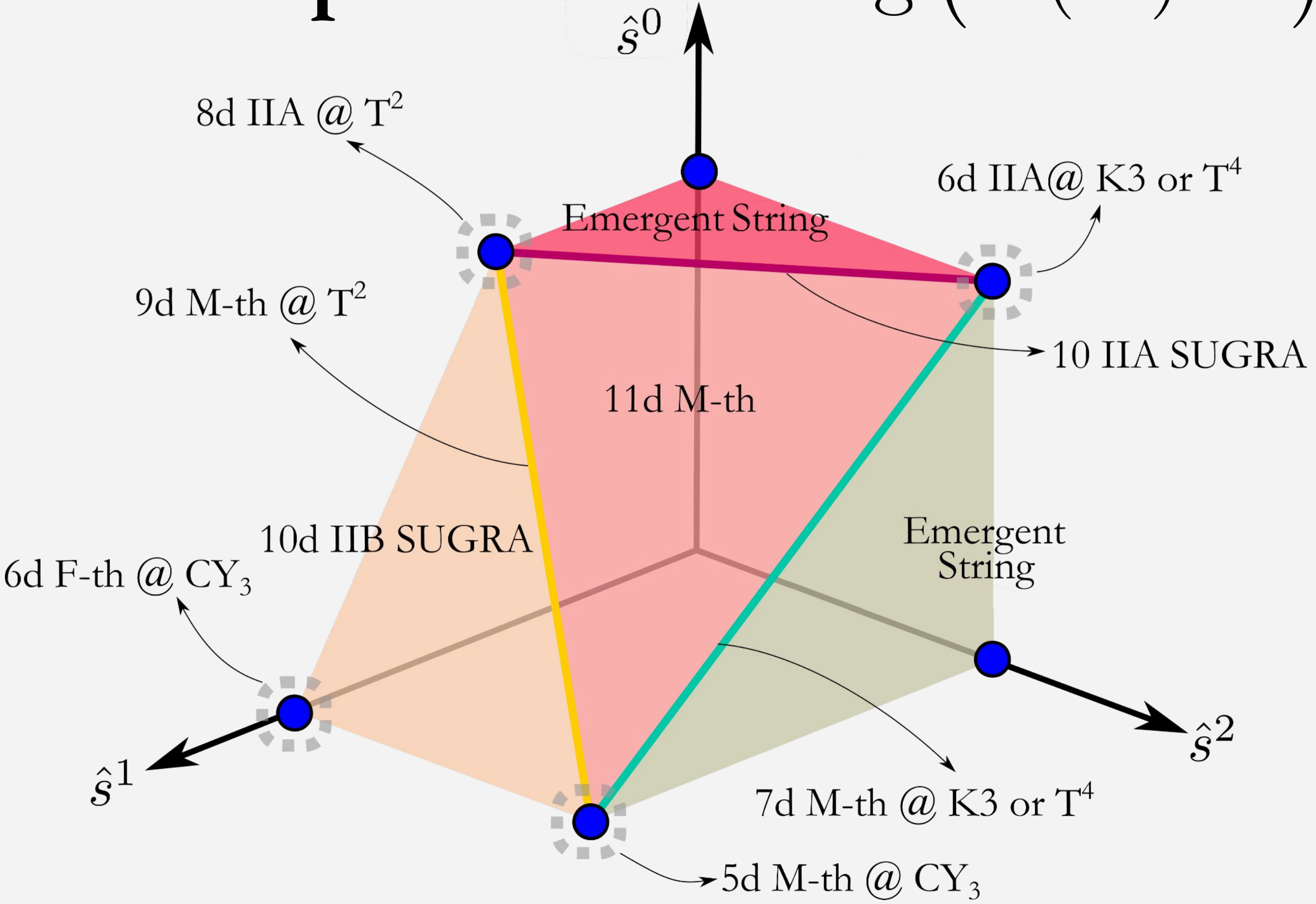

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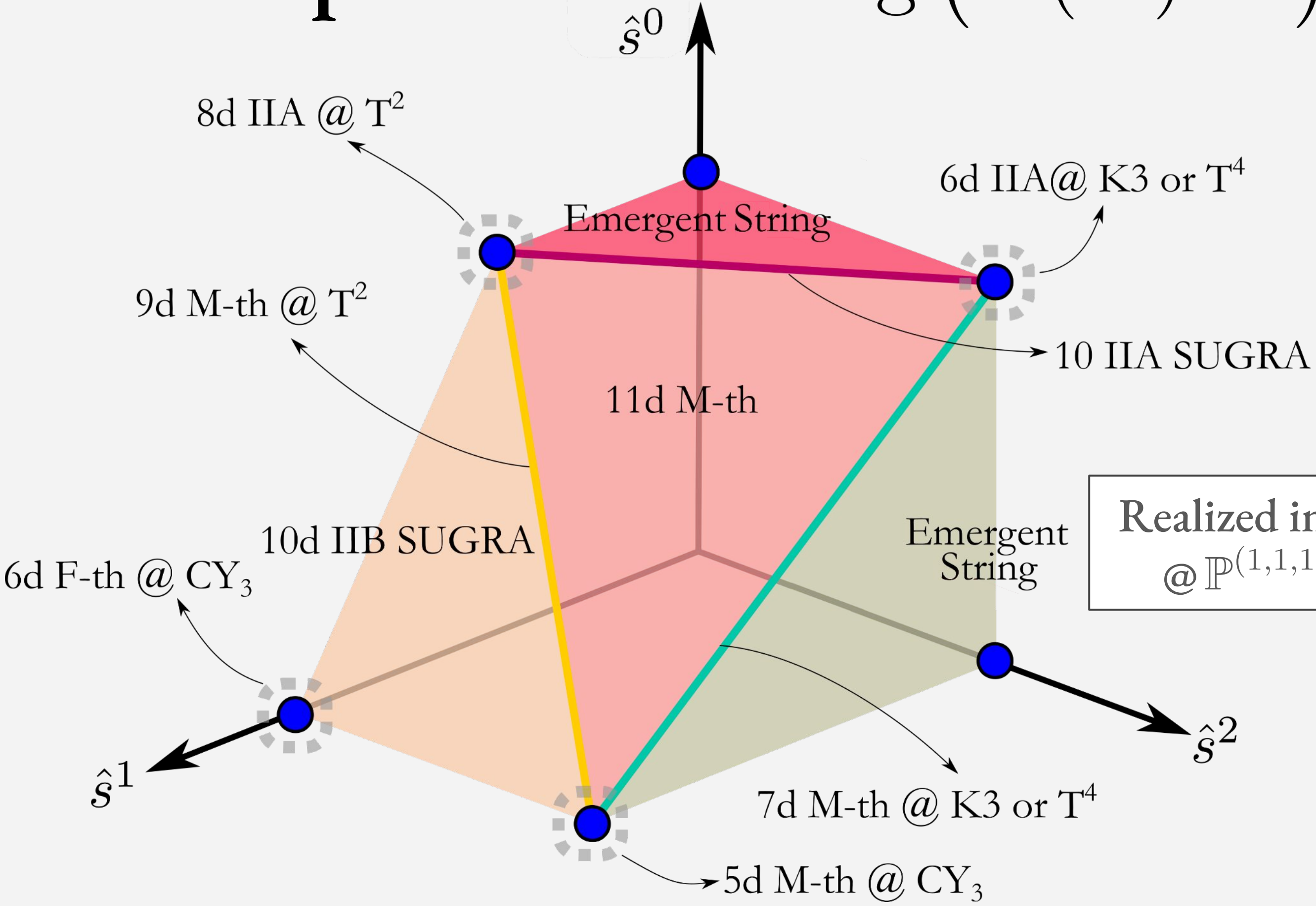

Realized in Heterotic @ $\mathbb{P}^{(1,1,1,6,9)}[18]$!

One moment!
Haven't we forgotten about something?

Extra ingredients in 4d $\mathcal{N}=1$

So far we have only considered **EFT strings** and **light towers**. We can have more!

Extra ingredients in 4d N=1

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- Runaway scalar potentials $V(\vec{\phi}) = e^K(\|DW\|^2 - 3|W|^2) > 0$ given in terms of super-potential $W = f_A \Pi^A + \ldots$. [Grimm,Li,Valenzuela,'19;Calderón-Infante,**I.R.**,Valenzuela,'22]

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- Extra (towers of) states becoming heavy in infinite distance limits! $m_I \sim e^{\alpha\phi} \to \infty$

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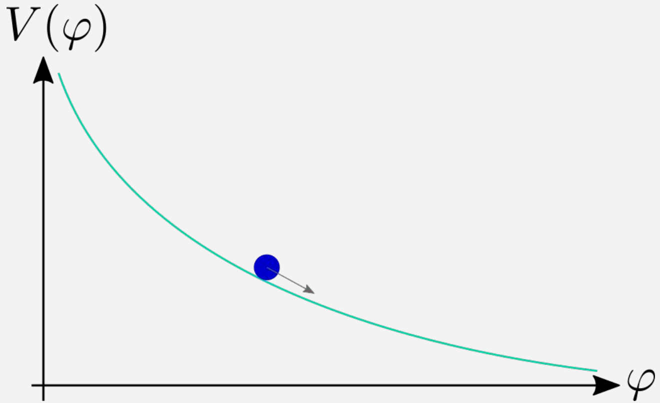

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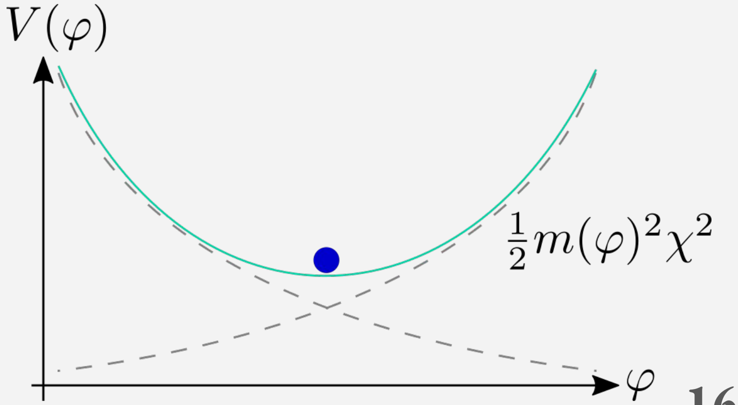
What if we combine both??

$$V_{\text{eff}}(\phi, t) = V_0 e^{-\lambda\phi} + \frac{1}{2} m_0^2 e^{2\mu\phi} \chi^2$$

Cosmological Chameleons: [Casas,Montero,**I.R.**,2406.07614]



See also
[Amendola,'99;Halyo,'01;Gomes,Hardy,Parameswara,'23]

Extra ingredients in 4d $\mathcal{N}$=1

So far we have only considered **EFT strings** and **light towers**. We can have more!

- Runaway scalar potentials $V(\vec{\phi}) = e^K(\|DW\|^2 - 3|W|^2) > 0$ given in terms of superpotential $W = f_A \Pi^A + \ldots$. [Grimm,Li,Valenzuela,'19;Calderón-Infante,**I.R.**,Valenzuela,'22]
- Extra (towers of) states becoming heavy in infinite distance limits! $m_I \sim e^{\alpha\phi} \to \infty$

Runaway potentials are **too steep** to accommodate (asymptotic) accelerated expansion.

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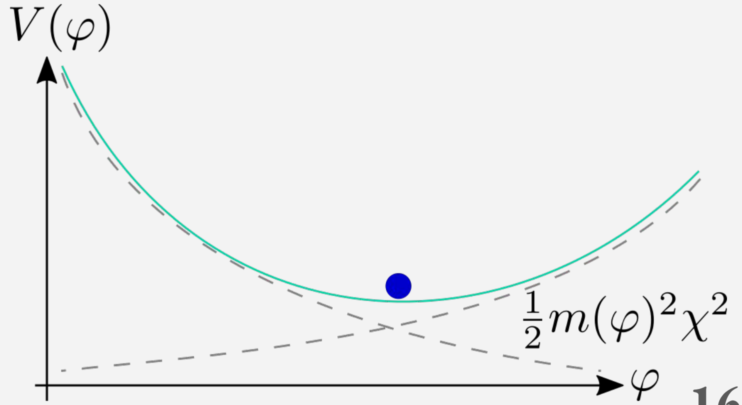

See also
[Amendola,'99;Halyo,'01;Gomes,Hardy,Parameswara,'23]

Extra ingredients in 4d N=1

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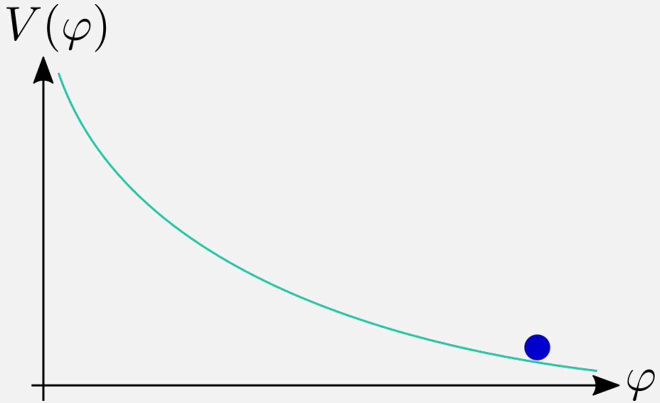


See also
[Amendola,'99;Halyo,'01;Gomes,Hardy,Parameswara,'23]

Transient quasi-dS with heavy states

In order to achieve transient dS epochs, we need terms states becoming heavy or potential terms growing in all directions:

$$V_{\text{eff}}(\vec{\phi},t) = \sum_J V_J(\vec{\phi}) + \frac{1}{2}\sum_I m_I(\vec{\phi})^2\chi_I^2(t)$$

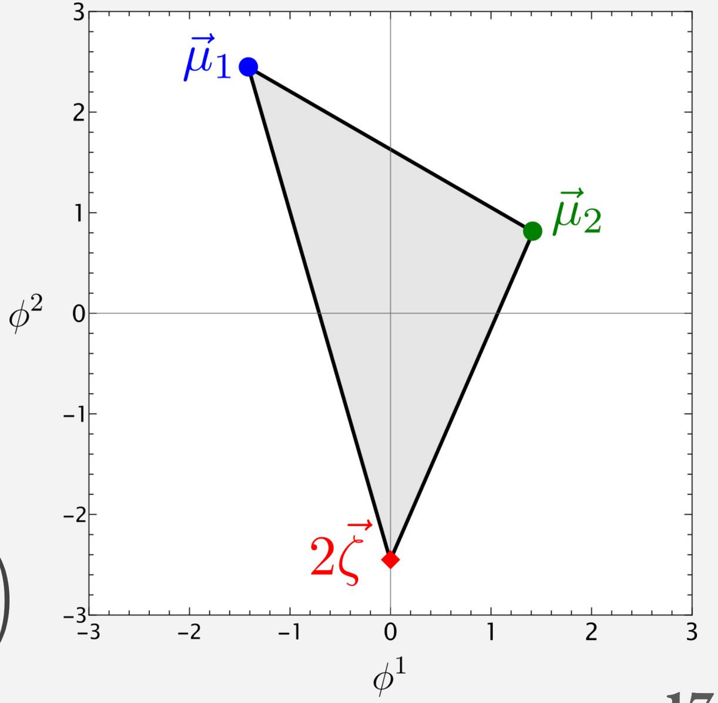
Given

$$\vec{\mu}_J = -\vec{\nabla}\log V_J(\vec{\varphi}) \quad \vec{\zeta}_I = -\vec{\nabla}\log m_I(\vec{\varphi})$$

[Calderón-Infante,**I.R.**,Valenzuela,'22]

transient minimum condition is given by

$$\vec{\nabla}V_{\text{eff}} = 0 \Leftrightarrow \vec{0} \in \text{Hull}\left(\{\vec{\mu}_J\}_J \cup \{2\vec{\zeta}_I\}_I\right)$$

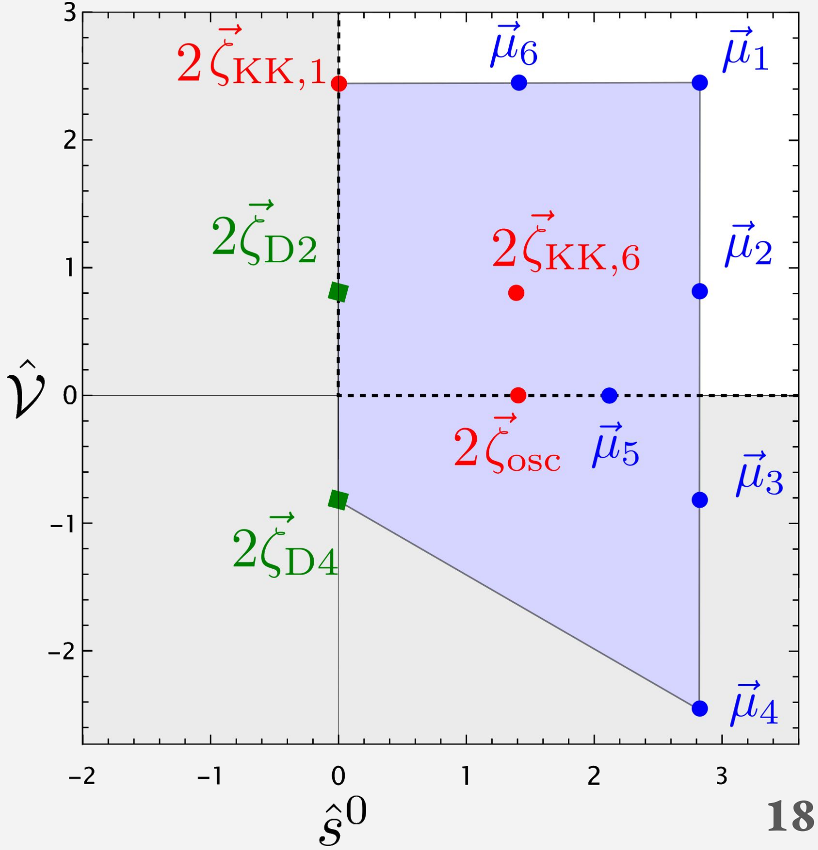


Transient quasi-dS with heavy states

Things tend to be slightly complicated on actual constructions:

Type IIA CY_3 orientifold compactification with flux potentials + light towers + heavy states:

Full stabilization not achieved!



Conclusions and Outlook

- In a similar manner to [2405.20332] Swampland arguments (**Integral Scaling Conjecture**) greatly constrains the asymptotic structure of 4d N=1 moduli spaces.
- All information can be read from EFT data (Kähler potential) → Universal structure given K?
- Easy generalization to $K = -\log P(s^j) + \ldots$ with more than one monomial.

Wrapping up!

Conclusions and Outlook

- In a similar manner to [2405.20332] Swampland arguments (**Integral Scaling Conjecture**) greatly constrains the asymptotic structure of 4d N=1 moduli spaces.
- All information can be read from EFT data (Kähler potential) → Universal structure given K?
- Easy generalization to $K = -\log P(s^j) + \ldots$ with more than one monomial.

Some work remains to be done!

- Microscopic understanding of ISC and origin of $w = 1, 2, 3$ scalings.
- Testing string universality on predicted duality structures.
- Extending this from single Kähler chamber → Let's allow for **flops**: This results in a new Kähler potential!
- General classification of heavy states from the UV.

Thanks your attention!

Back-up slides

Taxonomy rules:

$$\vec{\zeta}_a \cdot \vec{\zeta}_b = \frac{1}{d-2} + \frac{1}{n_a}\delta_{ab}$$

$$|\vec{\mathcal{Z}}|^2 = \frac{1}{d-2} + \frac{1}{d-2+n}$$

$$\vec{\zeta}_t \cdot \vec{\mathcal{Z}}_{\mathrm{QG}} = \frac{1}{d-2}$$

All geometry of $\vec{\zeta}$ and $\vec{\mathcal{Z}}_{\mathrm{QG}}$ arrangement is set with this!

Some assumptions

We can use the above expressions if the following is true:

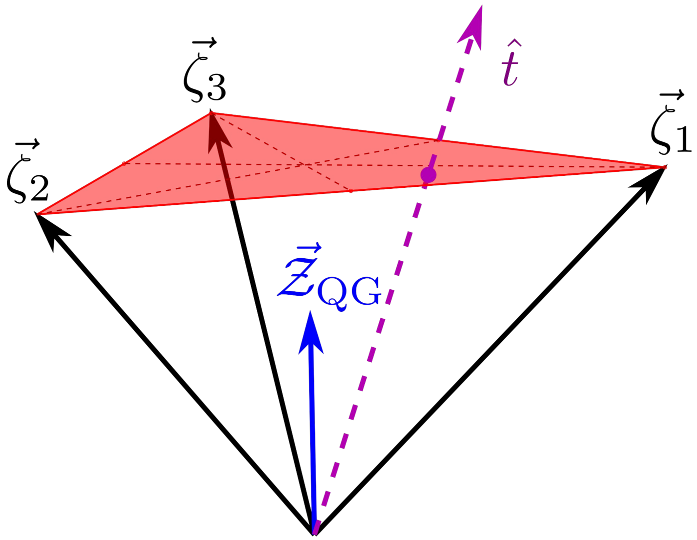
- The **Emergent String Conjecture** holds!
- In **decompactifications limits** the resulting spacetime manifold is **Ricci-flat** except in measure-zero regions (so **no defects** or **running solutions**).
- The above is true in the resulting EFT after decompactification: We can proceed in an **iterative manner**.

In order to be able to glue together the different **frames** we will need

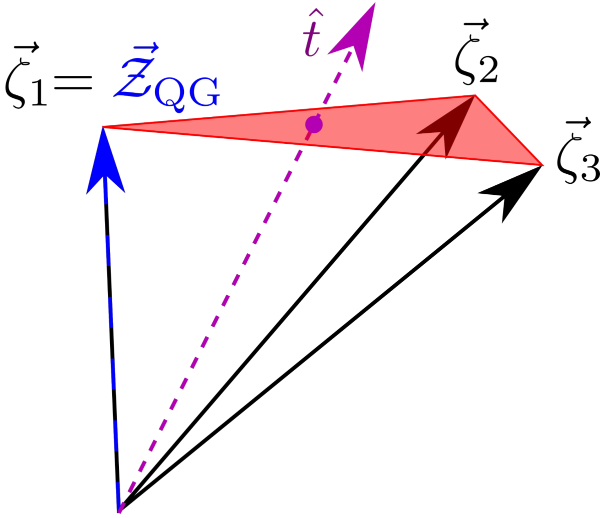
- There is an **asymptotically flat** slice of $\mathcal{M}$ to which the ζ-vectors are **tangent**.
- For generic limits the expression of the leading ζ-vectors is **constant** (so **no sliding**).

Putting things together

Neighboring tower vectors within a **duality frame** (same species scale) form a **frame simplex**:



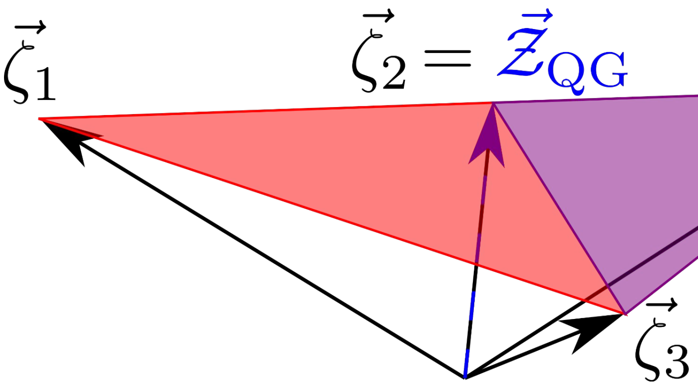
"Planckian" phase



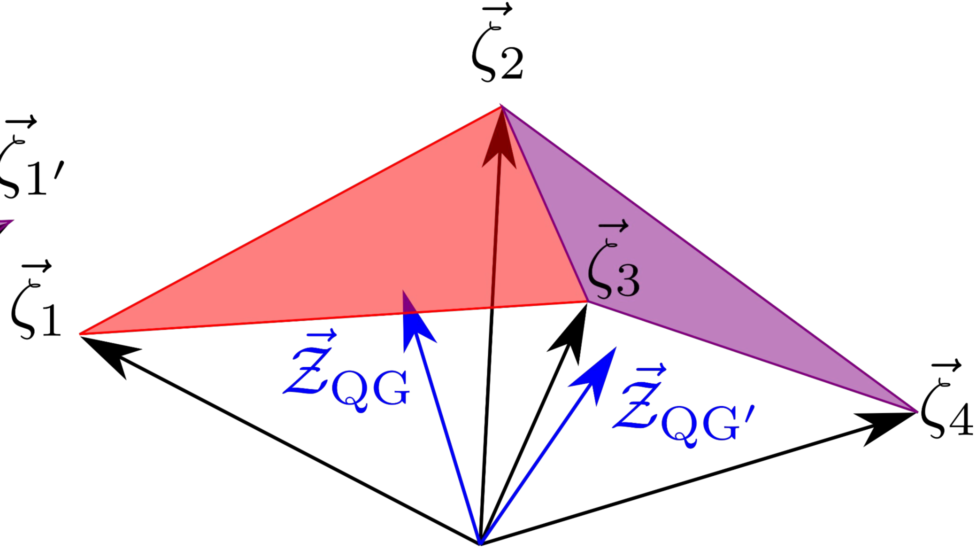
"Stringy" phase

Putting things together

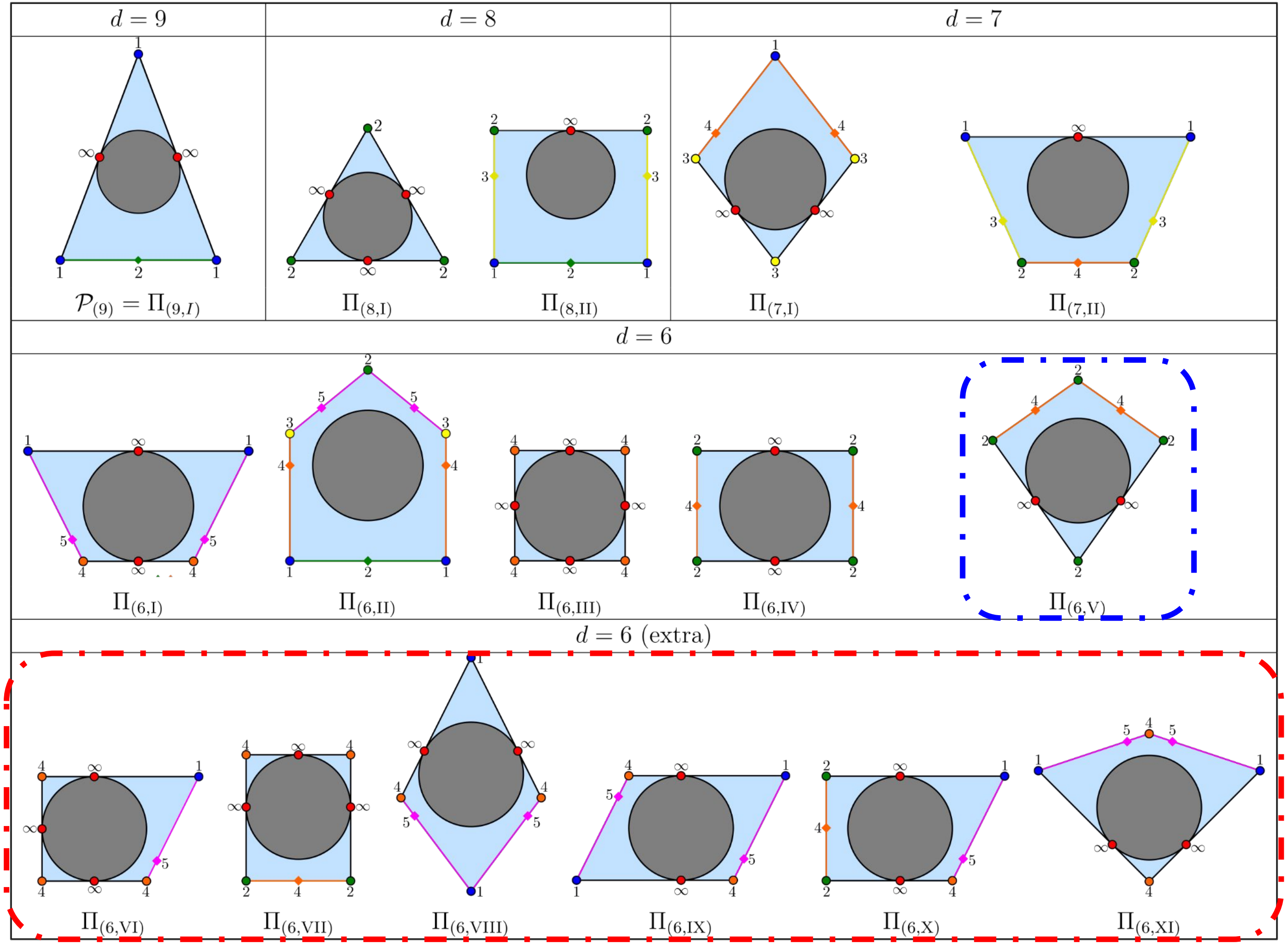
Under *relatively mild* assumptions we can **glue together** frame simplices into **frame complex**:



Generalized T-duality

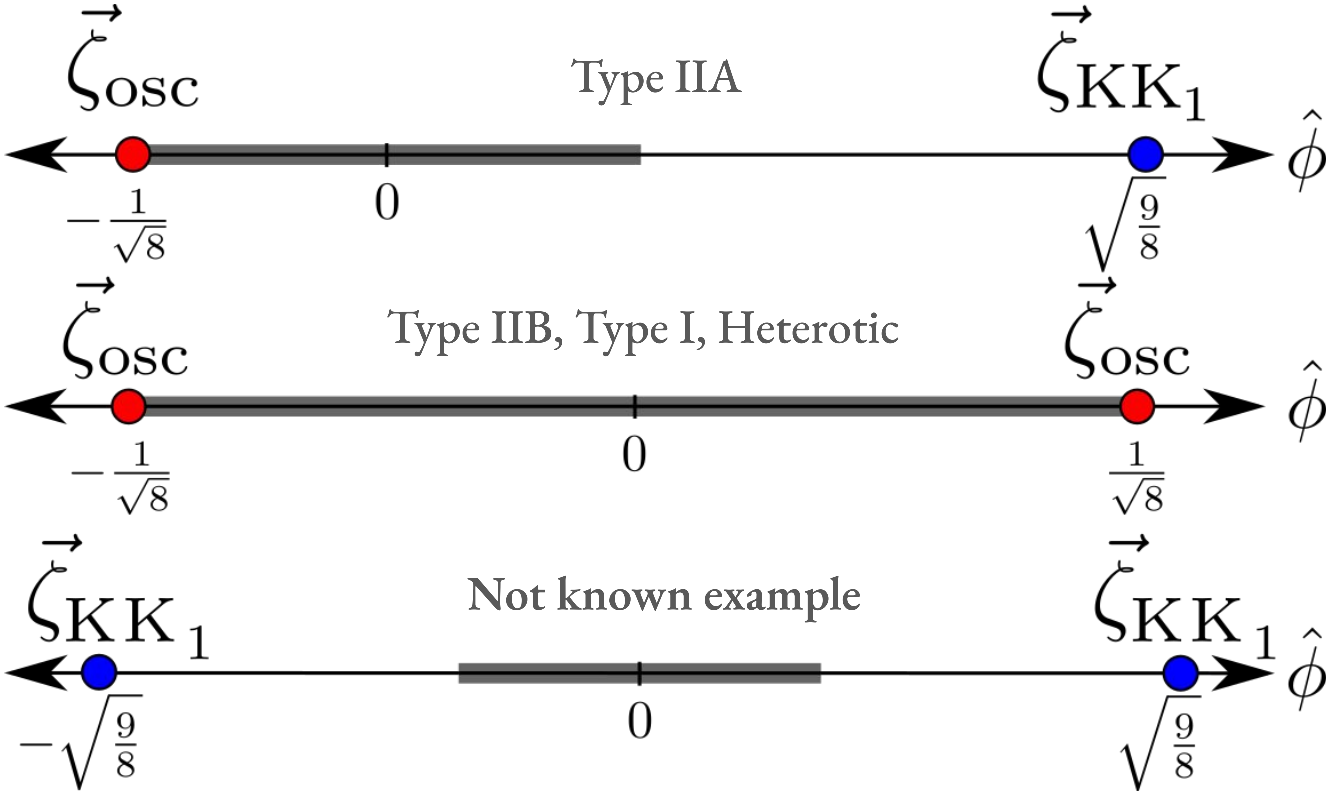


Generalized S-duality


d = 9
d = 8
d = 7
$\mathcal{P}_{(9)} = \Pi_{(9,I)}$
$\Pi_{(8,\mathrm{I})}$
$\Pi_{(8,\mathrm{II})}$
$\Pi_{(7,\mathrm{I})}$
$\Pi_{(7,\mathrm{II})}$
d = 6
$\Pi_{(6,\mathrm{I})}$
$\Pi_{(6,\mathrm{II})}$
$\Pi_{(6,\mathrm{III})}$
$\Pi_{(6,\mathrm{IV})}$
$\Pi_{(6,\mathrm{V})}$
d = 6 (extra)
$\Pi_{(6,\mathrm{VI})}$
$\Pi_{(6,\mathrm{VII})}$
$\Pi_{(6,\mathrm{VIII})}$
$\Pi_{(6,\mathrm{IX})}$
$\Pi_{(6,\mathrm{X})}$
$\Pi_{(6,\mathrm{XI})}$

Maximal SUGRA on $\mathbb{T}^n$

$d = 10$



Sliding of vertices

In general the expression of the ζ-vectors might not be constant, **sliding** as we vary the asymptotic geodesic.

Consider moving in a direction $\hat{t} = (\hat{t}^x, \hat{t}^y) = (1, 0)$ and $\vec{\zeta} = -\vec{\nabla} \log m(x, y)$. We expect a constant expression asymptotically:

$$\partial_x \zeta_x = \partial_x \zeta_y \to 0 \text{ as } x \to \infty \text{, so } \partial_y \zeta_x = \partial_x \zeta_y \to 0$$

Sliding only happens perpendicularly to asymptotic direction: depends on impact parameter

We expect sliding loci to be **measure-zero**, interpolating between regions following rules.

Non-complete moduli spaces: 4d $\mathcal{N}$=1 with $h^{1,1} = 2$

For lower dimensions, things get more complicated and in some cases some of the assumptions might no longer work.

Example: In CY3 compactifications we are required to stay within some (extended) Kähler cone.

$$\int_{\mathcal{C}} J > 0 \quad \int_{D} J \wedge J > 0 \quad \int_{CY_3} J \wedge J \wedge J > 0$$

$$B_2 + iJ = t^I w_I, \quad w_I \in H^2(CY3, \mathbb{Z}), \qquad t^I = \phi^I + is^I$$

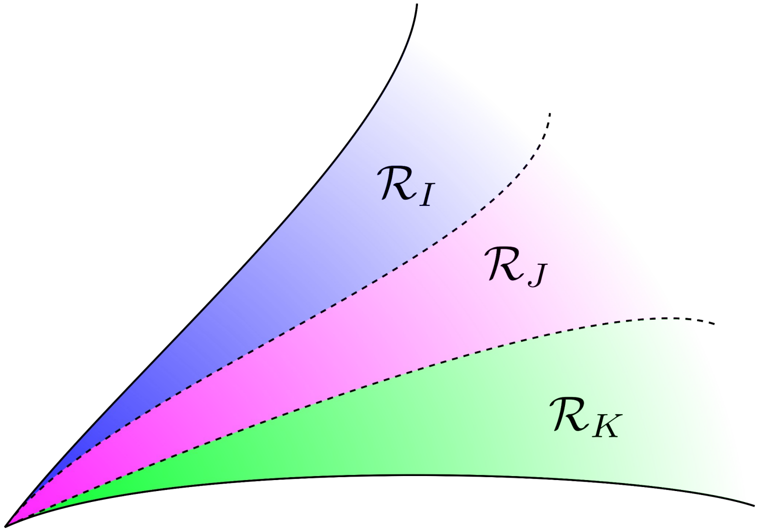
$$J = s^I w_I \in \mathcal{K}(CY3) \not\simeq \mathbb{R}^n$$

The relations between neighbouring ζ-vectors still apply. The rest of assumptions might apply locally: **We still can get some classification!**

Non-complete moduli spaces: 4d $\mathcal{N}$=1 with $h^{1,1} = 2$

We can divide our Kähler cone in **growth sectors**, where things greatly simplify:

$$\mathcal{R}_{i_1 \cdots i_n} = \left\{ t^{i_k} = \phi^{i_k} + i s^{i_k} : s^{i_1} \gg s^{i_2} \gg \cdots \gg s^{i_n} \gg 1, \phi^i < \delta \right\}$$



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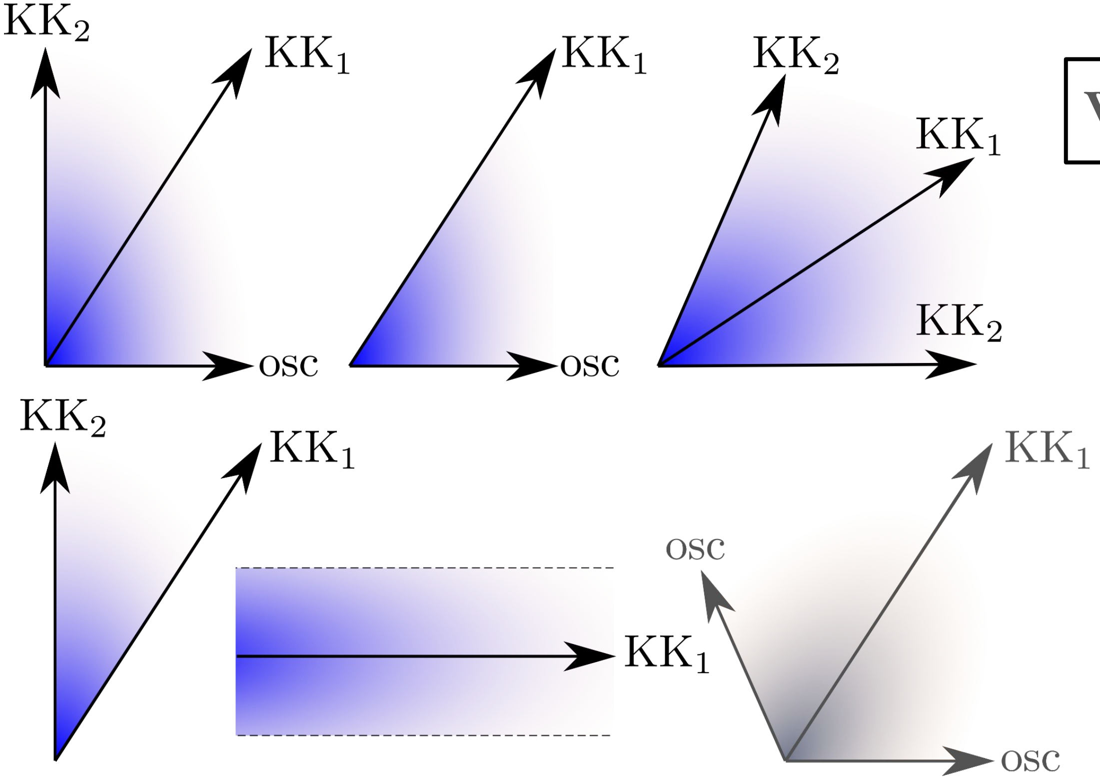
For $h^{1,1} = 2$ the saxionic metric is **always flat**: Transition functions are trivial!

In [Grimm, Ruehle, van de Heisteeg,'19] all possible leading towers when decompactifying saxions were classified to be

$$\vec{\zeta}_{\mathrm{KK},2} \qquad \vec{\zeta}_{\mathrm{KK},1} \qquad \vec{\zeta}_{\mathrm{osc}}$$

Non-complete moduli spaces: 4d N=1 with $h^{1,1} = 2$



We allow for flops!

[Gendler, Melotti, **I.R.**, Valenzuela, *WIP*]