- ABSTRACTS -

Operator related Function Theory
Erwin Schrödinger International Institute
for Mathematics and Physics
Vienna, AUSTRIA
April 8 to 12, 2019

Ameur, Yacin (University of Lund, Sweden)

A scale of boundary conditions for the Coulomb gas

In the theory of the Coulomb gas (in any dimension) it is natural to impose different kinds of boundary conditions. So far, it has been traditional to study either a "hard edge" where no particle is admitted to range outside of the droplet, or a "free boundary" where particles are admitted to range outside this set, but due to the influence of a strong confining potential, most of them will tend to stay in a close vicinity of it.

In my talk, I will discuss a new scale of boundary conditions which contains the free boundary and hard edge confinements as special cases. This scale has the interesting feature that it extends beyond the free boundary to produce boundary conditions that are more and more relaxed, i.e., the particles will be more and more likely to venture a bit outside of the droplet. We will in particular discuss the possibility of defining a limiting case of an "ultrasoft" boundary, when the droplet is just about ready to explode.

(This is joint work with Nam-Gyu Kang and Seong-Mi Seo.)

Badea, Catalin (University of Lille, France)

Kazhdan sets: between geometric group theory, harmonic analysis and operator theory

The notion of Kazhdan subset in a topological group comes from the study of groups with Kazhdan's Property (T). In my talk, I will survey some recent results about Kazhdan subsets of the integers obtained using operator-theoretical and harmonic analysis methods. In particular, I will discuss a counter-example to a conjecture of Russell Lyons (1988) inspired by the famous $\times 2$, $\times 3$ conjecture of Furstenberg. This is based upon joint works, one with Sophie Grivaux and one with Grivaux and Étienne Matheron.

Baranov, Anton (State University of St. Petersburg, Russia)

Differentiation invariant subspaces in the space of infinitely differentiable functions

In the space of all infinitely differentiable functions on an interval (a, b) consider a differentiation invariant subspace and assume that the restriction of differentiation onto this subspace has discrete spectrum. Is it true that in this case the subspace is generated by the exponential monomials it contains? In general, the answer is negative, since the subspace may have the so-called "residual" part (all functions vanishing on some subinterval). In 2007 A. Aleman and B. Korenblum posed the spectral synthesis problem: is any invariant subspace generated by its residual part and the corresponding exponentials? We give a complete description of subspaces which admit spectral synthesis in terms of their spectra. The talk is based on joint works with A. Aleman and Yu. Belov.

Belov, Yuri (State University of St. Petersburg, Russia)

Ordered structure for Cauchy-de Branges spaces

We obtain new versions of de Branges Ordering Theorem for nearly invariant subspaces in a class of Hilbert spaces of entire functions. In particular, we extend some results of M.G. Krein to the class of entire functions which can be represented as ratios of discrete Cauchy transforms in the plane. This is joint work with E. Abakumov and A. Baranov.

Borichev, Alexander (University of Marseille, France)

Szeg minimum problem

Given a finite positive measure μ on the unit circle, consider the distance $e_n(\mu)$ from z^n to the analytic polynomials of degree less than n in $L^2(\mu)$. We study the asymptotic behavior of $e_n(\mu)$ when the logarithmic integral of the density of μ diverges.

Brevig, Ole Fredrik (Norwegian Technical University, Trondheim)

Weissler's inequality for Dirichlet series

We explain how to obtain a "good" inequality by using a "bad" inequality an infinite number of times. The inequality enables a kind of interpolation between Hardy spaces of Dirichlet series using completely multiplicative weights. Several applications will be discussed, including high pseudomoments of the Riemann zeta function and Hardy–Littlewood inequalities.

Carlsson, Marcus (Lund University)

Nehari's theorem for convex domain Hankel operators in several variables

Nehari's theorem from 1957 gives an exact condition for boundedness of Hankel operatorswhich operate on L^2 of the positive real axis R^+. About 30 years later the result was extended by R. Rochberg to a more general class of Hankel operators, but still confined to one dimension. More precisely, he proves a version of Nehari's theorem for Hankel and Toeplitz operators on the Paley-Wiener space, which can be seen as a restriction of classical Hankel operators to an interval. The proof relies on Nehari's original theorem. The quest for a multivariable version was open for another 15 years until M. Lacey, S. Fergusson and E. Terwilleger proved an extension of Neharis theorem to L^2 of $(R^+)^n$, where n is an integer. Together with K-M. Perfekt we have generalized this result to the cube, in analogy with Rochbergs extension in the 1d setting. We also prove the theorem for more general convex polytopes, but the question of whether convex polytopes can be replaced with more general domains, remains open.

Haimi, Antti (Austrian Academy of Sciences)

Strict density inequalities for sampling and interpolation in weighted spaces of holomorphic functions

Answering a question of Lindholm, we prove strict density inequalities for sampling and interpolation in Fock spaces of entire functions in several complex variables defined by a plurisubharmonic weight. In particular, these spaces do not admit a set that is simultaneously sampling and interpolating. To prove optimality of the density conditions, we construct sampling sets with a density arbitrarily close to the critical density. The techniques combine methods from several complex variables (estimates for the dbar operator) and the theory of localized frames in general reproducing kernel Hilbert spaces (with no analyticity assumed). The abstract results on Fekete points and deformation of frames may be of independent interest.

Joint work with Karlheinz Gröchenig, Joaquim Ortega-Cerdà, José Luis Romero

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Harper, Adam J. (University of Warwick, UK)

High moments of random multiplicative functions

Random multiplicative functions $f(n)$ are a probabilistic model for certain interesting number theoretic functions, such as the Mobius function and Dirichlet characters. When $0 \leq q \leq 1$, it turned out that the moments $E|\sum_{n\leq x} f(n)|^{2q}$ were connected with the notion of multiplicative chaos from physics and probability. This connection led to new information on Helson's conjecture and the so-called embedding problem for Dirichlet polynomials. In this talk, I will discuss the behaviour of the moments in the other regime where $q > 1$. Here one sees some other interesting phenomena, especially for Rademacher random multiplicative functions where there is a "unitary to orthogonal" transition as q grows.

Hartz, Michael (FernUniversitaet-Hagen, Germany)

Smirnov class and common range of adjoint multipliers in the Drury-Arveson space

The classical Smirnov class in the unit disc consists of all quotients of bounded analytic functions whose denominator is an outer function. The dual space of the Smirnov class can be used to characterize the common range of all co-analytic Toeplitz operators on the Hardy space H^2.

I will talk about a generalization of the Smirnov class to the Drury-Arveson space H^2_d on the unit ball in $\mathbb{C}^d$. Moreover, I will explain how it sheds light on the common range of adjoints of multiplication operators on H^2_d. This is joint work with Alexandru Aleman, John McCarthy and Stefan Richter.

Heap, Winston (University College London, UK)

The maximum of the Riemann zeta function on the 1 - line

We consider upper bounds of the Riemann zeta function on the 1-line and demonstrate a link with the maximum of the function $S(t)$ - the remainder in the formula for the number of non-trivial zeros of height t in the critical strip. In particular, we show that a conjecture of Littlewood on the maximum of $\zeta(1 + it)$ follows from a folklore conjecture on the max of $S(t)$.

Hedenmalm, Hakan (KTH Stockholm)

Planar orthogonal polynomials and related determinantal processes: random normal matrices and arithmetic jellium

We obtain a new asymptotic expansion of planar orthogonal polynomials with respect to exponentially varying weights. This goes beyond the classical contributions by Carleman and Suetin. We apply the results to get the spectral boundary behavior of the eigenvalues of random normal matrices. We also introduce a new process: arithmetic jellium.

Jaming, Philippe (Universite de Bordeaux, France)

Mean convergence of prolate spheroidal wave function expansions

Prolate spheroidal wave functions form an orthonormal basis of the Paley-Wiener space that is optimaly concentrated in space. There are several extensions of them (weighted versions, Hankel prolates for higher dimensional radial analysis) The aim of this talk is to show that expensions in this basis converge in L^p for a certain range of ps and diverge outside this range.

This is base on joined work M. Boulsane, Ph. Jaming and A. Souabni, Mean convergence of prolate spheroidal series and their extensions, arxiv:1804.00851

Nicolau, Artur (Universitad Autonoma de Barcelona, Spain)

The Corona Theorem in Nevanlinna quotient algebras and interpolating sequences.

Let H be the algebra of bounded analytic functions in the unit disc and let I be an inner function. In 2007, Gorkin, Mortini and Nikolski studied the Corona problem in the quotient algebra H/IH and proved that there is no corona if and only if I satisfies the so called weak embedding property. We discuss an analogue problem for quotients of the Nevanlinna class and show that in contrast with the previous case, a complete answer can be given in terms of interpolating sequences. This is joint work with Xavier Massaneda and Pascal Thomas.

Nikolskii, Nikolai (University of Bordeaux, France)

On the sign distribution of Hilbert space frames

It is shown that the positive and negative parts $u_k(x)\pm$ of a frame in a real L^2 space with respect to a continuous measure have both infinite l^2masses: 1) always, the series of squares of $u_k(x)\pm$ diverge almost everywhere (in particular, there exist no positive frames, nor Riesz bases), but 2) the partial sums of the differences $(u_k(x)+) - (u_k(x)-)$ can grow locally as slow as we wish, and 3) it can happen that partial sums of $u_k(x)-$ are "much

less" than the sums of $u_k(x)+$ on a set of positive measure. Property 1) for orthonormal bases in $L^2(0,1)$ was settled earlier by V.Ya.Kozlov (1948) and (independently) for unconditional bases by F.Arutyunian (1966) and A.Powell and A.Spaeth (2016), each time using completely different (and more involved) arguments. Our elementary treatment includes also the case of unconditional bases in a variety of Banach spaces. For property 2), we show that, moreover, whatever is a decreasing sequence $c_k > 0$ does not belonging to l^2, there exists an orthonormal basis (u_k) in $L^2(0,1)$ such that $|u_k(x)| < A(x)c_k, 0 < A(x) < \infty$. The talk is based on a joint work with A.Volberg.

Nitzan, Shahaf (Georgia Institute of Technology, USA)

Uncertainty Principles for Fourier Multipliers

We quantify the admitable Sobolev regularity of functions w which have a zero in the d–dimensional torus and whose reciprocals $u = 1/w$ are (p, q)–multipliers. Our results address several aspects of this problem including zero–sets of positive Hausdorff dimension, matrix valued Fourier multipliers, and non–symmetric versions of Sobolev regularity. We apply these results on Fourier multipliers to refine and extend versions of the Balian–Low uncertainty principle for Gabor systems and shift–invariant spaces with various approximation properties. This is joint work with Michael Northington and Alex Powell.

Olsen, Jan-Fredrik (University of Lund, Sweden)

Balian-Low type theorems for finite sequences

In this talk, based on a joint work with Shahaf Nitzan, we formulate and prove finite dimensional analogues for the classical Balian-Low theorem as well as for a quantitative version previously obtained by Nitzan and Olsen.

Ortega-Cerdà, Joaquim (University of Barcelona, Spain)

A sequence of polynomials with optimal condition number

We find an explicit sequence of polynomials of arbitrary degree with optimal condition number. This solves a problem posed by Michael Shub and Stepen Smale in 1993. This is a joint work together with Carlos Beltran, Ujue Etayo and Jordi Marzo

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Poltoratskii, Alexei (Texas A&M University, USA)

Inverse spectral problems for canonical Hamiltonian systems

The Krein-de Branges theory was created in 1940-60s to apply methods of complex analysis to inverse spectral problems for second order differential equations. A recent approach based on the use of truncated Toeplitz operators has produced new results and formulas in the theory. In my talk I will discuss some of such results along with new examples of explicit solutions to inverse spectral problems. The talk is based on joint work with N. Makarov.

Pushnitski, Alexander (King's College London, UK)

An inverse spectral problem for Hankel operators

The main object of the talk is a finite rank Hankel operator H acting on the Hardy space in the upper half-plane. It turns out that a detailed spectral analysis of H displays interesting features related to the geometry of subspaces of the Hardy space. The Schmidt subspaces of H can be described as weighted model spaces in the Hardy space. I will describe various parameters (singular values, inner functions, weights, etc) appearing in the spectral analysis of H and indicate how the symbol of H can be reconstructed from these parameters. This is recent joint work with Patrick Gerard (Orsay).

Queffélec, Hervé (University of Lille, France)

Compactification, and beyond, of composition operators by weights

Weighted composition operators are often companions (e.g.adjoints) to composition operators. Recently, with G.Lechner, we examined the effect of a weight on the rate of decay of singular numbers. Here, in connection with the extreme, or exposed, points of the unit ball of H^∞, we will examine to which extent one can compactify, nuclearize, insert in a Schatten class, etc... a given composition operator on a Hardy space H^p thanks to a weight. This is joint work with P.Lefevre, D.Li, L.Rodriquez-Piazza.

Richter, Stefan (University of Tennessee, Knoxville, USA)

Free outer functions in complete Pick spaces

Let H be a Hilbert function space with normalized complete Pick kernel such as the Hardy or Dirichlet spaces of the unit disc, or the Drury-Arveson space of the unit ball of C^d. It is known that H can be isometrically embedded in the free Fock space, and via this embedding every function in H has an abstract "inner-outer" factorization. We express this factorization intrinsically in terms of the space H, and we show some applications.

Ross, William T. (University of Richmond, USA)

Range spaces of Toeplitz operators

In this joint work with Andreas Hartmann and Emmanuel Fricain we explore the range spaces of certain Toeplitz operators on the Hardy space. These range spaces make connections to the de Branges-Rovnyak spaces and model spaces. Along the way, we explore the multipliers from one range space to another.

Saksman, Eero (University of Helsinki, Finnland)

Decompositions of log-correlated fields with applications

We consider a simple idea to decompose of log-correlated Gaussian fields into two-parts, both of which behave well in suitable sense. Applications include Onsager type inequalities in all dimensions, analytic dependence and existence of critical chaos measures for a large class of log-correlated fields. Talk is based on joint work with Janne Junnila (EPFL) and Christian Webb (Aalto University).

Teschl, Gerald (Univ. Vienna)

Riemann–Hilbert problems for the Korteweg–de Vries equation

We discuss some general aspects and techniques associated with the long time asymptotics of steplike solutions for the Korteweg–de Vries equation via vector Riemann–Hilbert problems. We also discuss an ill-posedness of the matrix RiemannHilbert problem for the KdV case. Based on joint work with Iryna Egorova and Mateusz Piorkowski.

Vukotic, Dragan (Universidad Autonoma de Madrid)

Co-isometric and invertible weighted composition operators on general spaces of analytic functions

Weighted composition operators are a natural generalization of multiplication operators and composition operators and have close relationships with the description of isometries of various function spaces of Lebesgue type (in the spirit of Banach, Lamperti, Forelli, and Kolaski) and with the celebrated Brennan conjecture (as was shown by Shimorin and Smith). Our point of view is to avoid considering such operators on various individual spaces (in many similar contexts) and, thus, avoiding repetitions of results by considering -in one stroke- families of rather general spaces of analytic functions defined in an axiomatic way.

One part of the talk is a joint work with Maria J. Martin and Alejandro Mas. We consider a family of weighted Hardy spaces (as considered by Shields and others), i.e., Hilbert spaces of analytic functions in the unit disk whose reproducing kernels are assumed to have the usual natural form. Under such general assumptions, we obtain a

necessary and sufficient condition for a weighted composition operator to be co-isometric, which also turns out to be equivalent to the operator being unitary. As a consequence, we identify a specific family of weighted Hardy spaces as the unique spaces that support non-trivial operators of this kind. This extends various results by Le, Zorboska, and other authors.

In the remaining part of the talk (as time permits), we will also discuss a joint work with Irina Arevalo (possibly with some further new observations) regarding invertible weighted composition operators on general Banach spaces of analytic functions that satisfy only a handful of natural axioms. This complements or extends various recent results by Bourdon and by Lindstrom et al.