3+1-dimensional gravity as a quantum effect in the IKKT matrix model

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ESI Vienna, september 2023

consider the IKKT or IIB model as fundamental starting point

can we get $3+1$ D (near-)realistic physics from IKKT ?

insist: only finitely many d.o.f. in finite volume

$\Rightarrow$ no compactification ! (otherwise $\rightarrow$ field theory, ill-def)

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weakly coupled gauge theory on $3+1$ -dim. NC branes

physical modes on brane, nothing escapes into bulk (weak coupling!)
no compactification of target space!

quantum effects $\rightarrow$ IIB SUGRA in bulk (holographic dual, unphysical!)

= weak, short-range r^{-8} interaction on brane

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- novel mechanism for 3+1 gravity on branes $\mathcal{M}^{3,1} \times \mathcal{K}_N \subset \mathbb{R}^{9,1}$
  - 1-loop $\to$ induced E-H action on brane
    for eff. metric = open string metric
  - UV finite, reasonable cosmology without fine-tuning
  - $\mathcal{K}_N$ finite, gives structure to low-energy gauge theory
  - no compactification of target space, no landscape problem
- NC brane = quantized symplectic spaces = "fuzzy space(time)":
  - natural class of BG with $\int \Omega \sim \dim \mathcal{H}$
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IKKT = IIB Matrix Model

Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S = Tr([Y^a, Y^b][Y_a, Y_b] \ + \bar{\Psi}\Gamma_a[Y^a, \Psi])$$

maximal SUSY, closely related to IIB string theory

- gauge invariance $Y^a \to U^{-1} Y^a U$
- class. solutions / backgrounds = branes
  space-time = suitable 3+1d brane $\bar{Y}^a$
- geometric fluctuations $\to$ "pre-gravity" $\neq$ GR
- quantization:

$$Z = \int dY d\Psi e^{iS[Y,\Psi]}$$

1-loop $\to$ induced Einstein-Hilbert term, 3+1d gravity

unique model without pathological UV/IR mixing
in $3+1$ dimensions (maximal SUSY in UV)

outline:

- geometric interpretation of Yang-Mills matrix models
- geometrical structures: frame, metric, torsion
- **quantization**: 1-loop effective action
  $\to$ Einstein-Hilbert action (+ extras)
- covariant quantum space-time $\mathcal{M}_n^{3,1}$

introductory review: arXiv:1911.03162
quantization & E-H action: arXiv:2303.08012, 2110.03936

book "Quantum Geometry, Matrix Models, and Gravity" (very soon)

geometric interpretation of Yang-Mills matrix models:

$$S = Tr([Y^a, Y^b][Y_a, Y_b] \; + ...)$$

“almost-commuting” matrix config’s = quantized symplectic spaces

expect: dominant configs = “almost-commuting” matrix configurations

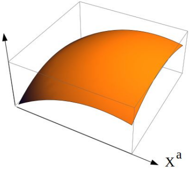
$$[Y^a, Y^b] \approx 0$$

Y^a generates algebra of functions $\mathrm{End}(\mathcal{H}) \sim \mathcal{C}(\mathcal{M})$

$$[Y^a, Y^b] \sim i\{y^a, y^b\}$$

matrix configuration / solution $Y^a \in \mathrm{End}(\mathcal{H})$ interpreted as

$$Y^a \sim y^a : \quad \mathcal{M} \to \mathbb{R}^D$$



$(\mathcal{M}, \omega)$ ... symplectic manifold (“brane”)

IR: semi-classical correspondence

$$\begin{aligned} \mathrm{End}(\mathcal{H}) &\sim \mathcal{C}(\mathcal{M}) \\ \Phi &\sim \phi(y) \\ [\Phi, \Psi] &\sim i\{\phi, \psi\} \\ \mathrm{Tr}\Phi &\sim \int_{\mathcal{M}} \Omega\phi, \qquad \Omega \ \dots \ \text{symp. volume} \end{aligned}$$

more precisely: approximate isometry below some scale

$$\begin{array}{ccc} \mathrm{End}(\mathcal{H}) & & \mathcal{C}(\mathcal{M}) \\ \cup & & \cup \\ \mathrm{Loc}(\mathcal{H}) & \cong & \mathcal{C}_{\mathrm{IR}}(\mathcal{M}) \end{array}$$

"almost-commutative" = sufficiently large semi-classical IR regime

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\end{array}
$$

"almost-commutative" = sufficiently large semi-classical IR regime

The effective metric in matrix models

consider transversal fluctuations = scalar fields $\phi \in \mathrm{End}(\mathcal{H})$

$$\begin{aligned} S[\phi] &= -Tr\eta_{ab}[Y^a,\phi][Y^b,\phi] \\ &\sim \int \rho_M\, \eta_{ab} E^{a\mu}\partial_\mu\phi E^{b\nu}\partial_\nu\phi \sim \int \sqrt{|G|}\, G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \end{aligned}$$

semi-classical frame & metric:

$$E^{\dot\alpha\mu} = \{Y^{\dot\alpha}, x^\mu\} \;\sim -i[Y^{\dot\alpha}, x^\mu]$$

divergence constraint $\nabla_\nu(\rho^{-2}E_{\dot\alpha}{}^\nu) = 0$ (Jacobi identity)

$$\begin{aligned} G^{\mu\nu} &= \rho^{-2}\eta_{ab}E^{a\mu}E^{b\nu} = \rho^{-2}\gamma^{\mu\nu} \\ \rho^2 &= \rho_M\sqrt{|\gamma^{\mu\nu}|} \qquad \text{....dilaton} \end{aligned}$$

governs all fluctuations in M.M, universal $\Rightarrow$ gravity !

no local Lorentz transformation of the frame!

coupling to fermions $\to$ talk by Battista

Weitzenböck connection:

$$\nabla^{(W)} E_{\dot\alpha} = 0 \qquad \text{(Weitzenböck)} \quad \Rightarrow \quad \nabla^{(W)} G^{\mu\nu} = 0$$

flat but torsion:

$$T_{\dot\alpha\dot\beta} \equiv T[E_{\dot\alpha}, E_{\dot\beta}] = \nabla_{\dot\alpha} E_{\dot\beta} - \nabla_{\dot\beta} E_{\dot\alpha} - [E_{\dot\alpha}, E_{\dot\beta}]$$

can show:

$$\boxed{T_{\dot\alpha\dot\beta}{}^{\mu} = \{\hat\Theta_{\dot\alpha\dot\beta}, x^\mu\},} \qquad \hat\Theta_{\dot\alpha\dot\beta} := -\{Y_{\dot\alpha}, Y_{\dot\beta}\}$$

$$T_{\dot\alpha} = dE_{\dot\alpha}, \qquad E_{\dot\alpha} = E_{\mu\dot\alpha} dx^\mu \qquad \text{...coframe}$$

torsion tensor encodes field strength of the NC gauge theory

(HS arXiv:2002.02742 , cf. Langmann Szabo hep-th/0105094)

geometric form of the matrix eom $\{X^a, \{X_a, X^b\}\} = m^2 X^b$

Weitzenböck connection:

$$\nabla_\nu^{(W)} T^\nu{}_{\rho\mu} + T_\nu{}^\sigma{}_\mu T_{\sigma\rho}{}^\nu = -m^2 \gamma_{\rho\mu}$$

HS arXiv:2002.02742 , cf. Hanada-Kawai-Kimura hep-th/0508211

Levi-Civita connection:

$$\nabla^{(G)\nu}\left(\rho^2 T_{\nu\mu}{}^{\dot a}\right) + \frac{1}{2} T^{(AS)\nu\sigma}{}_\mu T_{\nu\sigma}{}^{\dot a} = -m^2 E^{\dot a}{}_\mu$$

and

$$\star T^{(AS)} = \tilde T_\mu dx^\mu, \qquad \tilde T_\mu = \rho^{-2}\partial_\mu \tilde\rho$$

..." gravitational axion"

Fredenhagen, HS arXiv: 2101.07297

- E-H action in terms of torsion: identity

$$\int d^4x\sqrt{|G|}\mathcal{R} = -\int d^4x\sqrt{|G|}\Big(\frac{7}{8}T^{\mu}{}_{\sigma\rho}\,T_{\mu\sigma'}{}^{\rho}G^{\sigma\sigma'} + \frac{3}{4}\tilde{T}_{\nu}\tilde{T}_{\mu}G^{\mu\nu}\Big)$$

(cf. teleparallel gravity) S. Fredenhagen, H.S. arxiv:2101.07297

- on-shell Ricci tensor

$$\begin{aligned}\mathcal{R}_{\nu\mu} &= \tfrac{1}{4}T^{(AS)\sigma}{}_{\rho\mu}\,T^{(AS)}{}_{\sigma}{}^{\rho}{}_{\nu} - T_{\mu\sigma}{}^{\rho}T_{\nu}{}^{\sigma}{}_{\rho} + 2\rho^{-2}\partial_{\nu}\rho\partial_{\mu}\rho \\ &\quad + \tfrac{1}{4}G_{\nu\mu}\big(T^{\sigma}{}_{\nu\delta}\,T_{\sigma}{}^{\nu}{}_{\rho}\,G^{\delta\rho} - \tfrac{1}{3}T^{(AS)\sigma}{}_{\rho\mu}\,T^{(AS)}{}_{\sigma}{}^{\rho}{}_{\nu}\,G^{\mu\nu}\big)\end{aligned}$$

quadratic in T and $\partial\rho$ $\Rightarrow$ linearized on-shell metric fluctuations on flat background are Ricci-flat

pre-gravity from classical matrix model:

dynamical geometry, lin. Ricci-flat, differs from GR at non-lin level

- bare action: $S \sim \int \frac{1}{g^2}\Theta_{\dot\alpha\dot\beta}\Theta^{\dot\alpha\dot\beta}$ ... 2 derivatives less than E-H

$$\int d^4x\sqrt{|G|}\mathcal{R} = \int d^4x\sqrt{|G|}\Big(-\frac{3}{4}\tilde T_\nu \tilde T_\mu G^{\mu\nu} - \frac{7}{8}T^\mu{}_{\sigma\rho}T_{\mu\sigma'}{}^\rho G^{\sigma\sigma'}\Big)$$

since

$$T^{\dot\alpha\dot\beta\mu} = \{\Theta^{\dot\alpha\dot\beta}, x^\mu\} \sim \partial\Theta^{\dot\alpha\dot\beta} \qquad (\ \Theta^{\dot\alpha\dot\beta} = \{Y^{\dot\alpha}, Y^{\dot\beta}\}\ )$$

$\Rightarrow$ different from GR, expected to dominate on large scales

quantization is well-behaved!

- on covariant quantum spaces (later):
  - all gravitational dof, no ghosts, lin. Schwarzschild etc.

    Sperling, HS 1901.03522 , HS 1905.07255 ff
  - reasonable cosmology without any fine-tuning

    BBounce, $a(t) \sim \frac{3}{2}t$ at late times
  - Feynman propagator

    Karczmarek, HS 2207.00399; Battista, HS 2207.01295

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1-loop effective action and induced gravity

SUSY $\to$ mild quantum effects:

Idea:

Einstein-Hilbert action (+ extra) arises in the 1-loop effective action on $\mathcal{M}^{3,1}$ space-time (cf. Sakharov '67)

$$\Gamma_{1-\mathrm{loop}} \ni \int\limits_{\mathcal{M}} T_{\nu\lambda}{}^{\mu} T_{\nu\lambda}{}^{\mu} + \ldots \sim \int\limits_{\mathcal{M}} d^4x \sqrt{G}\, m_{\mathcal{K}}^2 \mathcal{R}[G] + \ldots$$

requires presence of fuzzy extra dimensions $\mathcal{K}$

finite, no UV divergence / cutoff !!

nonperturbative quantization of MM:

$$Z = \int dY d\Psi e^{iS[Y,\Psi]}, \qquad S = S_{\rm IKKT} + i\varepsilon Y^a Y^b \delta_{ab}$$

cf. numerical work (Nishimura, Tsuchiya, Anagnostopoulos etal.)

1-loop effective action

$$e^{i\Gamma_{\text{1-loop}}[Y]} = \int\limits_{1\,\text{loop}} d\mathcal{A} d\Psi e^{iS[Y+\mathcal{A},\Psi]}$$

$$\begin{aligned}\Gamma_{1\text{loop}}[Y] &= \frac{1}{2}\mathrm{Tr}\Big(\log(\Box - M_{ab}[\Theta^{ab},.]) - \frac{1}{2}\log(\Box - M_{ab}^{(\psi)}[\Theta^{ab},.]) - 2\log(\Box)\Big) \\ &= \frac{1}{2}\mathrm{Tr}\Big(\sum_{n=4}^{\infty}\frac{1}{n}\Big((\Box^{-1}M_{ab}[\Theta^{ab},.])^n - \frac{1}{2}(\Box^{-1}M_{ab}^{(\psi)}[\Theta^{ab},.])^n\Big)\Big)\end{aligned}$$

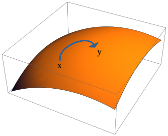
UV-finite on 4D backgrounds due to max. SUSY !!

evaluate trace use string mode formalism

$$\mathrm{Tr}_{\mathrm{End}(\mathcal{H})}\mathcal{O} = \frac{1}{(2\pi)^m}\int\limits_{\mathcal{M}\times\mathcal{M}} dxdy\, \left(\begin{smallmatrix}x\\y\end{smallmatrix}\right|\mathcal{O}\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)$$

string modes:

$$\boxed{\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right) := |x\rangle\langle y|} \qquad \in \mathrm{End}(\mathcal{H})$$



$|x\rangle$ ... coherent state on $\mathcal{M}$

... “string” from x to y, extreme UV but non-local on any NC space

H.S. arXiv:1606.00646, cf. Iso Kawai Kitazawa hep-th/0001027

H.S., J. Tekel arXiv:2203.02376

$\approx$ diagonalize kinetic operators:

$$[Y^a, \left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)] \approx (x^a - y^a)\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)$$

$$\Box\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right) \approx (|x-y|^2 + 2\Delta^2)\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)$$

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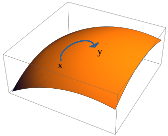

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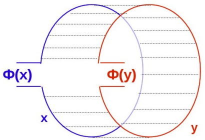
digression: UV/IR mixing in NC field theory

non-local string modes dominate loops in UV-divergent QFT
$\Rightarrow$ nonlocal effects UV/IR mixing, ~~renormalizability~~

"nonplanar" contribution:

$$\mathrm{Tr}(.(\Box + \mu^2)^{-1}\phi.\phi) \quad = \quad \int\limits_{\mathcal{M}\times\mathcal{M}} dxdy \frac{1}{|x-y|^2+\tilde{\mu}^2}\phi(x)\phi(y)$$



effective action obtained directly in position space!

back to 1-loop of IKKT model:

$$\Gamma_{1\text{loop};4}[Y] = \frac{1}{8}\mathrm{Tr}\left((\Box^{-1}(M_{ab}[\Theta^{ab},.])^4 - \frac{1}{2}(\Box^{-1}M^{(\psi)}_{ab}[\Theta^{ab},.])^4\right)$$
$$= \frac{1}{4}\frac{1}{(2\pi)^m}\int\limits_{\mathcal{M}\times\mathcal{M}} dxdy\,\frac{3S_4[\delta\Theta(x,y)]}{(|x-y|^2+2\Delta^2)^4}$$

where

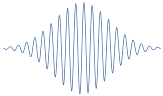
$$-S_4[\delta\Theta] = 4tr(\delta\Theta\delta\Theta\delta\Theta\delta\Theta) - (tr\delta\Theta\delta\Theta)^2$$
$$\delta\Theta = \Theta^{ab}(x) - \Theta^{ab}(y)$$

note:

- UV-finite (maximally SUSY) $\rightarrow$ short string modes dominate,
- short-distance regime requires refined analysis:

short string modes as localized Gaussian wave-packets:

$$\Psi^{(L)}_{k;y} := \int d^4z\, e^{-|y-z|^2/L^2} \left|{}^{z+\frac{k}{2}}_{z-\frac{k}{2}}\right) \cong e^{ikx} e^{-|x-y|^2/L^2}$$



locally diagonalize kinetic operators in IR:

$$\begin{aligned} \Box \Psi^{(L)}_{k;y} &\approx \gamma^{\mu\nu}(x) k_\mu k_\nu \Psi^{(L)}_{k;y} \\ [\theta^{ab}, \Psi^{(L)}_{k;y}] &\approx -\{\theta^{ab}, x^\mu\} k_\mu \Psi^{(L)}_{k;y} \end{aligned}$$

Trace formula for UV-finite traces on NC spaces:

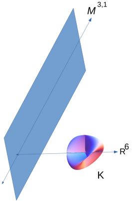
$$\mathrm{Tr}\mathcal{O} = \frac{1}{(2\pi)^m} \int\limits_{\mathcal{M}\times\mathcal{M}} \Omega_x \Omega_y \left({}^x_y\right| \mathcal{O} \left|{}^x_y\right) \approx \frac{1}{(2\pi)^m} \int\limits_{\mathcal{M}} \sqrt{G} dx \int \frac{1}{\sqrt{G}} dk \langle \Psi^{(L)}_{k,x}, \mathcal{O}\Psi^{(L)}_{k,x} \rangle$$

use this to evaluate 1-loop eff. action

a priori: 4-derivative action ☹

however: brane $\mathcal{M} \times \mathcal{K} \subset \mathbb{R}^{9,1}$ with fuzzy extra dim.

from 6 transversal directions $\langle \phi^i \rangle \neq 0$



mixed term $(\delta\Theta^{\alpha\beta}\delta\Theta^{\alpha\beta})(\delta\Theta^{ij}\delta\Theta^{ij})$ leads to induced E-H action ☺

$$\{\theta^{\alpha\beta}, \{\theta^{\alpha\beta}, \psi_{k;y}\}\} \approx -\{\theta^{\alpha\beta}, x^\mu\}\{\theta^{\alpha\beta}, x^\nu\} k_\mu k_\nu \psi_{k;y}$$
$$= -T^{\alpha\beta\mu} k_\mu T^{\alpha\beta\nu} k_\nu \psi_{k;y}$$

(torsion $T^{\alpha\beta\mu} = \{\theta^{\alpha\beta}, x^\mu\}$)

$$\Gamma_{\text{1loop}} \sim -\int_{\mathcal{M}} d^4x \sqrt{G}\, c_{\mathcal{K}}^2 m_{\mathcal{K}}^2\, T^\rho{}_{\sigma\mu} T_{\rho'}{}^\sigma{}_\mu G^{\mu\mu'}$$
$$\sim \int d^4x \sqrt{G}\, c_{\mathcal{K}}^2 m_{\mathcal{K}}^2 \left(8\mathcal{R}[G] + 6\tilde{T}_\nu \tilde{T}_\mu G^{\mu\nu}\right)$$

where $m_{\mathcal{K}}^2$ ... KK scale on $\mathcal{K}$ HS 2110.03936 bottom line:

- $\Gamma_{\rm 1loop}$ includes Einstein-Hilbert action, eff. Newton constant

$$\frac{1}{G_N} \sim c_{\mathcal{K}}^2 m_{\mathcal{K}}^2$$

set by Kaluza-Klein mass scale on $\mathcal{K}$

- large vacuum energy

$$\Gamma_{\rm 1loop}^{\mathcal{K}} \sim -\int_{\mathcal{M}} \Omega\, \rho^{-2} m_{\mathcal{K}}^4 \sum_{\Lambda s} \frac{V_{4,\Lambda}}{\mu_\Lambda^4} + \ldots$$

not c.c., leads to stabilization of $m_{\mathcal{K}}$ at one loop !

- $S \sim \int \Theta^{\alpha\beta}\Theta^{\alpha\beta} + S_{E-H}$

bare action dominates extreme IR (=cosm. !)

4D covariant quantum spaces & $\mathfrak{hs}$

issues:

- Poisson structure $\theta^{\mu\nu}$ breaks Lorentz / rotation invariance
- enough dof for metric, frame ?

quantized twistor space as brane:

$$\mathbb{C}P^{1,2}_N \overset{loc}{\cong} S^2 \times \mathcal{M}^{3,1} \quad \subset \mathbb{R}^{9,1}$$

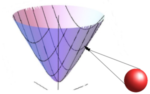
... sympl. equivariant S^2- bundle over space(time) $\mathcal{M}^{3,1}$

- $\langle\theta^{\mu\nu}\rangle_{\mathcal{M}} = 0$!
- price to pay: higher-spin theory, all dof for metric on $\mathcal{M}^{3,1}$
- vol.-preserving diffeos on $\mathcal{M} \subset$ higher-dim symplectomorphisms

HS : 1606.00769 , M. Sperling, HS 1806.05 ff, HS , T. Tran 2203.05436

MM description: 2-step procedure

- $\mathbb{C}P^{1,2}_n$ = quantized S^2_n -bundle over H^4_n
  equivariant under $SO(4,1)$



realized by MM background $Y^a := \frac{1}{R}\mathcal{M}^{a5}, \quad a = 0, ..., 4$

... minimal discrete unitary irrep $\mathcal{H}_n$ of $\mathfrak{so}(4,2)$

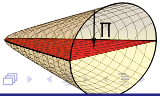
$$\mathrm{End}(\mathcal{H}_n) \cong \mathcal{C}(\mathbb{C}P^{1,2}) \cong \bigoplus_{s=0}^{n} \mathcal{C}^s$$

would-be KK modes

$\rightarrow$ spin s modes on H^4 taking values in $\mathfrak{hs} = \oplus \begin{array}{|c|c|c|}\hline & & \\ \hline & & \\ \hline\end{array}$

matrix model $\rightarrow$ higher spin gauge theory, truncated at n

- further projection $H^4 \rightarrow \mathcal{M}^{3,1}$ ... FLRW quantum space-time
  manifest homogeneous & isotrop, Big Bounce
  M. Sperling, HS 1901.03522



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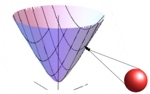

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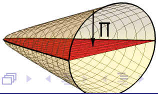
- further projection $H^4 \to \mathcal{M}^{3,1}$ ... FLRW quantum space-time
  manifest homogeneous & isotrop, Big Bounce
  M. Sperling, HS 1901.03522



summary & open questions

gravity arises as quantum effect on 3+1-dim. quantum space-time in the IKKT matrix model (maximally SUSY !)

- MM = "pre-gravity", suitable for quantization
- quantization $\rightarrow$ induced Einstein-Hilbert action, no c.c. problem (? TBC)
- cross-over GR $\leftrightarrow$ cosm. background (class.)
- covariant quantum spaces = twisted S^2 bundles over $\mathcal{M}^{3,1}$ $\rightarrow$ higher spin gauge theory rotation invariance manifest
- new physics (axion, dilaton, $\mathfrak{hs}$ ...)

IKKT = distinguished model for emergent near-realistic (?) physics string theory without compactification

Fuzzy extra dimensions $\mathcal{K}$

consider backgrounds with product structure

$$\mathcal{M}^{3,1} \times \mathcal{K} \qquad\qquad (\subset \mathbb{R}^{9,1}!)$$

$\mathcal{K}$ ... quantized compact symplectic space

realized by

$$Y^{\dot a} \sim y^{\dot a}: \qquad \mathcal{M} \hookrightarrow \mathbb{R}^{3,1}, \qquad \dot a = 0, ..., 3$$

$$Y^{i} \sim y^{i}: \qquad \mathcal{K} \hookrightarrow \mathbb{R}^{6}, \qquad i = 4, ..., 9$$

acting on $\mathcal{H} = \mathcal{H}_{\mathcal{M}} \otimes \mathcal{H}_{\mathcal{K}}$

matrix d'Alembertian decomposes as

$$\Box = [Y^{\dot a}, [Y_{\dot a}, .]] + [Y^{i}, [Y_{i}, .]] = \Box_{\mathcal{M}} + \Box_{\mathcal{K}} \; .$$

internal $\Box_{\mathcal{K}}$ has a positive spectrum

$$\Box_{\mathcal{K}} \lambda_\Lambda = m_\Lambda^2 \, \lambda_\Lambda$$

hence

$$\Box \phi_\Lambda = (\Box_{\mathcal{M}} + m_\Lambda^2)\phi_\Lambda \; , \qquad [\Theta^{ij}, [\Theta^{ij}, \lambda_\Lambda]] = m_{\mathcal{K}}^4 C_\Lambda^2 \lambda_\Lambda$$

$\mathcal{K}$ ... fuzzy space (quantized symplectic space), e.g. S^2_N, $\tilde{\mathbb{C}}P^2, \ldots$

stabilization of $\mathcal{K}$: either

- add cubic term $\mathrm{Tr} f_{ijk} Y^i [Y^j, Y^k]$ by hand (breaks SUSY..)

  cf. Chatzistavrakidis HS Zoupanos 1107.0265 ff, Andrews Dorey hep-th/0505107 etc.

- better: 1-loop effect (interaction $\mathcal{K} \leftrightarrow \mathcal{M}^{3,1}$) stabilizes radius !

$$V(m_{\mathcal{K}}^2) = -c^2 m_{\mathcal{K}}^2 + \frac{d^2}{g^2} m_{\mathcal{K}}^4$$

where $m_{\mathcal{K}} \sim r_{\mathcal{K}}$, nontriv. minimum

HS 2110.03936

gauge transformations as diffeos

... arise from $Y^a \to U^{-1} Y^a U$ on NC branes $\mathcal{M}$

scalar fields:

$$\delta_\Lambda \phi = \{\Lambda, \phi\} = \xi^\mu \partial_\mu \phi = \mathcal{L}_\xi \phi, \qquad \xi^\mu = \{\Lambda, x^\mu\}$$

vector fields (frame!):

$$\begin{aligned} \delta_\Lambda Y_{\dot\alpha} &= \{\Lambda, Y_{\dot\alpha}\} \\ \delta_\Lambda E_{\dot\alpha} &= \{\Lambda, \{Y_{\dot\alpha}, .\}\} - \{Y_{\dot\alpha}, \{\Lambda, .\}\} = \mathcal{L}_\xi E_{\dot\alpha} \qquad \text{(Jacobi)} \end{aligned}$$

hence

$$\boxed{\delta_\Lambda E_{\dot\alpha}{}^\mu = \mathcal{L}_\xi E_{\dot\alpha}{}^\mu, \qquad \delta_\Lambda G^{\mu\nu} = \mathcal{L}_\xi G^{\mu\nu}}$$

diffeos from NC gauge trafos!

$\{\Lambda, .\}$ ... Hamiltonian VF

on covariant quantum space $\mathcal{M}^{3,1}$: all dof for vol-preserving diffeos