(Quasi-)chiral higher-spin theories from twistor space

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What is higher-spin gravity (HSGRA)?

• Some of the most promising approaches toward a quantum theory of gravity involve higher-spin fields (string theory, bulk reconstruction ...)

• **Higher-spin gravities (HSGRA)** are theories where the massless spin-2 graviton becomes part of the unique higher spin multiplet of massless gauge fields with spin-$s = 0, 1, 2, \dots, \infty$.

• The main idea: the more massless fields, the more gauge symmetries. The more gauge symmetries, the fewer counter terms.

Higher-spin symmetry $\xrightarrow{?}$ quantum gravity

• In the context of the AdS/CFT correspondence: HSGRAs in AdS should be the dual theories of (large N) free or weakly coupled Vector Model (Ising) and Chern-Simons matter theories.

HSGRAs may help us to make CFT predictions

Are there local higher-spin theories that can avoid no-go theorems?

• There are two notable no-go theorems that forbid the existence of *interacting* massless higher-spin theories in flat space

1- **Weinberg's soft theorem**: ruled out the existence of low energy massless higher-spin fields in any local Lorentz-invariant theory by studying conservation laws from a simple but stringent relation

$$\boxed{\sum_i g_{s,i}\, p_i^{\mu_1} \dots p_i^{\mu_{s-1}} = 0}$$

2- **Coleman-Mandula theorem**: There is no higher-spin charge if the S-matrix of a finite number of particles is non-trivial and analytic.

$$\boxed{\mathbb{R}^{1,3} \rtimes SO(1,3) \times (\text{internal symmetry group})}$$

▷ The above theorems imply that

Higher-spin scattering amplitudes of massless fields in flat space should be trivial

If we insist on higher-spin theories, is there a way out?

- Higher-spin problem can be resolved with certain **prices**.

  1- **Light-cone approach.** Dealing directly with physical degrees of freedom and can be used to construct local higher-spin theories (Bengtsson, Bengtsson, Brink; Metsaev; Ponomarev-Skvortsov, ...).
     - **Not covariance**.

  2- **Go to 3d.** Typically topological and can be written in Chern-Simons form (Blencowe+(Berhshoeff-Stelle); Pope-Townsend; Fradkin-Linetsky; Kuzenko; Henneaux-Rey; Campoleoni-Fredenhagen-Pfenninger-Theisen; ...)
     - **No propagating degree of freedom**.

  3- **Higher-spinization of Weyl gravity.** One can write down covariant local action for conformal HSGRA (Tseylin-Segal; Grigoriev-Tseylin; Bakaert-Joung-Mourad;...)
     - **Non-unitary due to higher derivatives in the kinetic action**

If we insist on higher-spin theories, is there a way out?

4- **Twistor theory.** Locality is controllable from the beginning by working with chiral representations. All vertices match with the ones of the light-cone approach.

- Self-dual conformal HSGRA (Adamo-Hahnel-Mcloughlin)
- Self-dual HS Yang-Mills and gravity (Adamo-T; Herfray, Krasnov, Skvortsov)

• **Parity invariance is violated by construction.**

5- **Matrix model type HS.** Work with IKKT matrix model on quantized twistor space where higher-spin fields are introduced to mitigate the effect of Lorentz violation by the non-commutativity of matrices (Steinacker et. al.)

• **Parity invariance is violated by construction.**

▷ **In constructing higher-spin theories, there is no free lunch.**

- Unitary higher-spin theories are non-local. (Boulanger, Kessel, Skvortsov, Taronna ; Bakaert, Erdmenger, Ponomarev, Sleight ; Das, de Mello Koch, Jevicki-Rodrigues, Yoon)
- Local higher-spin theories are either non-unitary or non parity-invariant.

This talk

- **Part I.**

◇ Review of no-go theorems/results and some approaches that seem to work.

- **Part II.**

◇ Discuss Fronsdal and chiral representations.
◇ Revisit Weinberg's soft theorem.
◇ Some examples of local higher-spin theories obtained from twistor space.

- **Part III.**

◇ IKKT matrix model type higher-spin gauge theory.

- **Part IV.**

◇ Conclusion.

Fronsdal and chiral representations used in 4-dim higher-spin theories

◇ Any massless higher-spin field in $4d$ can be represented as $T^{\alpha(m)\,\dot{\alpha}(n)} \in S(m,n)$.

• Fronsdal rep (m=n):

- Fields are Lorentzian real. They can propagate on flat and (A)dS.

- Theories constructed from Fronsdal representation are unitary, parity-invariant but suffer from non-locality issues.

- Subject to Weinberg's soft theorem in flat space.

- Flat limit of interacting (A)dS theories are hard to achieved.

• Chiral rep ($m \geq n \geq 0\,,\, n = 0, 1$):

- Fields are not Lorentzian real. They can propagate on any self-dual background.

- Theories constructed from chiral rep are instrinsically chiral and local (vertices have bounded number of derivatives).

- Do not have problem with Weinberg's soft theorem (**parity violation**).

- Flat limit of interacting (A)dS theories can be taken smoothly.

Back to Weinberg's soft theorem

□ **Setup.** Let $\mathcal{M}_n$ be an n-point scattering amplitude between scalar fields ϕ_i with momentum $p_i^\mu \sim p_i^{\alpha\dot{\alpha}}$ and consider a soft emitting higher-spin field A_s with momentum $k_\mu \sim k_{\alpha\dot{\alpha}}$. Gauge invariance of the S-matrix implies the following constraints.

◇ **Fronsdal rep.** We have $\sum_i g_{s,i}\, p_i^{\mu_1} \ldots p_i^{\mu_{s-1}} = 0$. In particular,

- $s = 1$ we have $\sum_i g_i = 0$ [**charge conservation**]
- $s = 2$ we have $\sum_i g_i p_i^\mu = 0$ which can be satisfied if $g_i = \text{const}$ and $\sum_i p_i^\mu = 0$ [**low-energy equivalence principle**]
- $s > 2$ we have $g_i = 0 \quad \Rightarrow$ **No local parity-invariant higher-spin theory**

◇ **Chiral rep.** All constraints of Weinberg are accompanied by the soft momentum $k_{\alpha\dot{\alpha}}$. In particular, $\sum_m \left(\sum_i^n g_{s,m,i} \underbrace{p_{i\alpha\dot{\gamma}} \ldots p_{i\alpha\dot{\gamma}}}_{m\,\text{times}} \right) \underbrace{k_\alpha^{\ \dot{\gamma}} \ldots k_\alpha^{\ \dot{\gamma}}}_{m\,\text{times}} = 0.$

- **m is the number of derivatives in cubic vertices.**
- Weinberg's soft theorem is related to derivatives rather than spins.
- As $k \to 0$, there is no restriction on higher-spin interactions, which implies

It is possible to have local non-parity-invariant higher-spin theories

Examples of (quasi-)chiral HSGRAs

There are a few examples of (quasi-)chiral HSGRAs:

- **Chiral HSGRA and its contractions.** They are chiral theory with complex actions and trivial S-matrix.
  (Metsaev; [“Ponomarev”-({Skvortsov}]-T-Tsulaia)||
  ⟨Sharapov-Skvortsov⟩-Sukhanov-van Dongen ...)
- **HS-YM.** A quasi-chiral theory with non-trivial scattering amplitudes (Adamo-T)
- **HS-IKKT.** It is closely related to HS-YM and is also a quasi-chiral theory ((Sperling-[Steinacker)-T])

PART III - HS-IKKT model

- (HS-)IKKT in general
- Semi-classical limit
- Algebra of functions on H_N^4 and $\mathbb{P}^{1,2}$
- Hopf maps

• **Our result:** Local spinorial descriptions of HS-IKKT in Lorentzian and Euclidean signature.

HS-IKKT – Generality

⋄ **IKKT matrix model** (Ishibashi, Kawai, Kitazawa, Tsuchiya-96') is an alternative and constructive description of type IIB superstring theory with $SO(1,9)$-invariant action

$$S = \text{Tr}\Big([Y^{\boldsymbol{I}}, Y^{\boldsymbol{J}}][Y_{\boldsymbol{I}}, Y_{\boldsymbol{J}}] + \Psi_{\boldsymbol{\mathcal{A}}}(\tilde{\gamma}^{\boldsymbol{I}})^{\boldsymbol{\mathcal{A}\mathcal{B}}}[Y_{\boldsymbol{I}}, \Psi_{\boldsymbol{\mathcal{B}}}]\Big)\,, \quad \boldsymbol{I} = 0, 1, \ldots, 9$$

where $Y^{\boldsymbol{I}}$ are $N \times N$ hermitian matrices, and $\Psi^{\boldsymbol{\mathcal{A}}}$ are the $SO(1,9)$ matrix-valued Majorana-Weyl spinors. The action is invariant under $\delta Y^{\boldsymbol{I}} = U^{-1} Y^{\boldsymbol{I}} U$ with U being an arbitrary unitary matrix.

- Obtained by dimensional reduction of 10-dim SYM theory to a point
- Spacetime along with physical fields emerge from matrix dof. by considering the fluctuations of the background $\bar{Y}$ as $Y^{\boldsymbol{I}} = \bar{Y}^{\boldsymbol{I}} + \mathcal{A}^{\boldsymbol{I}}$
- Naturally induces a HS gauge theory (**HS-IKKT**) on fuzzy (quantized) twistor space (Steinacker et. al.) to mitigate the effect of Lorentz violation.
- HS-IKKT can be defined to be Lorentzian real but there is a price to pay

HS-IKKT – Semi-classical limit

◇ In the **large N limit**, matrices become effectively commutative and we have the dequantization rules (Review: 1911.03162)

Quantum/fuzzy geometry		Semi-classical/dequantized geometry	
(matrix) $Y^{\boldsymbol{I}}$	$\mapsto$	$y^{\boldsymbol{I}} = \langle y\vert Y^{\boldsymbol{I}}\vert y\rangle$	(function)
$[\,,]$	$\mapsto$	$i\{\,,\}$	(Poisson bracket)
Tr	$\mapsto$	$\int \mho$	(symplectic volume form)

◇ $y^{\boldsymbol{I}} = \{y^a, y^{\mathcal{I}}\}$ can be used to define certain variety (S^4_N, H^4_N) embedded in $\mathbb{R}^{1,9}$.

◇ To construct a higher-spin gauge theory in $4d$ spacetime with Lorentzian signature, we let $y^a \in \mathbb{R}^{1,4}$ where $y_a y^a = -R^2 = -\frac{\ell_p^2 N^2}{4}$. The Poisson bracket between y^a is

$$\boxed{\{y^a, y^b\} = \theta^{ab} = -\ell_p^2\, m^{ab}\,, \quad a,b = 0,1,2,3,4}$$

where m^{ab} are generators of $\mathfrak{so}(1,4)$

$$\{m_{ab}, m_{cd}\} = (m_{ad}\eta_{bc} - m_{ac}\eta_{bd} - m_{bd}\eta_{ac} + m_{bc}\eta_{ad})$$
$$\{m_{ab}, y_c\} = y_a\eta_{bc} - y_b\eta_{ac}$$

The above relations form an $\mathfrak{so}(2,4)$ algebra and describe a fuzzy 4-hyperboloid H^4_N.

HS-IKKT – Algebra of functions on H^4_N

- The algebra of functions on H^4_N consists of polynomials in terms of (y^a, m^{ab}) subjects to the self-duality relation $\boxed{\epsilon_{abcde} m^{ab} y^c = -4N/\ell_p \, m_{de}}$. Note that

The presence of ϵ_{abcde} breaks parity invariance of HS-IKKT

which has been already realized at linearized level [upon integrating out θ^{ab}] (Sperling-Steinacker)

- The space of functions on H^4_N reads

$$\mathscr{C}(y^a, m^{ab}) = \sum_{n,s} f_{a(n)c(s),b(n)} \theta^{ab} \dots \theta^{ab} y^c \dots y^c = \bigoplus_{n,s} \begin{array}{|c|}\hline \quad n+s \quad \\ \hline n \\ \hline \end{array}$$

Truncated higher-spin algebra as subspace of $\mathscr{C}$

$$\mathfrak{ths}(\mathfrak{so}(1,4)) = \sum^{N} g_{a(n),b(n)} \theta^{ab} \dots \theta^{ab} = \bigoplus_{n} \begin{array}{|c|}\hline n \\ \hline n \\ \hline \end{array}$$

HS-IKKT – Algebra of functions on $\mathbb{P}^{1,2}$

$\diamond$ It is useful to view H_N^4 as $\mathbb{P}^{1,2}$ since it is a 6 real dimensional coadjoint orbit of $SO(2,4) \simeq SU(2,2)$.

• Let Z^A (which are $\mathfrak{su}(2,2)$ vectors) be homogeneous coordinates on $\mathbb{P}^{1,2}$ with $A = 1,2,3,4$. The **Dirac conjugate** $\bar{Z}_A = Z_B^\dagger(\gamma^0)^B{}_A$ of Z^A which obeys the Poisson algebra

$$\boxed{\{Z^A, \bar{Z}_B\} = \delta^A{}_B}$$

can be used to defined the number operator $\mathcal{N} := \bar{Z}_A Z^A = N = 2R/\ell_p$, which gives the gradation

$$\{\mathcal{N}, Z^A\} = -Z^A\,, \qquad \{\mathcal{N}, \bar{Z}_A\} = +\bar{Z}_A$$

Then, the algebra of functions of $\mathbb{P}^{1,2}$

$$\boxed{\mathscr{C}(\mathbb{P}^{1,2}) = End(\mathcal{H}_N) = (N,0,0) \otimes (0,0,N) = \sum_n f_{A(n)}{}^{B(n)} Z^{A(n)} \bar{Z}_{B(n)}}$$

where $\mathcal{H}_N = (0,0,N) = (0,0,1)^{\odot N}$ is N-particle Fock space, consists of polynomials with equal number of Z and $\bar{Z}$.

HS-IKKT – Hopf maps

◇ There are two Hopf maps we consider for the case of H^4_N:

$$H^4: \quad Z^A \mapsto y^a := \frac{\ell_p}{2}\bar{Z}_A(\gamma^a)^A{}_B Z^B\,, \qquad a = 0,1,2,3,4$$

$$H^{2,2}: \quad Z^A \mapsto t_{\hat{a}} := \frac{\ell_p}{2}\bar{Z}_A(\Sigma_{\hat{a}4})^A{}_B Z^B\,, \quad \hat{a} = 0,1,2,3,5$$

where $t_{\hat{a}}$ transform as vectors under $SO(2,3)$ and

$$\{t^{\hat{a}}, t^{\hat{b}}\} = \frac{1}{R^2} m^{\hat{a}\hat{b}}\,,$$

$$t_{\hat{a}} t^{\hat{a}} = -t_0^2 + t_i t^i - t_5^2 = 1/\ell_p^2\,, \qquad i = 1,2,3\,,$$

$$y_{\hat{a}} t^{\hat{a}} = 0 = y_\mu t^\mu\,, \qquad \mu = 0,1,2,3$$

Due to the last relation, t^μ are understood as generators of the internal **space-like** S^2 which underlies the higher-spin structure of HS-IKKT in Lorentzian signature.

• To get a Lorentzian $SO(1,3)$-covariant spacetime $\mathcal{M}^{1,3}$, there are two projections one can consider

$$\pi_y: m^{ab} \mapsto y^\mu = \ell_p m^{\mu 5}\,, \qquad \eta_{\mu\nu} y^\mu y^\nu = -R^2 - y_4^2 = -R^2\cosh^2(\tau)$$

$$\pi_t: m^{\hat{a}\hat{b}} \mapsto t^\mu = \frac{1}{R} m^{\mu 4}\,, \qquad \eta_{\mu\nu} t^\mu t^\nu = \frac{1}{\ell_p^2} + \frac{y_4^2}{\ell_p^2 R^2} = +\ell_p^{-2}\cosh^2(\tau)$$

HS-IKKT - Spinorial description

• Decompose $Z^A = (\lambda^\alpha, \mu^{\dot\alpha})$ for $\alpha = 0, 1$ and $\dot\alpha = \dot 0, \dot 1$ where the spinors (λ, μ) transform in the fundamental rep of the compact subgroup $SU(2) \times SU(2) \subset SU(2,2)$. (λ, μ) are **not** Weyl spinors.

In spinor notation, the number operator reduces to

$$\mathcal{N} = \bar Z_A Z^A = \langle \lambda\, \bar\lambda \rangle - [\mu\, \bar\mu] = N$$

Here, $\langle a\, b \rangle = -a^\alpha b^\beta \epsilon_{\alpha\beta} = a^\alpha b_\alpha\,,\ [a\, b] = -a^{\dot\alpha} b^{\dot\beta} \epsilon_{\dot\alpha\dot\beta} = a^{\dot\alpha} b_{\dot\alpha}$ where $\epsilon^{01} = 1 = \epsilon_{01}$.

• The correspondence between $\mathbb{P}^{1,2}$ and H^4 is expressed via the incidence relations:

$$\mu^{\dot\alpha} = \mathrm{x}^{\alpha\dot\alpha} \lambda_\alpha \qquad \Leftrightarrow \qquad \mathrm{x}^{\alpha\dot\alpha} = \frac{\lambda^\alpha \bar\mu^{\dot\alpha} - \bar\lambda^\alpha \mu^{\dot\alpha}}{\langle \lambda\, \bar\lambda \rangle}$$

Substitute this to the Hopf maps, we obtain

$$y^0 = R + \ell_p [\mu\, \bar\mu]\,, \qquad y^i = -\frac{1}{4R}\left(\lambda^\dagger \sigma^i \mu + \mu^\dagger \sigma^i \lambda\right)\,, \qquad \textbf{(global)}$$

$$t^0 = \frac{1}{4R}\left(\mu^\dagger \lambda + \lambda^\dagger \mu\right)\,, \quad t^i = +\frac{1}{4R}\left(\lambda^\dagger \sigma^i \lambda + \mu^\dagger \sigma^i \mu\right)\,, \qquad \textbf{(local)}\,.$$

HS-IKKT – local spinor description of $SO(1,3)$-covariant spacetime $\mathcal{M}^{1,3}$

In contrast to $y^\mu = (y^0, y^i)$, $t^\mu = (t^0, t^i)$ are not defined globally. Using the $SO(1,3)$ isometry of $\mathcal{M}^{1,3}$, we can choose a reference point $p = (p^0, 0, 0, 0) \in \mathcal{M}^{1,3}$ where $t^\mu\Big|_p = (0, t^i)$. This spans the local $\mathbb{P}^1$ which can be described by $(\lambda, \bar{\lambda})$.

- It can be shown that at the reference point p, $t^i \sim \lambda^\dagger \sigma^i \lambda$.
- Any function supported by an open subset $U_p \subset \mathcal{M}^{1,3}$ can be effectively written as

$$\boxed{\varphi(y|t) = \sum_{s=0}^{\infty} \varphi_{i(s)} t^{i(s)} \simeq \sum_{s=0}^{\infty} \varphi_{\beta(2s)} \lambda^{\beta(s)} \bar{\lambda}^{\beta(s)}}$$

The 'derivative' wrt. to momenta background t^μ is defined by

$$\Big\{t^\mu, \varphi(y|\lambda, \bar{\lambda})\Big\}\Big|_{U_p} = \Big(\{t^\mu, y^\nu\}\partial_\nu + \{t^\mu, \lambda^\alpha\}\partial_\alpha + \{t^\mu, \bar{\lambda}^\alpha\}\bar{\partial}_\alpha\Big)\varphi(y|\lambda, \bar{\lambda})\Big|_{U_p}$$

The effective metric can be computed by considering the kinetic term $\{t^\mu, \varphi(y)\}\{t_\mu, \varphi(y)\}$ (Steinacker)

$$\gamma^{\mu\nu} = \eta^{\mu\nu} \sinh^2(\tau)$$

HS-IKKT – Gauge fixing and propagating dof. in Lorentzian HS-IKKT

◇ Unlike the standard massless, massive or conformal HS theories, the number of propagating dof. in HS-IKKT fall between the ones of massive and conformal theories.

- Starting with

$$\mathsf{a}_\mu = \sum_s \mathcal{A}_{\nu(s)|\mu} t^{\nu(s)}$$

which has a total $4(2s+1)$ off-shell dof., we can remove $(2s+1)$ degrees of freedom by removing the pure gauge modes with the gauge transformation

$$\delta \mathsf{a}_\mu = \{t_\mu, \xi\}$$

and removing an extra $(2s+1)$ dof. by choosing the gauge fixing condition to be

$$\{t_\mu, \mathsf{a}^\mu\} = 0$$

This leaves us with a total of $\sum_s 2(2s+1)$ physical dof. Thus,

A spin-s field has $2(2s-1)$ dof. in HS-IKKT

- **Remarks:**
  - All higher-spin fields are <u>not</u> divergence-less a priori.
  - If we can impose divergence-free condition, we get down to the standard massless higher-spin fields.

HS-IKKT – In progress

- On-going work with Lorentzian HS-IKKT:
  - Spinorial description for (HS-)IKKT model with Lorentzian signature is not evident as spinors are space-like and transform under $SU(2) \times SU(2)$ rather than $SL(2, \mathbb{C})$ [local Lorentz invariance is not manifest].
  - Understand more about these space-like spinors.

HS-IKKT – Euclidean case

◇ In Euclidean signature, things seem to be a bit better. In particular, we can obtain the full action of HS-IKKT in spinorial description

$$
\begin{aligned}
S = \int & \frac{1}{2}\mathsf{f}_{\alpha\alpha}\mathsf{f}^{\alpha\alpha} + \frac{1}{2}\{p^{\alpha\dot\alpha},\hat\phi\}\{p_{\alpha\dot\alpha},\hat\phi\} + \frac{1}{2}\{p^{\alpha\dot\alpha},\phi^{IJ}\}\{p_{\alpha\dot\alpha},\phi_{IJ}\} \\
& - \frac{i}{2}\bar\chi^{\alpha}{}_{\mathcal{I}}\{p_{\alpha\dot\beta},\tilde\chi^{\dot\beta\mathcal{I}}\} + \frac{i}{2}\chi^{\alpha}{}_{\mathcal{I}}\{p_{\alpha\dot\beta},\bar{\tilde\chi}_{\dot\beta}{}^{\mathcal{I}}\} + \frac{i}{2}\bar{\tilde\chi}_{\dot\alpha\mathcal{I}}\{y_0,\tilde\chi^{\dot\alpha\mathcal{I}}\} - \frac{i}{2}\bar\chi_{\alpha\mathcal{I}}\{y_0,\chi^{\alpha\mathcal{I}}\} \\
& + \frac{1}{2}\{y_0,p^{\alpha\dot\alpha}\}\{y_0,p_{\alpha\dot\alpha}\} + \frac{1}{2}\{y_0,\hat\phi\}\{y_0,\hat\phi\} + \frac{1}{4}\{y_0,\phi^{IJ}\}\{y_0,\phi_{IJ}\} \\
& + \frac{i}{2}\bar{\tilde\chi}_{\dot\alpha\mathcal{I}}\{\hat\phi,\tilde\chi^{\dot\alpha\mathcal{I}}\} - \frac{i}{2}\bar\chi_{\alpha\mathcal{I}}\{\hat\phi,\chi^{\alpha\mathcal{I}}\} - \frac{i}{2}\bar\chi_{\alpha}{}^{\mathcal{I}}\{\phi_{\mathcal{I}\mathcal{J}},\chi^{\alpha\mathcal{J}}\} + \frac{i}{2}\bar{\tilde\chi}_{\dot\alpha}{}^{\mathcal{I}}\{\phi_{\mathcal{I}\mathcal{J}},\tilde\chi^{\dot\alpha\mathcal{J}}\} \\
& + \frac{1}{2}\{\hat\phi,\hat\phi\}\{\hat\phi,\hat\phi\} + \frac{1}{2}\{\hat\phi,\phi^{IJ}\}\{\hat\phi,\phi_{IJ}\} + \frac{1}{2}\{\phi^{IJ},\phi^{MN}\}\{\phi_{IJ},\phi_{MN}\}
\end{aligned}
$$

where $\mathsf{f}^{\alpha\alpha} = \{p^{\alpha}{}_{\dot\alpha},p^{\alpha\dot\alpha}\} = \{y^{\alpha}{}_{\dot\alpha} + \mathsf{a}^{\alpha}{}_{\dot\alpha}, y^{\alpha\dot\alpha} + \mathsf{a}^{\alpha\dot\alpha}\}$. Our results are:

- Higher-spin modes can be shown to be completely disentangled in the YM sector (f^2 term) using spinorial formalism.
- We attempted to compute scattering amplitudes of the massless sector in f^2 term by imposing the divergence-less condition $\partial^{\alpha\dot\alpha}\mathsf{a}_{\alpha\dot\alpha} = 0$ to reduce the dof. to 2. We found that $\mathcal{M}_n = 0$ in this sector.

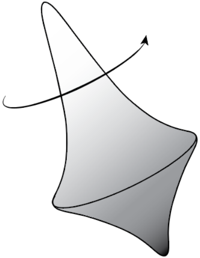
- **Take-home message.**
  - Unitary higher-spin theories are non-local.
  - Local higher-spin theories are either non-unitary or non parity-invariant.
  - Weinberg's soft theorem is related to derivatives rather than spins.
  - Local higher-spin theories constructed from chiral reps with higher-derivative interactions tend to have simple S-matrices.

Conclusion – Outlook

Some future directions:

- Explore the landscape of consistent local higher-spin theories by probing for quasi-chiral higher-spin theories.
- Search for a world-sheet description.
- Compute some observables of (HS)-IKKT theory.

And many more to come :)



Keep calm and work on higher-spin theories