Higher Spin Gravities: old puzzles and bits of Holography

HiSGRA: Conformal Structures, PDEs, and Q-manifold (ESI)

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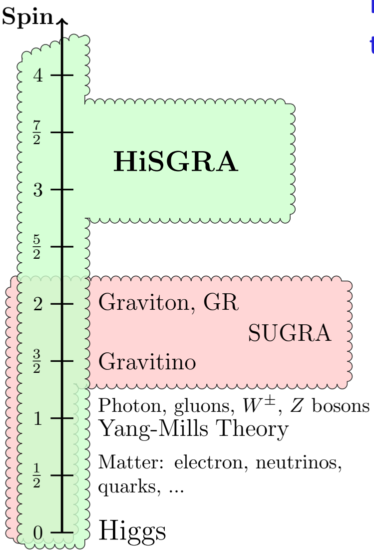
FREEDOM TO RESEARCH

Main Messages

- Higher Spin Gravities (HiSGRA) — the most minimal extensions of gravity with massless higher spin fields — toy models of Quantum Gravity. The idea is that massless fields $\rightarrow$ gauge fields; more gauge symmetries $\rightarrow$ less counterterms $\rightarrow$ Quantum Gravity. No free lunch: HiSGRA are hard to construct and there are very few (no-go's)
- Recent: an example of HiSGRA — Chiral HiSGRA, which we quantized and it turns out to be UV-finite, related to SDYM
- In AdS Chiral HiSGRA is related to physics via AdS/CFT and Chern-Simons Matter theories (Ising, etc.). It helps to prove the three-dimensional bosonization conjecture at the level of three-point functions, which leads to first predictions of HiSGRA
- New/old covariant formulations following SDYM/SDGRA and twistor-inspired description of higher spin fields

Why higher spins?



Different spins lead to very different types of theories/physics:

- $s = 0$: Higgs
- $s = 1/2$: Matter

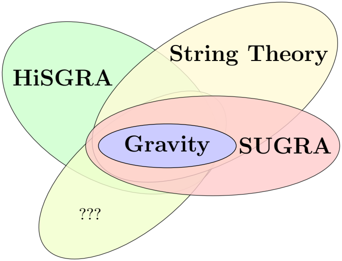
- $s = 1$: Yang-Mills, Lie algebras
- $s = 3/2$: SUGRA and supergeometry, graviton $\in$ spectrum
- $s = 2$ (graviton): GR and Riemann Geometry, no color

- $s > 2$: HiSGRA and String theory, ∞ states, graviton is there too!

Why higher spins?

Various examples (not all)

- string theory
- divergences in (SU)GRA's
- Quantum Gravity via AdS/CFT



seem to indicate that quantization of gravity requires

- infinitely many states
- the spectrum is unbounded in spin

HiSGRA is to find the most minimalistic extension of gravity by massless, i.e. gauge, higher spin fields. Vast gauge symmetry should render it finite.

Quantizing Gravity via HiSGRA =? Constructing Classical HiSGRA

Plan

- Some excuses not to work on HiSGRA (no-go's)
- Proof of the concept: one-loop finiteness of Chiral HiSGRA & $3d$ bosonization duality via holography
- New/old covariant formulation of HS fields from Twistors
- Gauge/gravitational interactions and manifestly covariant HS-theories that feature them both in flat and AdS

What Higher Spin Problem is: Field theory approach

A massless spin-s particle can be described by a rank-s tensor

$$\delta\Phi_{\mu_1\ldots\mu_s} = \nabla_{\mu_1}\xi_{\mu_2\ldots\mu_s} + \text{permutations}$$

which generalizes $\delta A_\mu = \partial_\mu \xi, \qquad \delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$
Fronsdal, Berends, Burgers, Van Dam, Bengtsson^2, Brink, ...

Problem: find a nonlinear completion (action, gauge symmetries)

$$S = \int (\nabla\Phi)^2 + \mathcal{O}(\Phi^3) + \ldots \qquad\qquad \delta\Phi_{\ldots} = \nabla_{.}\xi_{\ldots} + \ldots$$

and prove that it is UV-finite, hence a Quantum Gravity model.

Warning: brute force does not seem to work! (too many fields, indices, derivatives, vertices, ...) Most minimal = Hamiltonian +...

Between Scylla and Charybdis

The main HS-problem is that HiSGRA do not (want to) exist, which might be related to the complexity of the quantum gravity problem

The research is heavily constrained by many no-go theorems (there are many more no-go's than yes-go's )

It is important to know the basic ones as not to look in a dark room for a black cat that isn't there

Two types of no-go's:

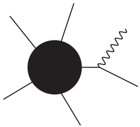
- Local: constraining options for having interesting actions $\mathcal{L}[\Phi]$ or Hamiltonians
- Global: constraining (holographic) S-matrix type observables

Check different space-times, e.g. flat and (anti)-de Sitter

Global & Flat: **asymptotic higher spin symmetry** in Minkowski

$$\delta\Phi_{\mu_1\ldots\mu_s}(x)=\partial_{\mu_1}\xi_{\mu_2\ldots\mu_s}$$

Weinberg low energy theorem (similarly, Coleman-Mandula theorem):



$$\sum_i g_i\, p^i_{\mu_1}\ldots p^i_{\mu_{s-1}}=0$$

more or less imply that $S=1$ for HiSGRA (irrespective of whether they exist or not as local field theories).

Global & AdS: **asymptotic higher spin symmetry** in anti-de Sitter

$$\delta\Phi_{\mu_1\dots\mu_s}(x) = \nabla_{\mu_1}\xi_{\mu_2\dots\mu_s} \qquad \Longleftrightarrow \qquad \partial^m J_{ma_2\dots a_s} = 0$$

$$\Updownarrow$$

free CFT

Given a CFT in $d \geq 3$ with stress-tensor J_2 and at least one higher-spin current J_s, one can prove that it is a free CFT in disguise Maldacena, Zhiboedov; Boulanger, Ponomarev, E.S., Taronna; Alba, Diab, Stanev

This is a generalization of the Weinberg and Coleman-Mandula theorems to AdS/CFT: higher spin symmetry implies

holographic $S =$ **free CFT**

Also, as AdS/CFT conjectures (Sundborg; Klebanov, Polyakov; Sezgin, Sundell; Leigh, Petkou)

Global & AdS (bonus): **asymptotic slightly-broken** (Maldacena, Zhiboedov) **higher spin symmetry** in anti-de Sitter

$$\delta\Phi_{\mu_1\ldots\mu_s}(x) = \nabla_{\mu_1}\xi_{\mu_2\ldots\mu_s} \qquad \Longleftrightarrow \qquad \partial^m J_{ma_2\ldots a_s} = \frac{1}{N}[JJ] \neq \mathbf{0}$$

Large-N critical vector model (Wilson-Fisher)

$$S = \int d^3x \left((\partial\phi^i)^2 + \frac{\lambda}{4!}(\phi^i\phi^i)^2\right)$$

should be dual to the same HiSGRA (Klebanov, Polyakov), which is 'kinematics' (Hartman, Rastelli; Giombi, Yin; Bekaert, Joung, Mourad).

holographic $S =$ **Large-N Ising**

This can be extended to Chern-Simons Matter theories, (Chang, Minwalla, Sharma, Yin, Giombi, Prakash, Trivedi, Wadia; Aharony; Maldacena, Zhiboedov, ...)

We see that **asymptotic higher spin symmetries** (HSS)

$$\delta\Phi_{\mu_1\ldots\mu_s}(x) = \nabla_{\mu_1}\xi_{\mu_2\ldots\mu_s}$$

seem to completely fix (holographic) S-matrix to be

$$S_{\text{HiSGRA}} = \begin{cases} 1^{***}, & \text{flat space} \\ \text{free CFT}, & \text{asymptotic AdS, unbroken HSS} \\ \textbf{Chern-Simons Matter}, & \textbf{asymptotic AdS}_4\textbf{, slightly-broken HSS} \end{cases}$$

Trivial/known S-matrix can still be helpful for QG toy-models. That S is fixed by symmetries is to be expected/sign of integrability, (Sharapov, E.S.) for a proof of the last line.

None of the above implies that there exist actual HiSGRA that deliver such S-matrices

Local & Flat: (Aragone, Deser, Boulanger, Leclercq, Zinoviev, ...) HS fields do not want to couple to gravity, no minimal gravitational coupling:

$$S_3 = \int T_{\mu\nu}\, g^{\mu\nu} \qquad\qquad T_{\mu\nu} = \partial_\mu \Phi_{...} \partial_\nu \Phi^{\cdots} + ...$$

instead there are some $2-s-s$ interactions with $2s-2$ derivatives

$$\int \partial^{2s-4} \Phi \Phi R_{\bullet\bullet,\bullet\bullet} \sim \int g^{\bullet\bullet} \partial^{2s-2} \Phi \Phi$$

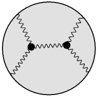
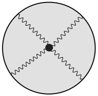
which do not induce 'diffeomorphism symmetry' $\delta\Phi_{...} = \xi^\nu \partial_\nu \Phi_{...} + ...$. Nevertheless, one can push the Noether procedure to higher orders. There are plenty of non-abelian interactions, which induce not only $\delta\Phi_{...} = \partial^{\cdots}\xi_{...}\Phi_{...}$, but also $[\delta_{\xi_1}, \delta_{\xi_2}] \neq 0$, (Bekaert, Boulanger, Sundell, ...):

The final result is 'no-go': there is no such deformation

N.B. One can solve Noether by abandoning locality (Barnich, Henneaux)

Between Scylla and Charybdis

Local & AdS: invert AdS/CFT and reconstruct the dual theory from free CFT (Bekaert, Erdmenger, Ponomarev, Sleight; Taronna, Sleight)

 $+\,t,u = \mathbf{2}\langle JJJJ\rangle$  $= -\langle JJJJ\rangle \sim \Phi^2 \dfrac{1}{\Box+\Lambda}\Phi^2$

Quartic vertex $\sim$ exchange. Field theory does not like that, which invalidates Noether procedure. No large gap, so as expected! (Heemskerk, Penedones, Polchinski, Sully)

(Maldacena, Zhiboedov, Simmons-Duffin): duals of vector models are closer to strings than to field theories, different from strongly coupled SYM

AdS/CFT allows one to get 'no-go's even quicker than in flat space, cf. (Bekaert, Boulanger, Leclerq; Roiban, Tseytlin; Taronna; Ponomarev, E.S.; ...) vs. ([Bekaert, Erdmenger, {Ponomarev}, (Sleight], Taronna))

Some issues

The actual question we want to address: **what are possible interactions among a given set of physical degrees of freedom?**

Instead, the question that is usually addressed: **what are possible interactions among a given set of Lorentz covariant fields?**

Subtlety 1. There is no(?) theorem that guarantees one can find the 'right' Lorentz covariant fields as to capture all of the required interactions.

Subtlety 2. Different formulations (dual formulations), which embed the same physical degrees of freedom into different covariant fields, capture only subsets of possible interactions, e.g. (Bekaert, Boulanger, Henneaux)

Updates on the basic HS folklore

Flat vs. AdS: in the past it looked like AdS is better than flat

For any triplet of helicities λ_i, $\lambda_1+\lambda_2+\lambda_3>0$ there is a unique interaction vertex (Brink, Bengtsson^2, Linden, 1983-87):

$$V_3 \sim C_{\lambda_1,\lambda_2,\lambda_3}[12]^{\lambda_1+\lambda_2-\lambda_3}[23]^{\lambda_2+\lambda_3-\lambda_1}[31]^{\lambda_3+\lambda_1-\lambda_2} \oplus \text{c.c}$$

$(\pm1,\pm1,\mp1)$ gives Yang-Mills vertex; $(\pm2,\pm2,\mp2)$ is Einstein-Hilbert

Surprise 1: $(\pm s,\pm2,\mp s)$ gives 2-derivative gravitational interaction! The same is via spinor-helicity, i.e. this is not a 'weird' light-cone feature.

Surprise 2: There is one-to-one between flat and AdS cubic vertices, (Metsaev; Nagaraj, Ponomarev). AdS and Flat kinematics are the same

In invariant terms: there is no difference between HS in Flat and AdS

HiSGRA's that survived

Quantizing Gravity via HiSGRA = Constructing Classical HiSGRA

Therefore, HiSGRA can be good probes of the Quantum Gravity Problem

Four classes of local HiSGRA in 2021 so far

3d massless and partially-massless (Blencowe; Bergshoeff, Blencowe, Stelle; Campoleoni, Fredenhagen, Pfenninger, Theisen; Henneaux, Rey; Gaberdiel, Gopakumar; Grumiller; Grigoriev, Mkrtchyan, E.S.; ...), $S = S_{CS}$ for a higher spin extension of $sl_2 \oplus sl_2$

$$S = \int \omega d\omega + \tfrac{2}{3}\omega^3$$

3d conformal (Pope, Townsend; Fradkin, Linetsky; Kuzenko; Grigoriev, Lovrekovic, E.S.), $S = S_{CS}$ for higher spin extension of $so(3,2)$

4d conformal (Tseytlin, Segal; Bekaert, Joung, Mourad; Kuzenko, ...), higher spin extension of Weyl gravity, local Weyl symmetry tames non-localities

$$S = \int \sqrt{g}\,(C_{\mu\nu,\lambda\rho})^2 + ...$$

4d massless chiral (Metsaev; Ponomarev, E.S.; Ponomarev; E.S., Tran, Tsulaia; E.S.). The smallest higher spin theory with propagating fields. **This talk!**

The theories avoid all no-go's. Surprisingly, all of them have simple actions and are clearly well-defined, as close to Field Theory as possible

Living on Light Front

Unless we want/can bootstrap the S-matrix directly, we may resort to local field theory methods: we can take the physical d.o.f. and attempt to construct generators of translations P_μ and Lorentz transformations $J_{\mu\nu}$ directly

Gauge symmetry is just a redundancy

The light-cone gauge eliminates all unphysical d.o.f., e.g. $SU(N)$ YM is the theory of (N^2-1) $\Phi_{\pm 1}$ scalars and gravity is a theory of two scalars $\Phi_{\pm 2}$

$$[J^{a-}, J^{b-}] = 0 \qquad\qquad [J^{a-}, P^-] = 0$$

e.g. $P^i = \int \Phi_{-s} p^i \Phi_{+s}$. Many important results have been first obtained in the light-cone gauge: quantization of strings (Goddard, Goldstone, Rebbi, Thorn), finiteness of $\mathcal{N}=4$ SYM (Mandelstam; Brink, Lindgren, Nilsson)

$$4d: \quad \Phi_{\mu_1\ldots\mu_s}(x) \qquad \Longrightarrow \qquad \Phi_{\pm s}(x)$$

Self-dual Yang-Mills in Lorentzian signature is a useful analogy

- the theory is non-unitary due to the interactions ($\boldsymbol{A_\mu \to \Phi^\pm}$)

$$\boldsymbol{\mathcal{L}_{\text{YM}} = \frac{1}{4}\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu}}$$

$$\wr\wr$$

$$\boldsymbol{\mathcal{L}_{\text{SDYM}} = \Phi^- \Box \Phi^+ + V^{++-} + {\color{red}V^{--+} + V^{++--}}}$$

- tree-level amplitudes vanish, $A_{\text{tree}} = 0$
- one-loop amplitudes do not vanish, are rational and coincide with $(+ + ... +)$ of pure QCD

Self-dual Theories

- actions are not real in Minkowski space
- actions are simpler than the complete theories
- integrability, instantons (Atiyah, Hitchin, Drinfeld, Manin; ...)
- **SD theories are consistent truncations**, so anything we can compute will be a legitimate observable in the full theory; **any solution of SD is a solution of the full**; ...
- different expansion schemes: instantons instead of flat, MHV, etc.

In general: amplitudes (MHV, BCFW, double-copy, ...), strings, QFT, Twistors, ... encourage to go outside Minkowski

In higher spins: little explored (Adamo, Hähnel, McLoughlin; E.S., Ponomarev; Ponomarev), can be the only reasonably local theories

Chiral HiSGRA (Metsaev; Ponomarev, E.S.) has all $s = 0, 1, 2, 3, ...$

- the theory is 'non-unitary' due to $\lambda_1 + \lambda_2 + \lambda_3 > 0$ in the vertex

$$\mathcal{L} = \sum_{\lambda} \Phi^{-\lambda} \Box \Phi^{+\lambda} + \sum_{\lambda_i} \frac{\kappa\, l_{\mathrm{Pl}}^{\lambda_1+\lambda_2+\lambda_3-1}}{\Gamma(\lambda_1+\lambda_2+\lambda_3)} V^{\lambda_1,\lambda_2,\lambda_3}$$

light-cone gauge is very close to the spinor-helicity language

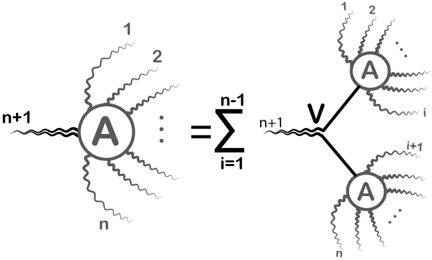
$$V^{\lambda_1,\lambda_2,\lambda_3} \sim [12]^{\lambda_1+\lambda_2-\lambda_3} [23]^{\lambda_2+\lambda_3-\lambda_1} [13]^{\lambda_1+\lambda_3-\lambda_2}$$

The relation to SDYM is deeper than it seems (Ponomarev), also similar to self-dual Gravity (Siegel; Krasnov)

This is the smallest higher spin theory and it is unique.
Graviton and scalar field belong to the same multiplet

Self-dual Theories

- the tree-level amplitudes can be computed a la Berends-Giele and found to vanish, $A_{\text{tree}} = 0$, just like in SDYM (E.S., Tsulaia, Tung)



$$\sim \boldsymbol{p}^2 \sim 0$$

No UV Divergences! One-loop finiteness

The interactions are naively non-renormalizable, the higher the spin the more derivatives:

$$\boldsymbol{V^{\lambda_1,\lambda_2,\lambda_3} \sim \partial^{|\lambda_1+\lambda_2+\lambda_3|}\Phi^3}$$

but there are **no UV divergences!** (E.S., Tsulaia, Tung). Some loop momenta eventually factor out, just as in $\mathcal{N}=4$ SYM, but ∞ many more times. One-loop amplitudes do not vanish, but are rational

At one loop we find three factors: (1) SDYM or all-plus 1-loop QCD; (2) higher spin dressing to account for λ_i; (3) total number of d.o.f.:

$$\boldsymbol{A}^{\text{1-loop}}_{\text{Chiral}} = \boldsymbol{A}^{++\ldots+}_{\text{QCD,1-loop}} \times \boldsymbol{D}^{\text{HSG}}_{\boldsymbol{\lambda_1,\ldots,\lambda_n}} \times \underbrace{\sum_{\lambda} 1}_{\begin{subarray}{c} \| \\ 0 \end{subarray}}$$

where # d.o.f.$=\sum_\lambda 1 = 1+2\sum_{\lambda>0} 1 = 1+2\zeta(0)=0$ to comply with no-go's, (Beccaria, Tseytlin) and agrees with many results in AdS, where $\neq 0$

Chiral HSGRA in Minkowski

- **stringy 1**: the spectrum is infinite $s = 0, (1), 2, (3), 4, \ldots$
- **stringy 2**: admit Chan-Paton factors, $U(N)$, $O(N)$ and $USp(N)$
- **stringy 3**: we have to deal with spin sums $\sum_s$ (worldsheet takes care of this in string theory) and ζ-function helps
- **stringy 4**: the action contains parts of YM and Gravity
- **stringy 5**: higher spin fields soften amplitudes
- consistent with Weinberg etc. $S = 1^{***}$ (in Minkowski)
- gives all-plus QCD or SDYM amplitudes from a gravity

The Minkowski background is not the only one for HiSGRA. If we can jump to AdS then all drawbacks will turn into virtues.

Chiral HSGRA in AdS

With the help of Metsaev, 2018 it is possible to uplift Chiral Theory to AdS_4. Now it is less trivial

- it is not obstructed by nonlocalities (follows from Lorentz symmetry)
- flat space story should guarantee the absence of UV-divergences in AdS: Chiral HSGRA should be a consistent quantum gravity toy-model
- **holographic three-point function are known and are not trivial and do not belong to any free CFT**
- using the uniqueness of the Chiral block one can prove the three-dimensional bosonization duality at 3-point level, which is gives concrete formulas for the correlators (to be checked from CFT side)

What can be done?

- All HSGRA in $4d$ need all possible vertices save for scalar self-coupling, **but some couplings are not writable with Fronsdal's $\Phi_{\mu_1\ldots\mu_s}$ fields**
- There are some contractions of Chiral HSGRA, which have 1- and 2-derivative interactions — gauge and gravitational (Ponomarev)
- It would be great to find Lorentz covariant approach/form for all the vertices, especially, the gravitational ones in flat space (gauge symmetry is a redundancy till you ask about geometry or need to find an exact solution, etc.)
- It cannot be Fronsdal's, but there are other people to choose from. Let's 'ask' Penrose and pals, (Krasnov, E.S., Tung; Krasnov, E.S.)

Vector-spinor dictionary

Each μ equals AA' where $A, B, \ldots = 1,2$ and $A', B', \ldots = 1,2$

$$\sigma^{AA'}_{\mu} v^{\mu} = v^{AA'} \qquad\qquad v = \begin{pmatrix} t+x & y+iz \\ y-iz & t-x \end{pmatrix}$$

In general we have $V^{A(n),A'(m)}$ and all indices are symmetric. The only anti-symmetric object is invariant $\epsilon_{AB} = -\epsilon_{BA}$, idem. for $\epsilon_{A'B'}$. Abstract Penrose notation:

$$\begin{aligned} \text{Maxwell}: &\qquad F_{\mu\nu} = F_{AB}\epsilon_{A'B'} + \epsilon_{AB}F_{A'B'} \\ \text{Weyl}: &\qquad C_{\mu\nu,\lambda\rho} = C_{ABCD}\epsilon_{A'B'}\epsilon_{C'D'} + \epsilon_{AB}\epsilon_{CD}C_{A'B'C'D'} \\ \text{Traceless}: &\qquad \Phi_{\mu(s)} = \Phi_{A(s),A'(s)} \end{aligned}$$

Any of $V^{A(n),A'(m)}$ with $n+m=2s$ can describe a spin-s field. For $n=m=s$ we have a symmetric/Hermitian description. For $m=2s$, $n=2s$ we have (conjugate) Weyl tensors $\Psi^{A(2s)}$, $\Psi^{A'(2s)}$.

Twistor-inspired approach

Twistors treat positive and negative helicities differently:

$$\nabla_B{}^{A'}\,\Psi^{BA(2s-1)} = 0 \qquad \text{(Penrose, 1965)}$$

$$\nabla^A{}_{B'}\,\Phi^{A(2s-1),B'} = 0 \qquad \delta\Phi^{A(2s-1),B'} = \nabla^{AB'}\xi^{A(2s-2)}$$

Known since (Eastwood, Penrose, Wells, 1981), (Hitchin, 1980) almost derived an action

$$S = \int \sqrt{g}\,\Psi^{BA_2\ldots A_{2s}}\nabla_B{}^{B'}\,\Phi_{A_2\ldots A_{2s},B'}$$

Feature: allow us to put higher spins on any self-dual background, not just flat or (A)dS, c.f. Conformal HS (Adamo, Hähnel, McLoughlin)

N.B: for $s=1$ we have Ψ^{AB} and $A^{CC'}$, for $s=2$ Ψ^{ABCD} and $\Phi^{AAA,A'}$

Q: what about other interactions?

Twistor-inspired approach

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$$\nabla_B{}^{A'}\,\Psi^{BA(2s-1)} = 0 \qquad \text{(Penrose, 1965)}$$

$$\nabla^A{}_{B'}\,\Phi^{A(2s-1),B'} = 0 \qquad \delta\Phi^{A(2s-1),B'} = \nabla^{AB'}\xi^{A(2s-2)}$$

(Hitchin, 1980) entertains a possibility to introduce a connection

$$\omega^{A(2s-2)} \ni e_{BB'}\Phi^{A(2s-2)B,B'} \qquad \delta\omega^{A(2s-2)} = \nabla\xi^{A(2s-2)}$$

where $e_{AA'}$ is the vierbein and with $H^{AB} \equiv e^A{}_{C'} \wedge e^{BC'}$ we can write

$$\boldsymbol{S = \int \Psi^{A(2s)} \wedge H_{AA} \wedge \nabla\omega_{A(2s-2)}}$$

which is also invariant under $\delta\omega^{A(2s-2)} = e^A{}_{C'}\,\eta^{A(2s-3),C'}$ to get rid of the extra component. The simplest action for HS.

Surprise: presymplectic-AKSZ (Grigoriev et al) naturally contains the same action (E.S., Sharapov) from Hochschild cohomology of HS algebra

Self-dual Yang-Mills

With $F^2_{\mu\nu} = F^2_{AB} + F^2_{A'B'}$ and with $F \wedge F = F^2_{AB} - F^2_{A'B'}$ being topological we can massage YM action

$$S_{YM} = \frac{1}{g^2} \int F^2_{\mu\nu} \sim \frac{1}{g^2} \int F^2_{AB} \sim \int \boldsymbol{\Psi^{AB} F_{AB} - \frac{g'}{2} \Psi^2_{AB}} \,,$$

which is not manifestly real! The first part is the SDYM action

$$S_{\mathsf{SDYM}}[\Psi, \omega] = \int \Psi^{CD} F_{CD}(\omega) = \int \Psi^{CD} H_{CD} \wedge d\omega + \ldots$$

where we see the familiar action

As different from the flat space perturbation theory, we find an expansion of YM over SDYM, which is quite useful (Adamo et al; Chicherin et al; ...)

Higher Spin Self-dual Yang-Mills

Let's take $\omega^{A(2s-2)}$ and $\Psi^{A(2s)}$ and let them take values in some (matrix) Lie algebra, then the action

$$S = \sum_n \mathrm{tr} \int \Psi^{A(2s)} H_{AA} \wedge F_{A(2s-2)}$$

where all A's are symmetrized inside F

$$F = d\omega + \omega \wedge \omega \qquad\qquad \omega = \sum_n (\omega^{A(2s)})^i{}_j \; y_A ... y_A$$

is invariant under (thanks to $H_{AA} e_{AB'} \equiv 0$)

$$\delta\omega = \nabla\xi + [\omega, \xi] \qquad\qquad \delta\omega^{A(2s-2)} = e^A{}_{C'} \, \eta^{A(2s-3),C'}$$

Feature: describes gauge, one-derivative, interactions of higher spin fields that are inaccessible via Fronsdal's approach, c.f. Chalmers-Siegel

Self-dual Gravity

Let us start with the frame-like gravity ($H^{AB} \equiv e^{A}{}_{C'} \wedge e^{BC'}$)

$$S = \int H_{AA} \wedge R^{AA} + \tfrac{1}{2}\Lambda\, H_{AA}H^{AA}\,, \qquad R = d\omega^{AA} + \omega^{A}{}_{B} \wedge \omega^{BA}$$

If we want to make H^{AA} an independent field, we have to remember $H^{AA} \wedge H^{AA} = 0$, which can be imposed via

$$S[\omega, H, \Psi] = \int H_{AA} \wedge R^{AA} + \tfrac{1}{2}\Lambda\, H_{AA}H^{AA} + \tfrac{1}{2}\Psi^{AAAA} H_{AA}H_{AA}$$

Now we solve for H via $R + (\Lambda + \Psi)H = 0$ and expand in Ψ to get

$$S[\omega, \Psi] = \int R^{AA} \wedge R_{AA} + \boldsymbol{\Psi^{AAAA}\, R_{AA} \wedge R_{AA}} + \ldots$$

The first term is topological, the second is SDGRA (Krasnov). Dropping ω^2 we get SDGRA in flat $\int \boldsymbol{\Psi^{AAAA}\, d\omega_{AA} \wedge d\omega_{AA}}$ (E.S., Krasnov)

HS-SDGRA: Flat Space

In flat space we can simply write

$$S = \sum_{m,n} \int \Psi^{A(n+m)}\, d\omega_{A(n)} \wedge d\omega_{A(m)}$$

There is a special vacuum $\omega^{AA} = x^{A}{}_{C'}\, dx^{AC'}$, $d\omega^{AA} = H^{AA}$ and we find

$$S = \sum_{n} \int \Psi^{A(n+2)}\, H_{AA} \wedge d\omega_{A(n)} + \sum_{m,n} \int \Psi^{A(n+m)}\, d\omega_{A(n)} \wedge d\omega_{A(m)}$$

The theory is invariant under (new inner product $\overrightarrow{i_\eta} d\omega$):

$$\delta\omega^{A(n)} = d\xi^{A(n)} + \eta^{A(n-k);\mu} \frac{\partial}{\partial\, dx^\mu} (d\omega^{A(k)})$$

Feature: describes gravitational, two-derivative, interactions of higher spin fields that are inaccessible via Fronsdal's approach, c.f. Siegel

HS-SDGA: AdS Space

Instead of star-product we take Poisson algebra on y^A:

$$\{f,g\} = \partial_C f(y)\,\partial^C g(y) = \sum f_{A(n-1)C}\, g^C{}_{A(m-1)}\; y^A ... y^A$$

Curvature $F = d\omega + \frac{1}{2}\{\omega,\omega\}$ contains the Riemann two-form. The action

$$S = \sum_{m,n} \int \boldsymbol{\Psi}^{A(n+m)}\, \boldsymbol{F}_{A(n)} \wedge \boldsymbol{F}_{A(m)} = \langle \Psi \mid F \wedge F \rangle$$

is invariant under

$$\delta\omega = d\xi + \{\omega,\xi\} + \overrightarrow{i_\eta} F \qquad\qquad F = d\omega + \tfrac{1}{2}\{\omega,\omega\}$$

$$\delta\Psi = \Psi \circ \xi + R\overleftarrow{i_\eta} \qquad\qquad R = d\Psi - \Psi \circ \omega$$

where there are some dual operations

$$\langle f \mid \{\xi, g\}\rangle = \langle f \circ \xi \mid g\rangle \qquad\qquad \left\langle R \,\middle|\, \overrightarrow{i_\eta} X \right\rangle = \left\langle R\overleftarrow{i_\eta} \,\middle|\, X \right\rangle$$

HS-SDGRA: AdS Space

$$\boldsymbol{S = \langle \Psi \mid F \wedge F \rangle}$$

can be expanded over AdS to find

$$S = \left\langle \Psi \,\middle|\, y^A y^A H_{AA} \wedge \nabla\omega \right\rangle + \langle \Psi \mid \nabla\omega \wedge \nabla\omega \rangle + \ldots + \text{quintic terms}$$

Feature 1: describes gravitational, two-derivative, interactions of higher spin fields that are inaccessible via Fronsdal's approach, but $\in$ FV-vertex

Feature 2: this is a complete, Lorentz and gauge invariant action for HS

Noether vs. Geometry: Crucially, there is an off-shell shift symmetry with HS i_η, which is hard to capture via Noether or FDA (unfolding):

$$\delta\omega : \qquad e^A{}_{C'}\, \eta^{A(2s-3),C'} \quad \text{vs.} \quad i_\eta F = \eta^{A(k);\nu} F^{A(n)}_{\nu\mu}$$

The equations do not have an FDA-friendly form yet!

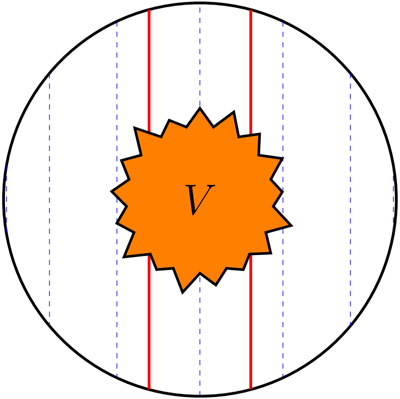
Concluding Remarks

- There are some free spots on the HS no-go's minefield
- Proof of the HiSGRA-concept: Chiral HiSGRA is local, one-loop finite theory with a graviton; AdS/CFT correlators give new predictions for Chern-Simons matter theories and $3d$ bosonization
- Twistor-inspired formulation of HS has nice features. We can construct manifestly Lorentz covariant interactions that are indispensable for consistency of HS and are inaccessible via Fronsdal's (flat)
- There are some HS theories that extend SDYM and SDGRA (Ponomarev), now written manifestly covariantly. Flat limit: Poisson $\rightarrow$ Commutative, vanishing coupling
- Future: quantization and AdS/CFT (expectation: UV-finite and subsector of Chern-Simons matter correlators); other interactions in the new approach; Twistor actions; SD-HS in other d; ...

Thank you for your attention!

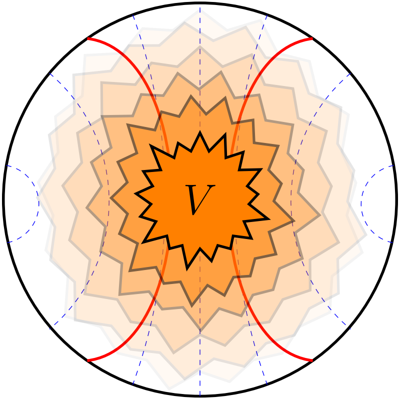
May the higher spin force be with you

Flat space



$$S = \text{QCD}^{+++}_{\text{1-loop}} \times \text{HS} \times \sum_{\lambda} 1$$

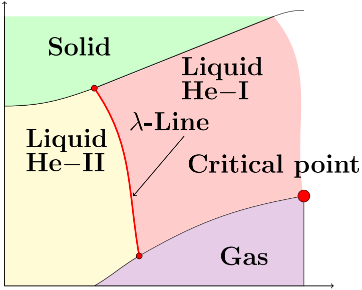
(anti)-de Sitter

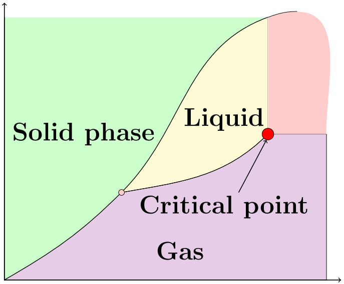


$$S \subset \text{Chern-Simons Matter}$$

The smallest higher spin extension of graviton

Chern-Simons Matter Theories and bosonization duality





Chern-Simons Matter theories and dualities

In AdS_4/CFT_3 one can do much better — there exists a large class of models, Chern-Simons Matter theories (extends to ABJ(M))

$$\frac{k}{4\pi}S_{CS}(A) + \text{Matter}\begin{cases}(D\phi^i)^2 & \textbf{free boson}\\ (D\phi^i)^2 + g(\phi^i\phi^i)^2 & \textbf{Wilson-Fisher (Ising)}\\ \bar{\psi}\not{D}\psi & \textbf{free fermion}\\ \bar{\psi}\not{D}\psi + g(\bar{\psi}\psi)^2 & \textbf{Gross-Neveu}\end{cases}$$

- describe physics (Ising, quantum Hall, ...)
- break parity in general (Chern-Simons)
- two parameters $\lambda = N/k$, $1/N$ (λ continuous for N large)
- exhibit remarkable dualities, e.g. ***3d* bosonization duality** (Aharony, Alday, Bissi, Giombi, Karch, Maldacena, Minwalla, Prakash, Seiberg, Tong, Witten, Yacobi, Yin, Zhiboedov, ...)

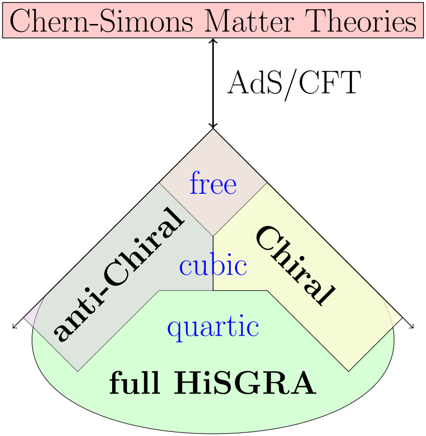
Chern-Simons Matter theories and dualities

$$
\begin{tikzcd}
\textbf{free boson} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"'] & \textbf{Gross-Neveu} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"] \\
\textbf{Wilson-Fisher} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] & \textbf{free fermion} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"]
\end{tikzcd}
$$

The simplest gauge-invariant operators are $J_s = \phi D ... D\phi$ or $J_s = \bar{\psi}\gamma D ... D\psi$, which are dual to higher spin fields.

$\gamma(J_s)$ at order $1/N$ (Giombi, Gurucharan, Kirillin, Prakash, E.S.) confirm the duality. 4, 5-loop $1/N^2$ results in Gross-Neveu and Wilson-Fisher (Manashov, E.S., Strohmaier) seem hard to extend in λ.

Chiral HiSGRA and Chern-Simons Matter



(anti)-Chiral Theories are rigid;

they must be closed subsectors;

just need to glue them together to get all 3-pt functions in CS-Matter

gluing depends on one parameter, which is introduced via simple EM-duality rotation $\Phi_{\pm s} \to e^{\pm i\theta}\Phi_{\pm s}$

Bosonization is manifest!

(anti)-Chiral Theories provide a complete base for 3-pt amplitudes

$$\boldsymbol{V_3 = V_{chiral} \oplus \bar{V}_{chiral} \quad \leftrightarrow \quad \langle JJJ \rangle}$$

Maldacena, Zhiboedov found out/conjectured the $3pt$-functions in CS-Matter theories to be (θ is related to N, k in a complicated way):

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle \sim \cos^2\theta \langle JJJ\rangle_b + \sin^2\theta \langle JJJ\rangle_f + \cos\theta\sin\theta \langle JJJ\rangle_o$$

Follow from slightly-broken higher spin symmetry: $\partial \cdot J = \frac{1}{N}[JJ]$

We get all the (missing) three-point functions, which is the first prediction of HiSGRA that is ahead of the CFT side

The free parameter θ is related to $U(1)$ EM duality rotations, $\Phi_{\pm s} \rightarrow e^{\pm i\theta}\Phi_{\pm s}$

This approach has good chances to prove the $3d$ bosonization duality provided extended to higher point functions — the correlators of J_s's get fixed irrespective of what the constituents are (bosons or fermions)!

Helicity in AdS and CFT

A massless spin-s field in AdS_4 is equivalent to two scalars

$$\Phi_{\mu_1\ldots\mu_s}(x,z) \qquad \Longleftrightarrow \qquad \Phi_{\pm s}(x,z)$$

A conserved spin-s tensors in CFT$_3$ is equivalent to two scalars

$$\partial^m J_{ma_2\ldots a_s}(x)=0 \qquad \Longleftrightarrow \qquad J_{\pm s}(x)$$

Thanks to the light-cone gauge we have the following relation

$$\begin{tikzcd}
\boxed{\begin{array}{c}3d\ \textbf{Minkowski}\\ \text{3-pt correlators, } \langle JJJ\rangle\end{array}} \arrow[rr, leftrightarrow, "\text{AdS/CFT}"] \arrow[dr, leftrightarrow, bend right] & & \boxed{\begin{array}{c}4d\ \textbf{anti-de Sitter}\\ \text{cubic vertices of } \Phi_s\end{array}} \arrow[dl, leftrightarrow, bend left] \\
& \boxed{\begin{array}{c}4d\ \textbf{Minkowski}\\ \text{amplitudes, } [12]^\bullet[32]^\bullet[31]^\bullet,\ \langle 12\rangle^\bullet\langle 32\rangle^\bullet\langle 31\rangle^\bullet\end{array}} &
\end{tikzcd}$$

Helicity is a useful concept for $3d$ CFT's, especially if we consider conserved currents, also recently (Caron-Huot, Li)