ESI **The Erwin Schrödinger International Institute for Mathematical Physics**

Boltzmanngasse 9/2
A-1090 Vienna, Austria

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Quantitative Studies of Nonlinear Wave Phenomena (follow-up) January 23-29, 2011

Organizers: P. C. Aichelburg (Vienna) and P. Bizoń (Cracow)

The main goal of this one-week meeting is to discuss the progress in projects that have been initiated during the workshop in February 2010.

Monday, January 24

Birgit Schörkhuber (Vienna) 14:30-15:00

On linear stability of self-similar blow-up for co-rotational wave maps

Abstract: Co-rotational wave maps from (3+1)-Minkowski space into the three-sphere (the SU(2)-Sigma Model) are known to exhibit finite time blow-up via self-similar solutions. Based on numerical investigation the self-similar ground state solution, which is known in closed form, is supposed to describe the generic blow-up behaviour of the system. We present a rigorous linear perturbation theory around the ground state solution and show that it is linearly stable if it is mode stable. The proof is based on the formulation of the linearized equation in adapted coordinates as an ODE in a suitable function space, which is then investigated by means of strongly-continuous one-parameter semigroups supplemented by spectral analysis of the generator.

Roland Donninger (Lausanne) 15:10-16:00

Stable self-similar blow-up for co-rotational wave maps

Abstract: I will present some recent results, partly in collaboration with B. Schörkhuber and P. C. Aichelburg, on the blow-up characteristics for corotational wave maps from physical Minkowski space to the 3-sphere (the SU(2) sigma model). The equation is energy supercritical and is known to exhibit self-similar solutions. I show that the blow-up described by the ground state self-similar solution is stable. This result only holds in a sufficiently strong topology, strictly stronger than the energy, which is a manifestation of the energy supercritical character of the equation. The argument relies on a novel semigroup formulation of the linearized problem in adapted coordinates and the implicit function theorem on Banach spaces.

Tuesday, January 25

Piotr Bizoń (Cracow) 10:30-11:20

Continuation beyond blowup in the heat flow for harmonic maps

Abstract: I will describe recent joint work with Pawel Biernat on singularity formation in the heat flow for harmonic maps into spheres. In this work we construct global weak solutions having the form of shrinking and expanding self-similar solutions glued together across a singularity. We show that in the generic case such a weak solution is unique.

Pawel Biernat (Cracow) 11:20-11:40

Numerical simulations of blowup in the heat flow for harmonic maps

Abstract: I will present numerical evidence supporting the conjectures formulated in Piotr Bizoń's talk.

Wednesday, January 26

Nikodem Szpak (Duisburg-Essen) 13:15-14:15

Dynamics near the threshold for blowup in nonlinear Klein-Gordon equations

Abstract: We consider a class of focusing nonlinear Klein-Gordon equations in one space dimension allowing for blowup of regular data in finite time. The dynamics near the borderline between blowup and dispersion is attracted by a single static solution. Linear analysis and numerical calculations show existence of three dynamical regimes depending on the power of the nonlinearity: fast, slow, or very slow convergence to the attractor. The key role is played by resonances, standing waves, and other spectral properties of the linearized evolution operator. We derive analytic formulae for the frequencies and decay powers as well as spatial profiles of various modes. This is joint work with Tadeusz Chmaj and Piotr Bizoń.

Thursday, January 27

Alan Rendall (AEI) 14:00-15:00

Self-similar collapse of collisionless matter

Abstract: In this talk I will explain work done in collaboration with Juan Velazquez on the existence of self-similar solutions of the Einstein-Vlasov system. This is motivated by the wish to understand more about the formation of singularities in solutions of this system and the issue of cosmic censorship. By assuming a suitable form of the solution the existence problem is reduced to the construction of a particular kind of solution of a four-dimensional system of ODE. This solution is constructed by means of a shooting argument, a kind of argument which is common in numerics but not used so often for analytical purposes. I will explain some of the main points of the existence proof in this example and use it to throw some light on the use of the shooting technique in general.

Joanna Jalmużna (Cracow) 15:30-16:00

Focusing semilinear wave equations on the Schwarzschild spacetime

Abstract: We consider semilinear wave equations with a focusing power nonlinearity on the Schwarzschild background. We show that a static solution with one unstable mode acts as an intermediate attractor in the evolution of initial data lying at the threshold of blowup. We describe in detail the saddle-point dynamics near this attractor using linear perturbation analysis and numerical simulations.

Andrzej Rostworowski (Cracow) 16:10-16:40

Numerical investigation of a perturbed AdS spacetime

Abstract: We investigate numerically small perturbations of the AdS spacetime, hoping to get insight into the problem of nonlinear stability/instability of the AdS spacetime. This is work in progress so the purpose of the talk is mainly to present the project rather than give any firm results.

All lectures take place in the ESI Boltzmann Lecture Hall.