Constraints on Kahler moduli space of 6d N=1 Supergravity

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POSTECH

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The Landscape vs. the Swampland

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Yuta's talk

String Landscape of 6d N=1 supergravities

- F-theory compactified on elliptic CY 3-folds [Vafa 96], [Morrison, Vafa 96],...
- Non-geometric constructions : asymmetric or quasicrystalline orbifolds [Baykara, Hamada, Tarazi, Vafa 23], [Baykara, Tarazi, Vafa 24], ...

Classifications of anomaly free 6d theories

- Number of 6d N=1 supergravities is finite when $T < 9$ [Kumar, Morrison, Taylor 10]
- T=0 theories with SU(N) gauge algebra [Kumar, Park, Taylor 10]
- Systematic classifications for generic gauge algebra and matter representations [Hamada, Loges 23, 24]

Non-perturbative constraints from string probes

- Bounds on rank of gauge algebra [HCK, Shiu, Vafa 19], [Lee, Weigand19]
- Matter representations [Tarazi, Vafa 21]

I will talk about new constraints on 6d supergravities, based on

5d Compactification

Higgsing

Unitarity of BPS strings

Classification of 5d SCFTs

All together give new constraints and predictions of QG

6d Supergravity

Yuta's talk

Consider 6d $\mathcal{N} = (1, 0)$ supergravity theories (preserving 8 SUSY)

Matter content :

- Gravity multiplet $(g_{\mu\nu}, B^+_{\mu\nu}, \Psi_\mu)$
- T tensor multiplets $(B^-_{\mu\nu}, j, \psi)$
- V vector multiplets (A_μ, λ)
- H hypermultiplets (q, Ψ)
- Matter representations are constrained by anomaly cancellations.

6d Supergravity

Yuta's talk

Consider 6d$\mathcal{N}=(1,0)$ supergravity theories (preserving 8 SUSY)

Matter content : gravity + tensor (T) + vector (V) + hyper (H)

- Matters are constrained by anomaly cancellations.

[Green, Schwarz 84], [Sagnotti 92]

$$H-V=273-29T\ ,\quad a\cdot a=9-T\ ,$$

$$B^i_{\mathbf{adj}}=\sum_{\mathbf{r}}n^i_{\mathbf{r}}B^i_{\mathbf{r}}\ ,\quad a\cdot b_i=\frac{\lambda_i}{6}\left(A^i_{\mathbf{adj}}-\sum_{\mathbf{r}}n^i_{\mathbf{r}}A^i_{\mathbf{r}}\right)\ ,$$

$$b_i\cdot b_i=\frac{\lambda_i^2}{3}\left(\sum_{\mathbf{r}}n^i_{\mathbf{r}}C^i_{\mathbf{r}}-C^i_{\mathbf{adj}}\right)\ ,\quad b_i\cdot b_j=2\lambda_i\lambda_j\sum_{\mathbf{r},\mathbf{s}}n^{ij}_{\mathbf{r},\mathbf{s}}A^i_{\mathbf{r}}A^j_{\mathbf{s}}\quad i\neq j$$

with anomaly coefficients $a,b_i\in\mathbb{R}^{1,T}$ and group theory factors

$$\mathrm{tr}_{\mathbf{r}}F^2=A_{\mathbf{r}}\mathrm{tr}F^2\ ,\quad \mathrm{tr}_{\mathbf{r}}F^4=B_{\mathbf{r}}\mathrm{tr}F^4+C_{\mathbf{r}}\left(\mathrm{tr}F^2\right)^2$$

for gauge group $G=G_1\times G_2\times\cdots\times G_k$

Circle compactification

Circle compactification leads to 5d N=1 supergravity :

- 6d gravity multiplet $(g_{\mu\nu}, B^{+}_{\mu\nu})$ $\longrightarrow$ gravity $(g_{\mu\nu}, A^{0}_{\mu})$ + vector
- tensor $B^{-}_{\alpha,\mu\nu}$ + vector A^{i}_{μ} $\longrightarrow$ vector multiplets $A^{\alpha}_{\mu} + A^{i}_{\mu}$
- hypermultiplets $\longrightarrow$ hypermultiplets

Effective Chern-Simons (CS) Lagrangian on Coulomb branch (parametrized by real scalar VEVs in 5d vector multiplets) :

$$\mathcal{L}^{5d}_{CS} = -\frac{9-T}{24} A^0 \wedge F^0 \wedge F^0 - \frac{1}{2} A^0 \wedge (\Omega_{\alpha\beta} F^{\alpha} \wedge F^{\beta}) + \frac{1}{4} \sum_i b^i_{\alpha} \cdot (A^{\alpha} - \tfrac{1}{2} a^{\alpha} A^0) \wedge \mathrm{tr}(F^i \wedge F^i) + \mathcal{L}_{\mathrm{z.m}}$$

$$F^{\alpha} = dA^{\alpha} \ (\alpha = 0, 1, \cdots, T)$$
$$F^{i} = dA^{i} \ (i = 1, \cdots, \mathrm{Rk}G_i)$$

$$\mathcal{L}_{\mathrm{z.m}} = -\frac{1}{6} C^{\mathrm{z.m}}_{abc} A^a \wedge F^b \wedge F^c \ , \quad C^{\mathrm{z.m}}_{abc} = \partial_a \partial_b \partial_c \mathcal{F}_{\mathrm{z.m}}(\phi)$$

$$\mathcal{F}_{\mathrm{z.m}} = -\frac{1}{12} \left(\sum_{e \in \mathbf{R}} |e \cdot \phi|^3 - \sum_f \sum_{w \in \mathbf{w}_f} |w(\phi)|^3 \right) \ ,$$

[Intriligator, Morrison, Seiberg 97], [Bonetti, Grimm 11], [Bonetti, Grimm, Hohenegger 13], …

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KK 1-loop

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$$F^\alpha = dA^\alpha \quad (\alpha = 0, 1, \cdots, T)$$
$$F^i = dA^i \quad (i = 1, \cdots, \mathrm{Rk}G_i)$$

GS-terms

$$\mathcal{L}_{\mathrm{z.m}} = -\frac{1}{6} C^{\mathrm{z.m}}_{abc} A^a \wedge F^b \wedge F^c \,, \quad C^{\mathrm{z.m}}_{abc} = \partial_a \partial_b \partial_c \mathcal{F}_{\mathrm{z.m}}(\phi)$$

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$$F^\alpha = dA^\alpha \ (\alpha = 0, 1, \cdots, T)$$
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KK zero modes

$$\mathcal{L}_{\mathrm{z.m}} = -\frac{1}{6} C^{\mathrm{z.m}}_{abc} A^a \wedge F^b \wedge F^c \ , \quad C^{\mathrm{z.m}}_{abc} = \partial_a \partial_b \partial_c \mathcal{F}_{\mathrm{z.m}}(\phi)$$

$$\mathcal{F}_{\mathrm{z.m}} = -\frac{1}{12}\left(\sum_{e \in \mathbf{R}} |e \cdot \phi|^3 - \sum_f \sum_{w \in \mathbf{w}_f} |w(\phi)|^3 \right) ,$$

[Intriligator, Morrison, Seiberg 97], [Bonetti, Grimm 11], [Bonetti, Grimm, Hohenegger 13], …

Coulomb branch of rank-1 supergravity

Generic cubic prepotential on rank-1 Coulomb branch (or Kahler cone)

$$6\mathcal{F} = c_1\, t_1^3 + 3c_2\, t_1^2 t_2 + 3c_3\, t_1 t_2^2 + c_4\, t_2^3 \qquad \text{with} \quad c_i \in \mathbb{Z}$$

- t_1, t_2 are called Kahler cone generators with $t_1,\, t_2 \geq 0$
- Massless BPS particle states at each boundary $t_1 = 0$ and $t_2 = 0$
- Constraints on CS coefficients

  - Monopole string tension positivity : $c_i \geq 0$

  - Metric positivity :

$$\Delta_1 \equiv c_2^2 - c_1 c_3 \geq 0\ , \quad \Delta_2 \equiv c_3^2 - c_2 c_4 \geq 0$$

(same conditions from unitarity of worldsheet CFT on monopole strings [Katz, HCK, Tarazi, Vafa 20])

Coulomb branch of rank-1 supergravity

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$$\Delta_1 \equiv c_2^2 - c_1 c_3 \geq 0\ , \quad \Delta_2 \equiv c_3^2 - c_2 c_4 \geq 0$$

Near $t_2 \to 0$ boundary

1. $c_1, c_2 = 0,\ c_3 > 0\ :\ \mathcal{F} \sim t_2^2 \sim 0$, light string oscillator modes
2. $c_1 = 0,\ c_2 > 0\ :\ \mathcal{F} \sim t_2 \sim 0$, light Kaluza-Klein modes
3. $c_1 > 0,\ \Delta_1 = 0\ :\ \mathcal{F} \sim \text{finite}$, 5d CFT fixed point
4. $c_1 > 0,\ \Delta_1 > 0\ :\ \mathcal{F} \sim \text{finite}$, flop transition or Weyl reflection

[Lee, Lerche, Weigand 19], [Etheredge, Heidenreich, Kaya, Qiu, Rudelius 22], [Rudelius 23], [Marchesano, Melotti, Paoloni 23], [HCK, Vafa 24], …

Coulomb branch of rank-1 supergravity

$$6\mathcal{F} = c_1\, t_1^3 + 3c_2\, t_1^2 t_2 + 3c_3\, t_1 t_2^2 + c_4\, t_2^3$$

$$\Delta_1 \equiv c_2^2 - c_1 c_3 \geq 0 \;, \quad \Delta_2 \equiv c_3^2 - c_2 c_4 \geq 0$$

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6d supergravity with T=0 and V=0 (geometrically, F-theory on elliptic P2)

$$6\mathcal{F} = 9t_0^3 + 9t_0^2 t_1 + 3t_0 t_1^2$$

- $t_0 \to 0$ limit : KK states become light
- $t_1 \to 0$ limit : ground states of 6d BPS strings become light

$t_0 \backslash t_1$	0	1	2	3	4
0	0	3	−6	27	−192
1	540	−1080	2700	−17280	154440
2	540	143370	−574560	5051970	−57879900
3	540	204071184	74810520	−913383000	13593850920

String ground states ← (row $t_0 = 0$)

↑ KK states (column $t_1 = 0$)

Genus-0 GV invariants of elliptic P2

6d supergravity with T=0 and V=0 (geometrically, F-theory on elliptic P2)

$$6\mathcal{F} = 9t_0^3 + 9t_0^2 t_1 + 3t_0 t_1^2$$

- $t_1 \to 0$ limit corresponds to 5d SCFT limit (with $\Delta_1 = 0$)

  - Coulomb branch metric has zero eigenvalue w.r.t vector $v = (1, -3)$
  Prepotential can be rewritten in terms of CFT Kahler parameter $\tilde{t}_1$.

$$6\mathcal{F} = 9\tilde{t}_0^3 + 9\tilde{t}_1^3 \qquad \text{with } t_0 = \tilde{t}_0 + \tilde{t}_1,\ t_1 = -3\tilde{t}_1$$

  - CS Coefficient of $\tilde{t}_1^3$ tells us that the 5d CFT is E_0 theory !

Note) 5d rank-1 SCFTs are classified, and CS coefficients are $1 \leq C_{\mathrm{cft}} \leq 9$

[Seiberg 96], [Seiberg, Morrison 96], [Intriligator, Morrison, Seiberg 97], [Jefferson, HCK, Vafa, Zafrir 17], [Jefferson, Katz, HCK, Vafa 18], [Bhardwaj 19]

6d SU(2) theories

6d SU(2) gauge theory with T=0, adjoint & fundamental hypers

$$n_{\mathbf{2}} = 2b(12-b)\ , \quad n_{\mathbf{3}} = \frac{(b-1)(b-2)}{2}$$

with $a = -3, \quad 1 \leq b \leq 12$

[Kumar, Park, Turner 10]

- F-theory construction for $1 \leq b \leq 8$
- Automatic symmetry enhancement in F-theory

$$b = 9 \ : \ (SU(2) \times U(1))/\mathbb{Z}_2\ ,$$
$$b = 10, 11 \ : \ (SU(2) \times SU(2))/\mathbb{Z}_2$$
$$b = 12 \ : \ SO(3)$$

[Raghuram, Taylor, Turner 20], [Morrison, Taylor 21]

5d compactification

5d effective prepotential

$$6\mathcal{F}_{SU(2)_b} = 9t_0^3 + 9t_0^2 t_1 + 3t_0 t_1^2 + 54 t_0^2 t_2 + 6t_1^2 t_2 + 36 t_0 t_1 t_2$$
$$+ 18(6-b)t_0 t_2^2 + 6(6-b)t_1 t_2^2 + 2(6-b)^2 t_2^3 \ ,$$

- t_0, t_1, t_2 : Kahler parameters for three primitive BPS states

$$(q_{KK}, q_{SU(2)}, q_S) = (1,-2,0) \ \rightarrow \ t_0 \ , \quad (0,0,1) \ \rightarrow \ t_1 \ , \quad (0,1,0) \ \rightarrow \ t_2$$

KK adj. hyper — string ground state — fund. hyper

Higgsings to rank-1 theory

5d effective prepotential

$$6\mathcal{F}_{SU(2)_b} = 9t_0^3 + 9t_0^2t_1 + 3t_0t_1^2 + 54t_0^2t_2 + 6t_1^2t_2 + 36t_0t_1t_2 + 18(6-b)t_0t_2^2 + 6(6-b)t_1t_2^2 + 2(6-b)^2t_2^3 \ ,$$

- t_0, t_1, t_2 : Kahler parameters for three primitive BPS states

$$(q_{KK}, q_{SU(2)}, q_S) = (1,-2,0) \ \rightarrow \ t_0 \,, \quad (0,0,1) \ \rightarrow \ t_1 \,, \quad (0,1,0) \ \rightarrow \ t_2$$

1) Higgsing $t_2 \to 0$: VEV to (0,1,0) hyper

- Results in 6d theory with T=0 and V=0 (on a circle).

2) Higgsing $t_0 \to 0$: VEV to KK adj. hyper state of (1,-2,0)

$$6\mathcal{F} = 6t_1^2t_2 + 6(6-b)t_1t_2^2 + 2(6-b)^2t_2^3$$

negative when $b > 6$

Kahler cone generators from worldsheet chiral primaries

$$6\mathcal{F} = 6t_1^2 t_2 + 6(6-b)t_1 t_2^2 + 2(6-b)^2 t_2^3$$

negative when $b > 6$

Negative CS coefficient implies

- Kahler cone is non-simplicial $=$ more than 3 Kahler cone generators.
- Actual Kahler cone is smaller than cone generated by t_0, t_1, t_2 .
- Extra primitive BPS states from primary fields in 2d CFT on 6d strings.

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- WZW model for worldsheet SU(2) current algebra at level $k = Qb$ can have primary fields with conformal weights

$$h_j = \frac{j(j+2)}{4(k+2)} \quad \text{for } SU(2) \text{ spin } j/2 \text{ where } 0 \leq j \leq k$$

$\Longrightarrow$ 5d BPS state of charge $(q_{KK}, q_{SU(2)}, q_S) = (n, \ell, Q)$

$n \geq h_j \ , \quad -j \leq \ell \leq j \ , \quad n, \ell \in \mathbb{Z}$

Q : string charge

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Ex) GV-invariants for $b = 8$

chiral primary of $\ell = -8,\ n = 3 \geq h_j = 2$

	n_0											
n_2	0	1	2	3	0	1	2	3	0	1	2	3
-2								-4				
-1								256				
0		40			3	-80	780	46224	-6	200	-3120	29640
1	128	128				-256	5120	572672		640	-20480	299520
2	40	204	40			-408	16128	3211024		1020	-64592	1433520
3		128	128			-256	30720	10856448		640	-123136	4334208
4		40	204	40		-80	37874	24626000		200	-151904	9127368
5			128	128			30720	39596032			-123136	14073984
6			40	204			16128	46253880			-64592	16214040
7				128			5120	39596032			-20480	14073984
	$n_1 = 0$				$n_1 = 1$				$n_1 = 2$			

Kahler cone of 5d Higgsed theory is parametrized by $\tilde{t}_1, t_2 \geq 0$ where $\tilde{t}_1$ is additional Kahler cone generator dual to the worldsheet primary,

- $$6\mathcal{F}_{b=7} = 6t_1^2 t_2 - 6t_1 t_2^2 + 2t_2^3 \xrightarrow{\tilde{t}_1 = t_1 - t_2} 6\tilde{t}_1^2 t_2 + 6\tilde{t}_1 t_2^2 + 2t_2^3$$ $\longrightarrow$ E_7 CFT at $\tilde{t}_1 \to 0$

- $$6\mathcal{F}_{b=8} = 6t_1^2 t_2 - 12t_1 t_2^2 + 8t_2^3 \xrightarrow{\tilde{t}_1 = t_1 - 2t_2} 6\tilde{t}_1^2 t_2 + 12\tilde{t}_1 t_2^2 + 8t_2^3$$ $\longrightarrow$ E_1 CFT at $\tilde{t}_1 \to 0$

- Theories for $b = 9, 10, 11$ could satisfy positivity constraints on 5d CS coefficients provided that there exists a worldsheet primary field having small enough conformal dimension.

b=12 theory and Swampland

6d SU(2) theory with T=0 and 55 adjoint hypermultiplets

- $\mathbb{Z}_2$ 1-form symmetry in massless spectrum
- F-theory construction has $SO(3) = SU(2)/\mathbb{Z}_2$ gauge symmetry

[Morrison, Taylor 21]

Constraints $\Delta_1, \Delta_2 \geq 0$ on 5d prepotential after (2nd) Higgsing can be satisfied only if 2d CFT on 6d BPS strings has a chiral primary field with charge

$$(q_{KK}, q_{SU(2)}, q_S) = (h_j, -j, Q) \quad \text{with } j = 12Q \quad \text{for some } Q$$

Assuming this and using Kahler parameter t_1 for this primary state, the Higgsed prepotential is given by

$$6\mathcal{F}_{b=12} = 6t_1^2 t_2 + 36 t_1 t_2^2 + 72 t_2^3 \qquad (\text{and } \Delta_2 = 0\ )$$

b=12 theory and Swampland

Higgsed prepotential

$$6\mathcal{F}_{b=12} = 6t_1^2 t_2 + 36 t_1 t_2^2 + 72 t_2^3$$

- Zero eigenvalue of metric with vector $v=(-6,1)$ at $t_1 \to 0$.
  This implies 5d rank-1 SCFT at the boundary.

- Prepotential can be rewritten by CFT Coulomb parameter $\tilde{t}_1$

$$6\mathcal{F}_{b=12} \quad \xrightarrow[]{\substack{t_1=-6\tilde{t}_1,\\ t_2=\tilde{t}_2+\tilde{t}_1}} \quad 72\tilde{t}_1^3 + 72\tilde{t}_2^3$$

- Coefficient 72 of $\tilde{t}_1^3$ term exceeds the bound '9' for CS coefficient of any rank-1 SCFT

Inconsistent and falls into Swampland

b=12 theory and Swampland

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- Coefficient 72 of $\tilde{t}_1^3$ term exceeds the bound '9' for CS coefficient of any rank-1 SCFT

Note, t_2 here is normalized assuming minimal $U(1) \subset SU(2)$ charge is '1'.

b=12 theory and Swampland

Higgsed prepotential

$$6\mathcal{F}_{b=12} = 6t_1^2t_2 + 36t_1t_2^2 + 72t_2^3$$

- If $\mathbb{Z}_2$ 1-form symmetry in massless spectrum is promoted to symmetry of the full theory and gauged, we need rescaling $t_2 \to t_2/2$

b=12 theory and Swampland

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$$6\mathcal{F}_{b=12} \quad \xrightarrow{t_2 \to t_2/2} \quad 3t_1^2 t_2 + 9 t_1 t_2^2 + 9 t_2^3$$

(= $\mathcal{F}$ of theory with T=V=0)

- SO(3) gauge group is consistent, whereas SU(2) is inconsistent.

- Massless Charge Sufficiency Conjecture

Massless states generates the full charge lattice of gauge group with $a \cdot b < 0$

[Morrison, Taylor 21]

6d SUGRA with 1-form symmetry

6d theories with T=0 and rank-1 gauge group

- SU(2) theories with integral SU(2) spin rep. hypers having $\mathbb{Z}_2$ 1-form symm.

Ex) $b = 32 \ : \ n_{\mathbf{7}} = 1, \ n_{\mathbf{5}} = 13, \ n_{\mathbf{3}} = 66$

$b = 56 \ : \ n_{\mathbf{9}} = 1, \ n_{\mathbf{7}} = 2, \ n_{\mathbf{5}} = 36, \ n_{\mathbf{3}} = 15$

- Infinite family of U(1) gauge theories having $\mathbb{Z}_n$ 1-form symm.

$$b = 6(p^2 + pr + p^2) \ , \quad 54 \times (\pm p) + 54 \times (\pm r) + 54 \times (\pm(p+r))$$

$$\mathrm{GCD}(p, r) = n, \ n \geq 2$$

Constraints on 5d Higgsed prepotentials of these theories show

- 1-form symm. is gauge symmetry of the full theory
- Massless Charge Sufficiency Conjecture (and also No Global Symmetry Conjecture) holds

Summary and Outlooks

- Constraints on CS coefficients on Coulomb branch of 5d supergravity.
- Kahler cone generators from primary fields on BPS strings as well as 6d massless states.
- 1-form symmetries of massless spectrum in 6d supergravity with rank-1 gauge algebra must be promoted to a gauge symmetry of the full theory.

- Higher rank generalization ?
- More constraints from worldsheet CFTs on BPS strings ?
- Prove Massless Charge Sufficiency Conjecture ?

Thank you !