

# Tropical Integration for Gauge Theory Feynman Integrals

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**Introduction:** We provide a new program for **fast numerical integration of multi-loop/multi-leg Feynman integrals.**

• **Current frontier using tropical method:** `feyntrop` [Borinsky, Munch, Tellander]

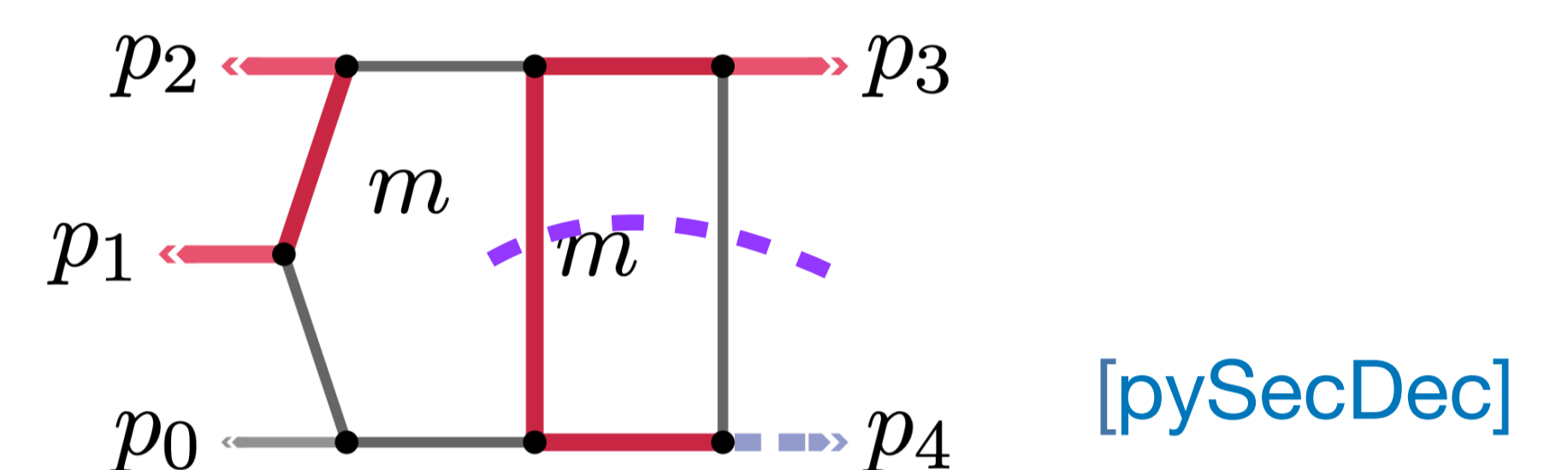
• **Extended method to “gauge theories”:** `trillo` [Wang, SQZ]

• Support Feynman integrals with “numerators”. **Benchmark: 8L integrals with 5 numerators**

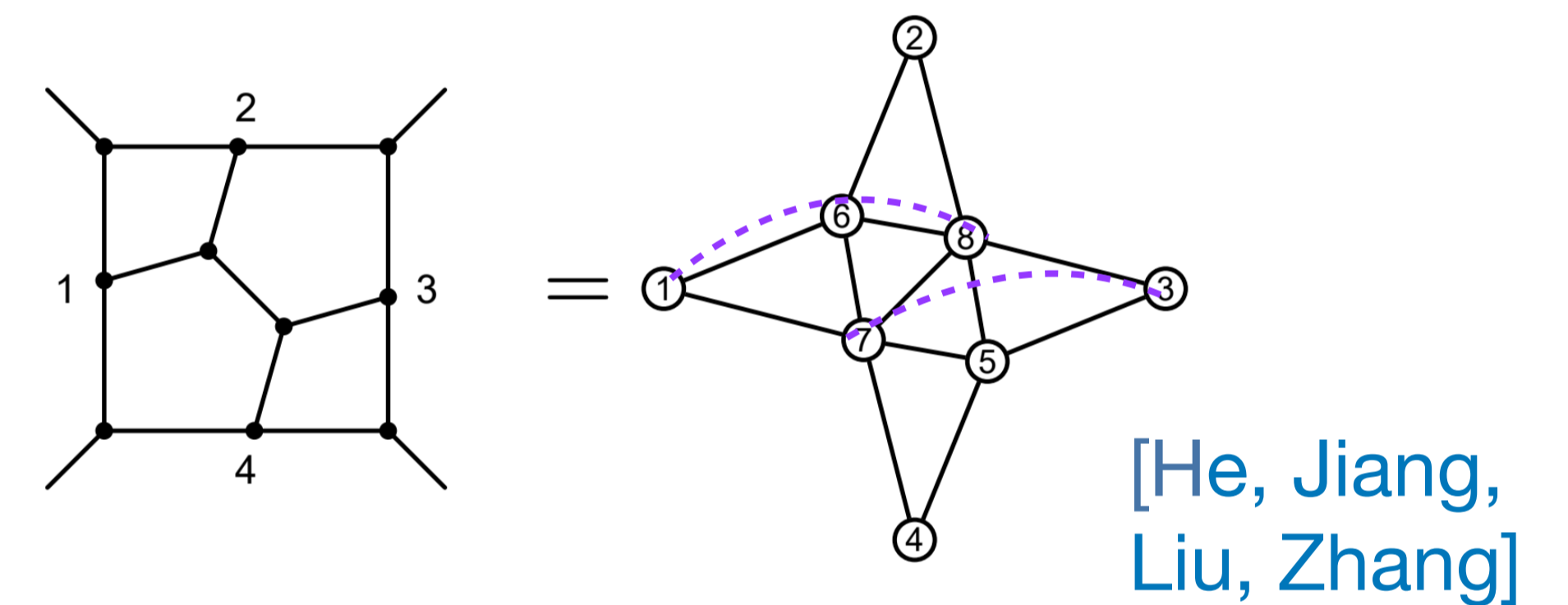
• Dual momentum space Feynman integrals for **planar** and **non-planar** (optimal for N=4 SYM)

**Applications:** The **new program** applies to the integrals below.

**Collider physics:** applications to Feynman integrals with **many scales** (such as massive quarks or electroweak physics). Current numerical tools [pySecDec, AMFlow]

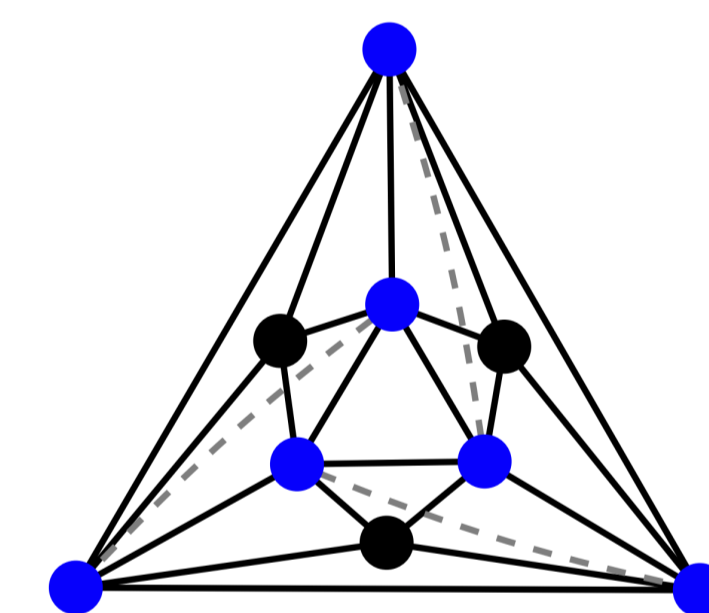


**N=4 SYM:** High-loop computations of scattering amplitudes and correlation functions in super Yang-Mills theory. Integrands up to **12 loops**. [Eden, Heslop, Korchemsky, Sokatchev; Bourjaily, He, Shi, Tang]



**Periods:** Numbers and functions in QFT [Schnetz] and the recent generalization to N=4 SYM.

**Localization** [1] gives constraints on Feynman integrals (new relations among **multiple zeta values**).



$$= 120\zeta(3)^2\zeta(5) + 400\zeta(5)^2 - 10410\zeta(11) - \frac{8\pi^6}{21}\zeta(5) + \frac{24\pi^4}{5}\zeta(7) + 1080\pi^2\zeta(9) + 144\zeta(5,3,3)$$

Predicted in [2], now verified

## Outlook:

• (Finite) **Observables** in N=4 SYM, e.g. **Wilson loops with Lagrangian insertion**, [Carrolo, Chicherin, Henn, Yang, Zhang; Chicherin, Henn, Xu, Zhang, SQZ] and **Negative geometry expansion** [Arkani-Hamed, Henn, Trnka], and their **Positivity properties** in Euclidean region.

• **Correlation functions** at higher loops, e.g. **Elliptic sector** at four loops [He, Huang, Kuo] and higher charges using **10D symmetry** [Caron-Huot, Coronado]

• **Gravitational wave scattering processes** [Bern, Cheung, Roiban, Shen, Solon, Zeng]

[1] D. Dorigoni, M. B. Green, and C. Wen, “Exact properties of an integrated correlator in N = 4 SU(N) SYM,” JHEP **05** (2021) 089, arXiv: 2102.09537.

[2] C. Wen and S.-Q. Zhang. “Integrated correlators in N = 4 super Yang-Mills and periods”. JHEP **05** (2022), 126. arXiv: 2203.01890.