The Sampling - Reconstructing Dual

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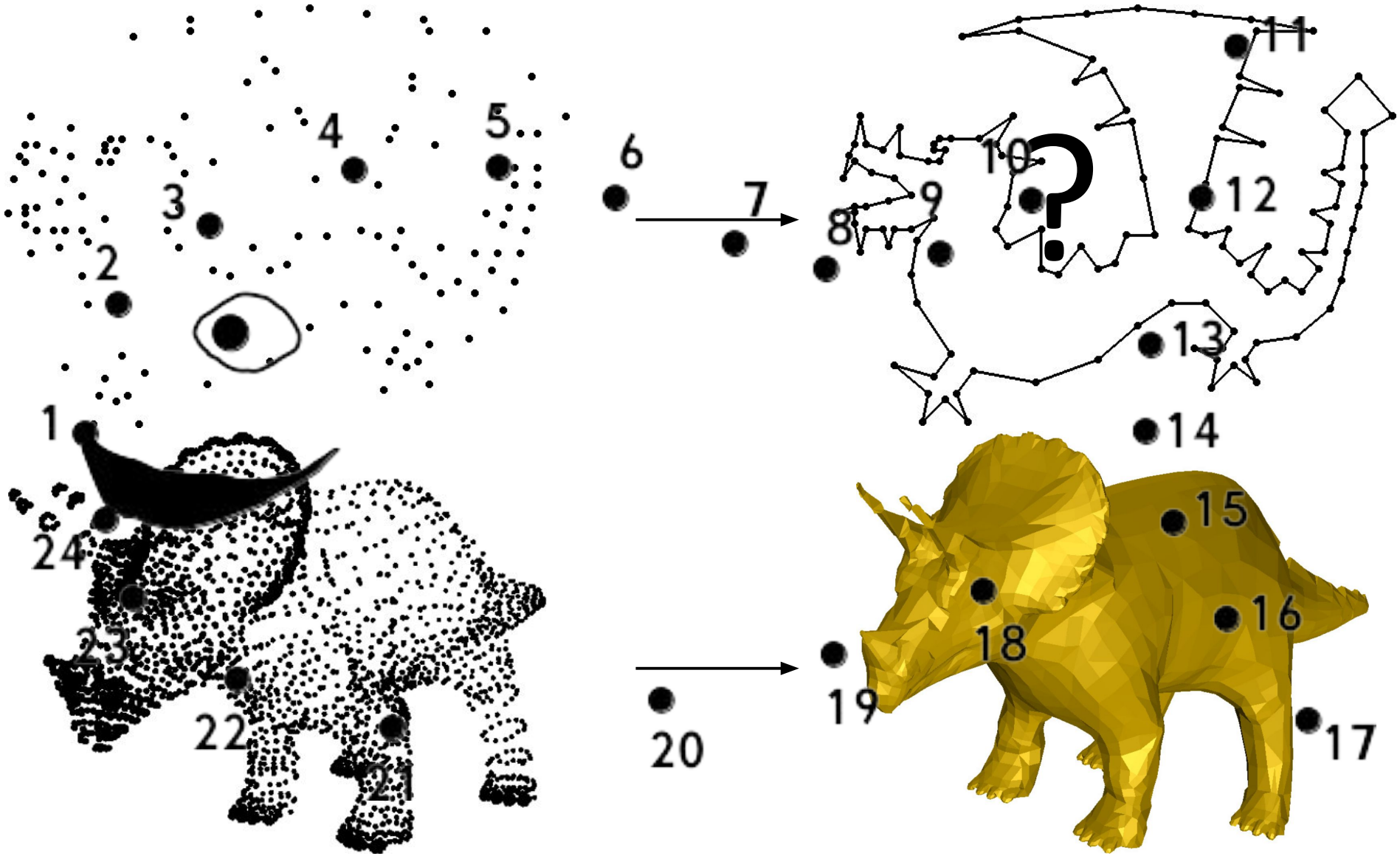
Problem: Reconstruct Boundary from Points



Examples of Virtual Models

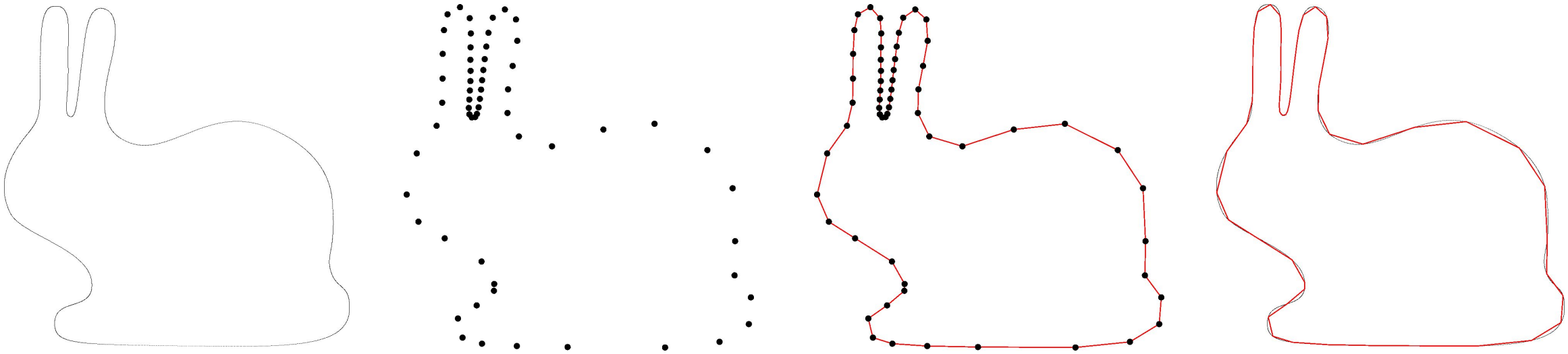
Retz town

Scan with 50 million points

Armadillo model

173.000 points

Boundary -> Sampling -> Reconstruct Connectivity



sample **reconstruct**

Boundary ⟶ Points ⟶ Connectivity Side-by-Side

As few samples as required for connectivity

-> defines sampling condition for reconstruction

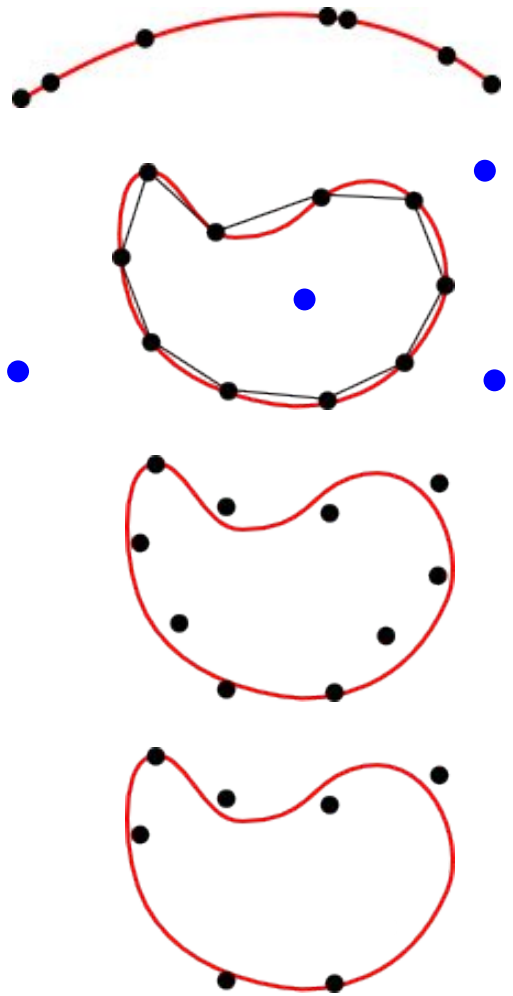
Challenges in Reconstructing the Real World

Non-uniform

Outliers

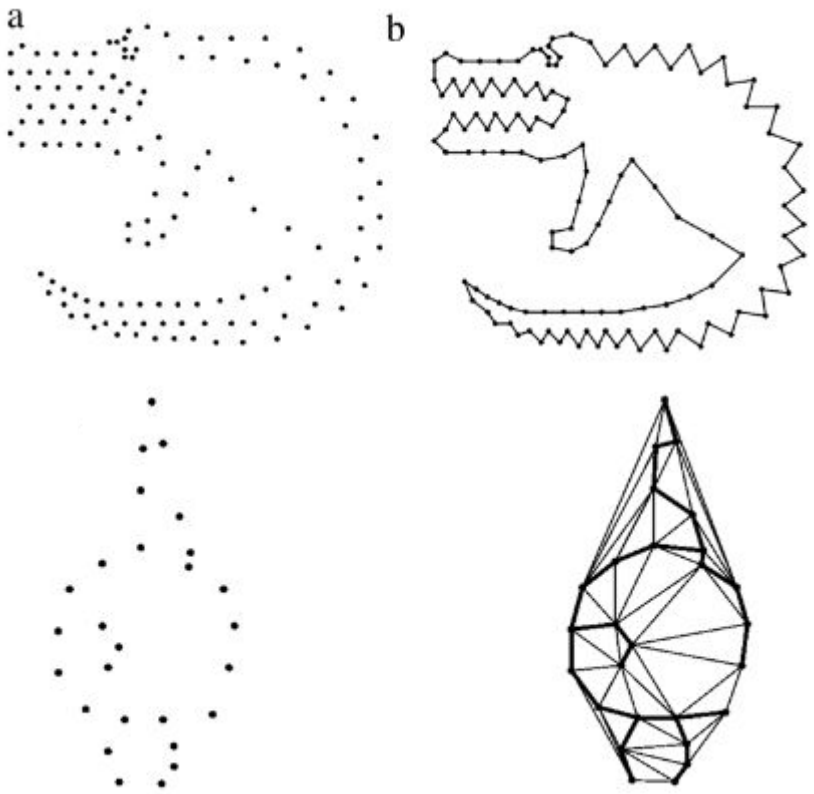
Noise

Missing data



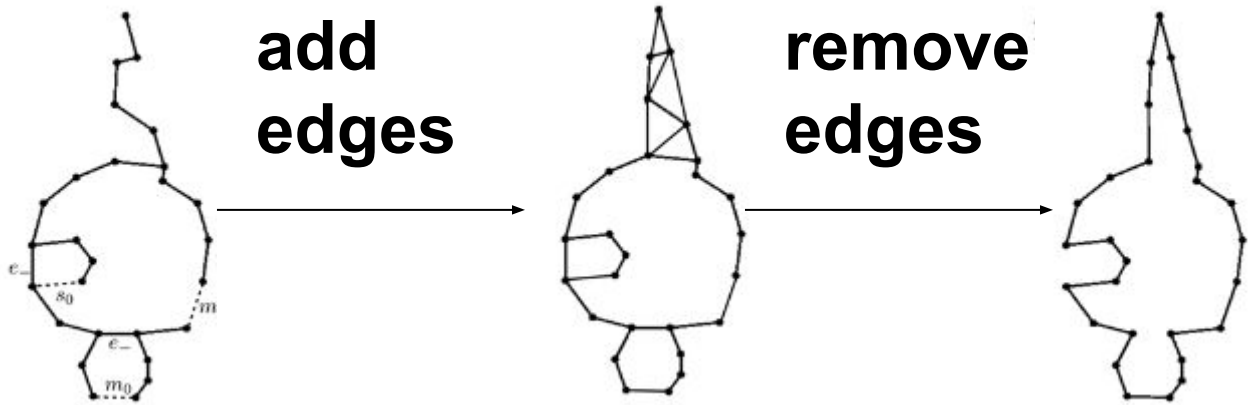
Google Scholar: “surface reconstruction”: >500.000 results

Curve Reconstruction: Minimum Boundary B_{min}



Hypothesis: Minimum boundary is best

→Traveling Salesman Problem, is NP-hard

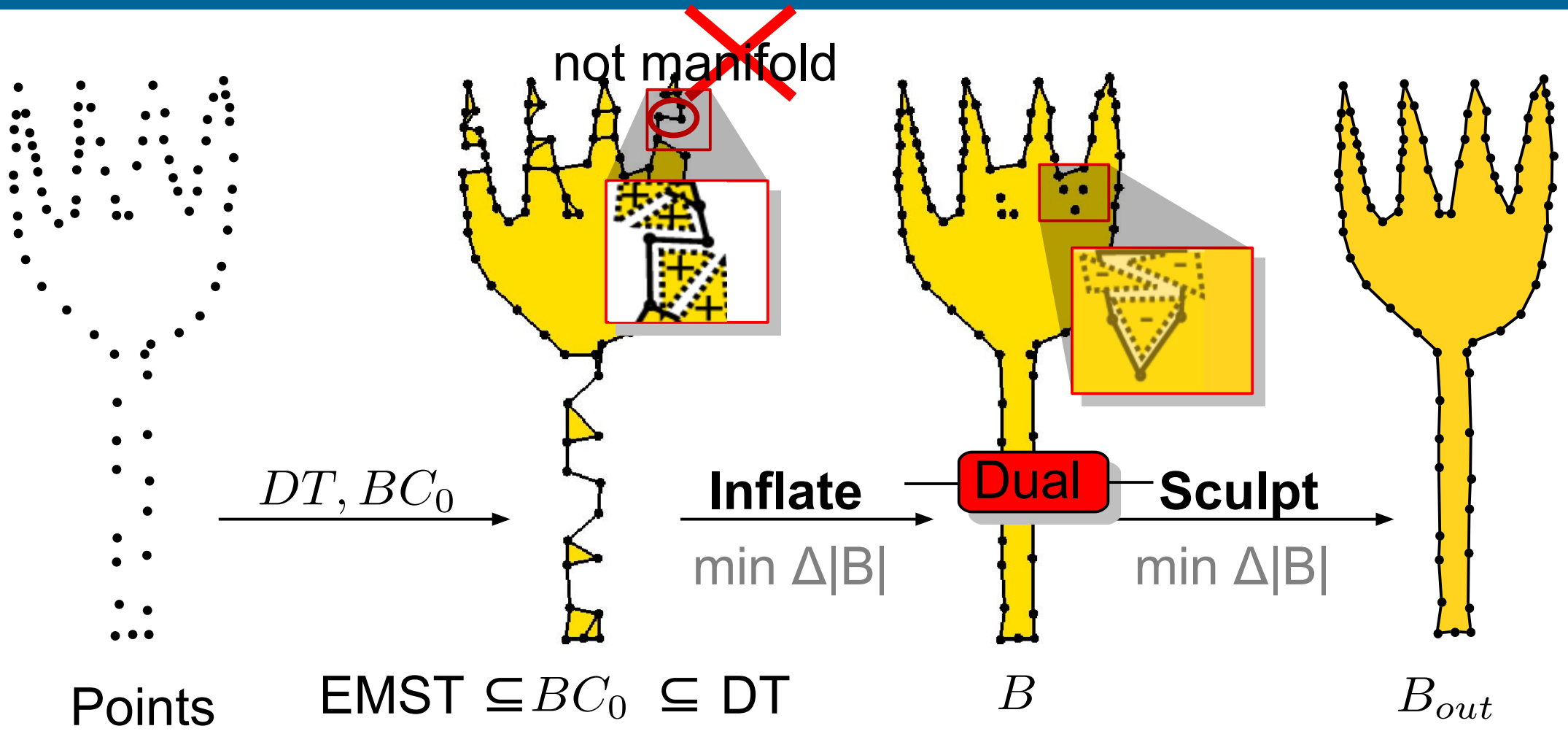


Points Delaunay Tri. (DT) EMST Manifold hull Manifold

Theorem: Our algorithm reconstructs B_{min} - but it is still too slow

Stefan Ohrhallinger, Sudhir Mudur: Interpolating an unorganized 2D point cloud with a single closed shape, CAD 2011

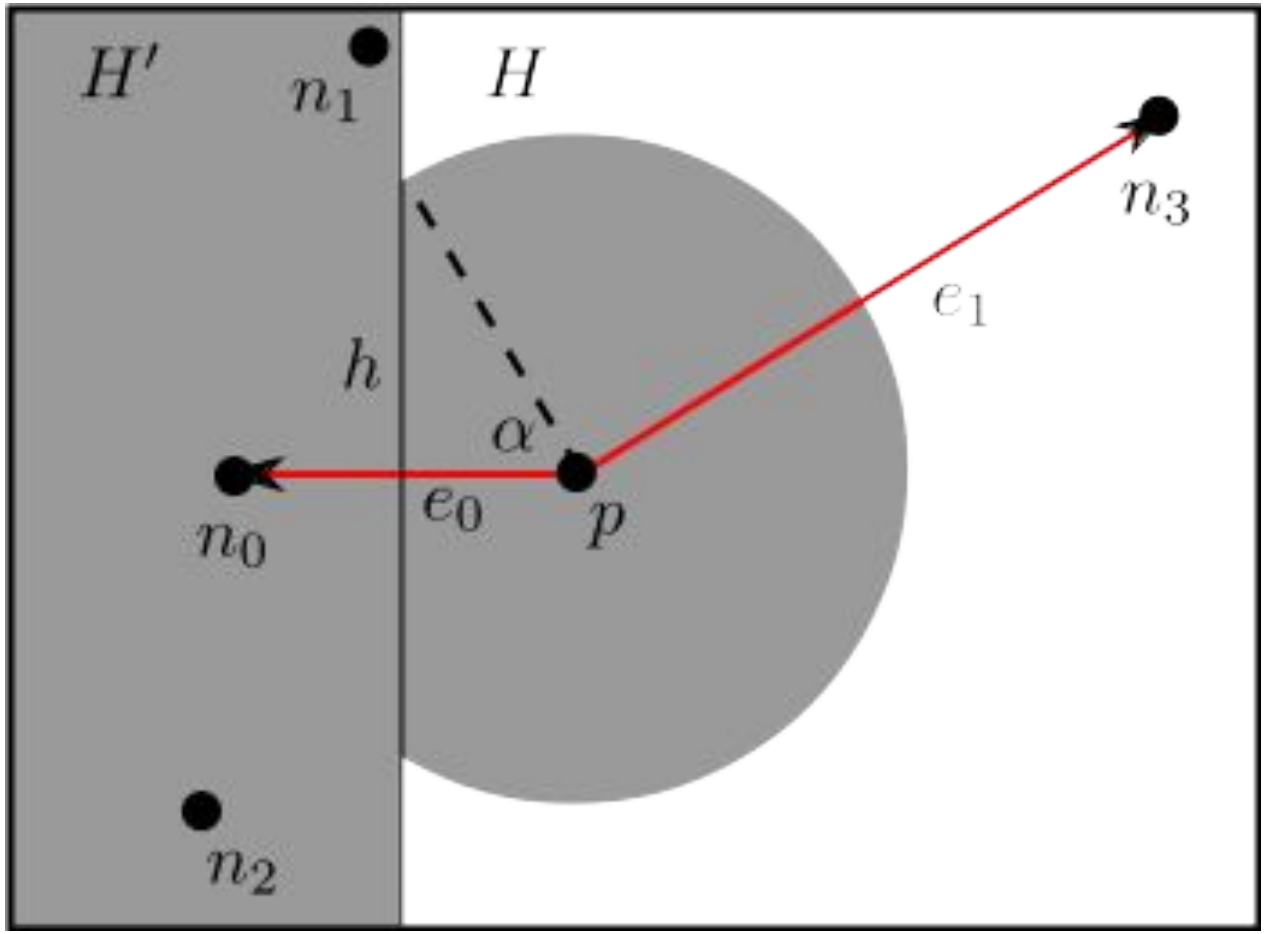
Curve Reconstruction: Fast & often Min. Boundary



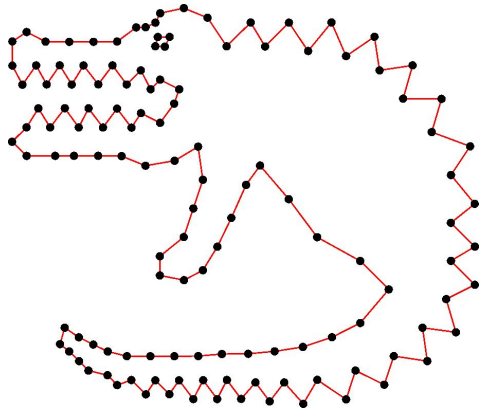
Theorem: Our algorithm constructs B_{out} in O(n log n) time.

Stefan Ohrhallinger, Sudhir Mudur: An Efficient Algorithm for Determining an Aesthetic Shape Connecting Unorganised 2D Points, CGF 2013

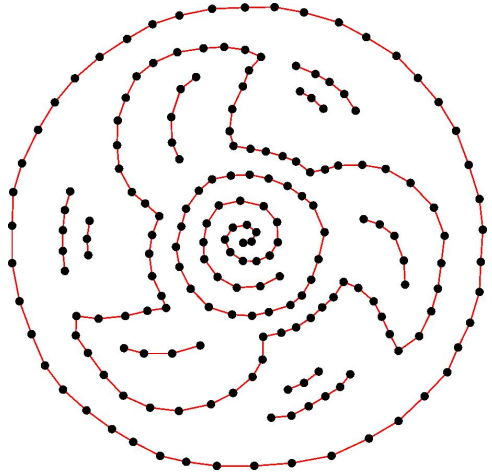
Curve Reconstruction: Local Connectivity



Sharp Angles



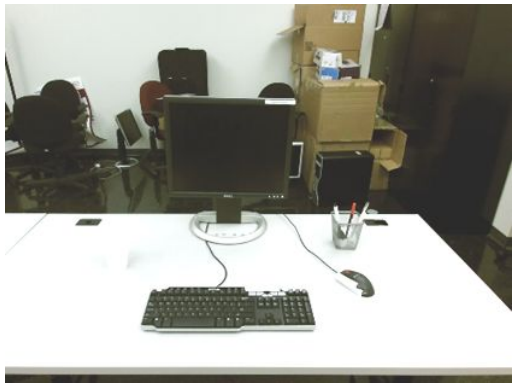
Open Curves

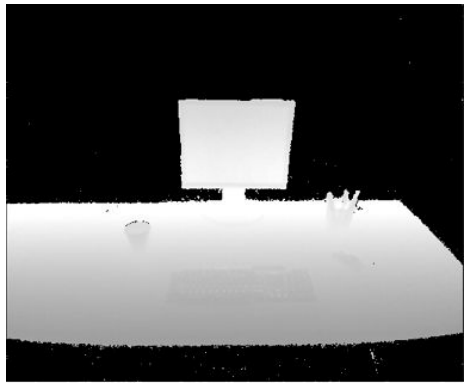


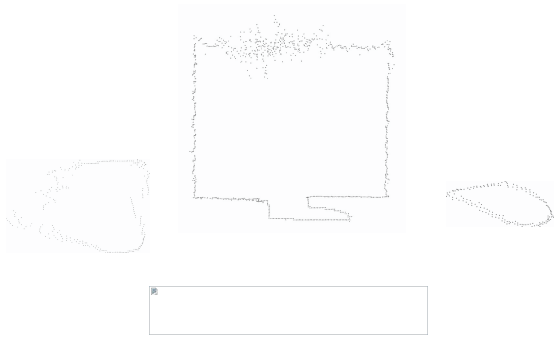
Theorem: Reconstructs for a Sampling w.r.t. Local Feature Size

Stefan Ohrhallinger, Scott A. Mitchell, Michael Wimmer: Curve Reconstruction with Many Fewer Samples, Computer Graphics Forum, 35(5):176-176, 2016.

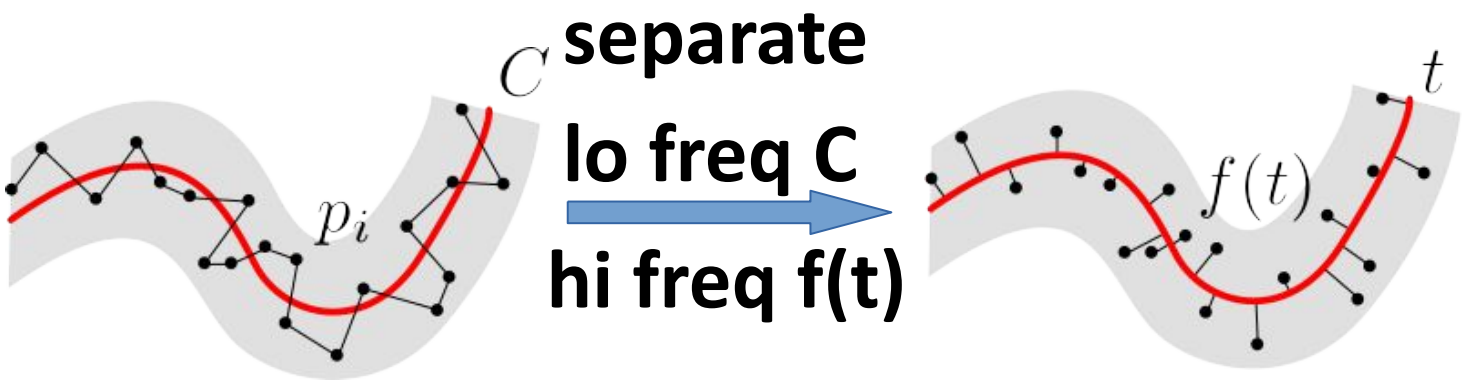
Curve Reconstruction: Fit Noisy Points







Color image Depth image Segmented parts Noisy boundaries



separate

lo freq C

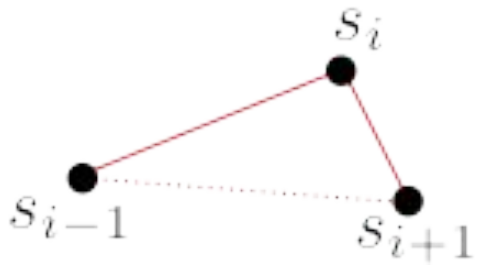
hi freq f(t)

Hypothesis: can recover features > noise (as in the Sampling Theorem)

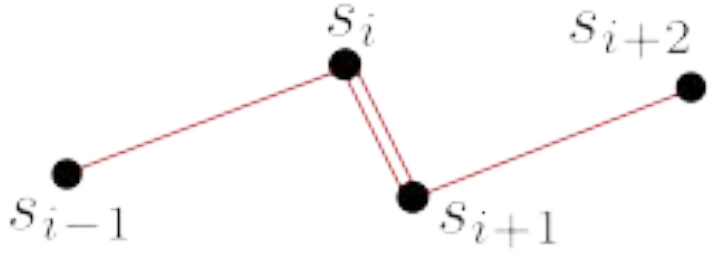
Step 1: Connect p_i on C Step 2: Smooth noise f(t) along C

Stefan Ohrhallinger, Michael Wimmer: FitConnect: Connecting Noisy 2D Samples by Fitted Neighborhoods, CGF 2019

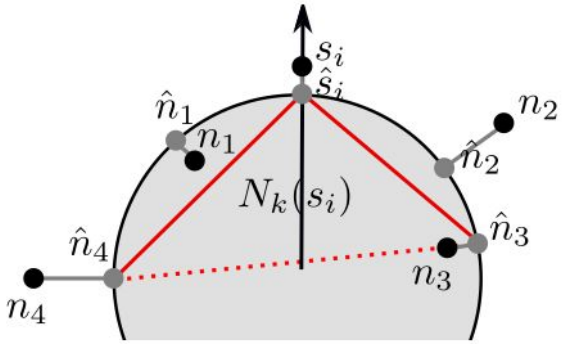
Curve Reconstruction: Fit Noisy Points



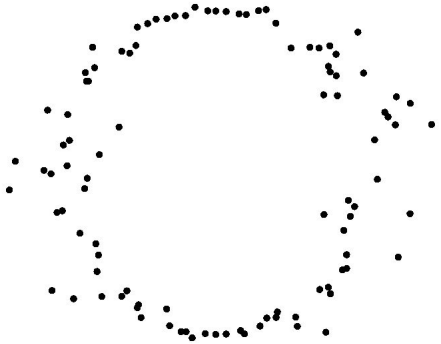
Locally connect



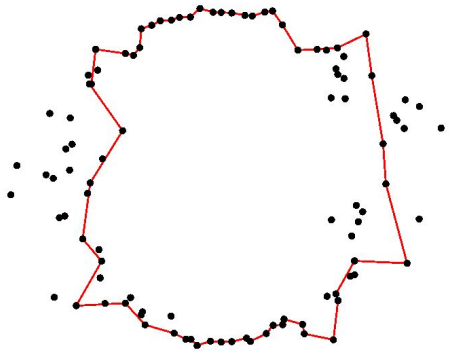
Neighbor Consistency



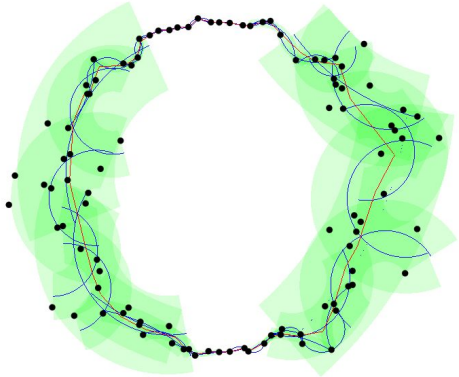
Noisy: Locally project



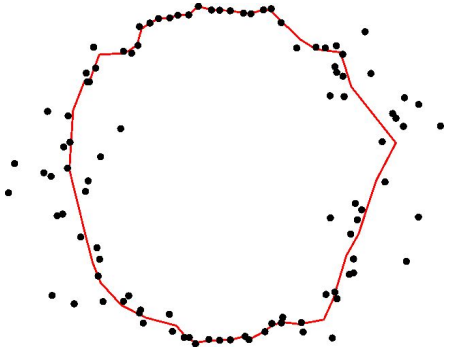
Noisy points



Connectivity



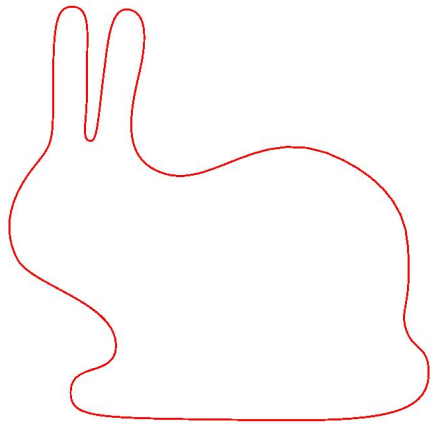
Noise variance



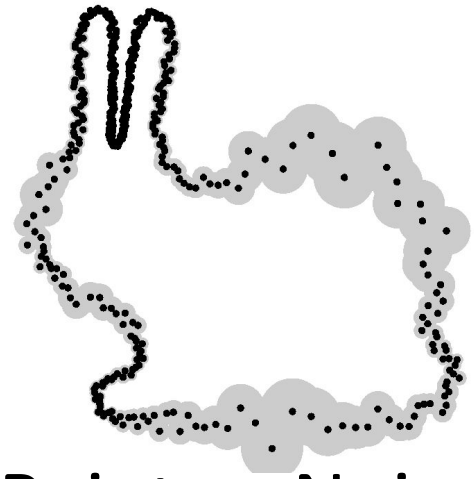
Blended result

Stefan Ohrhallinger, Michael Wimmer: FitConnect: Connecting Noisy 2D Samples by Fitted Neighborhoods, CGF 2019

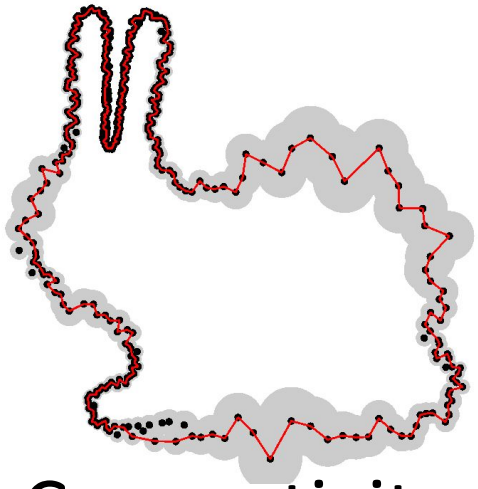
Curve Reconstruction: Remove Noise from Points



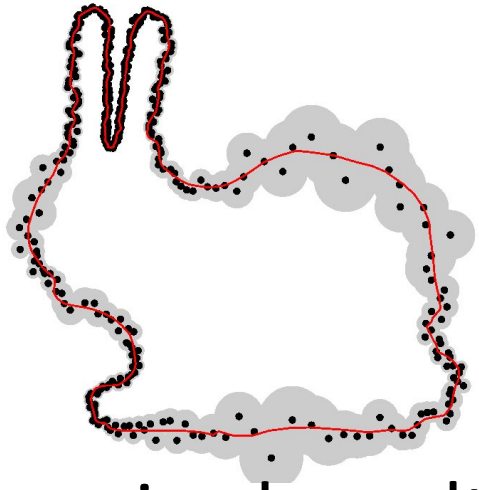
Original curve



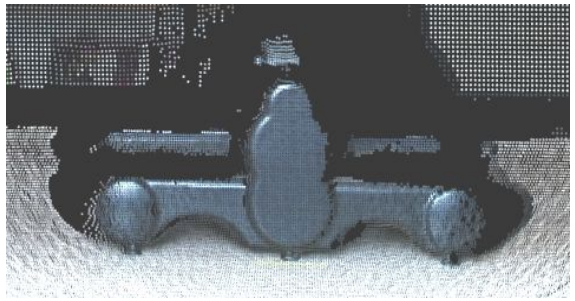
Points + Noise



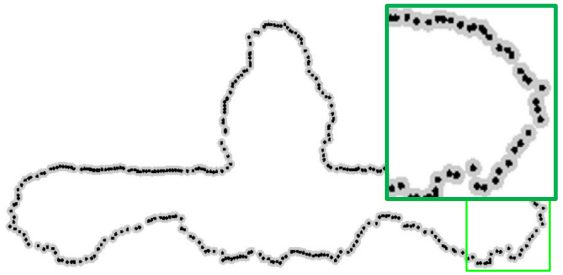
Connectivity



Denoised result



Color depth image



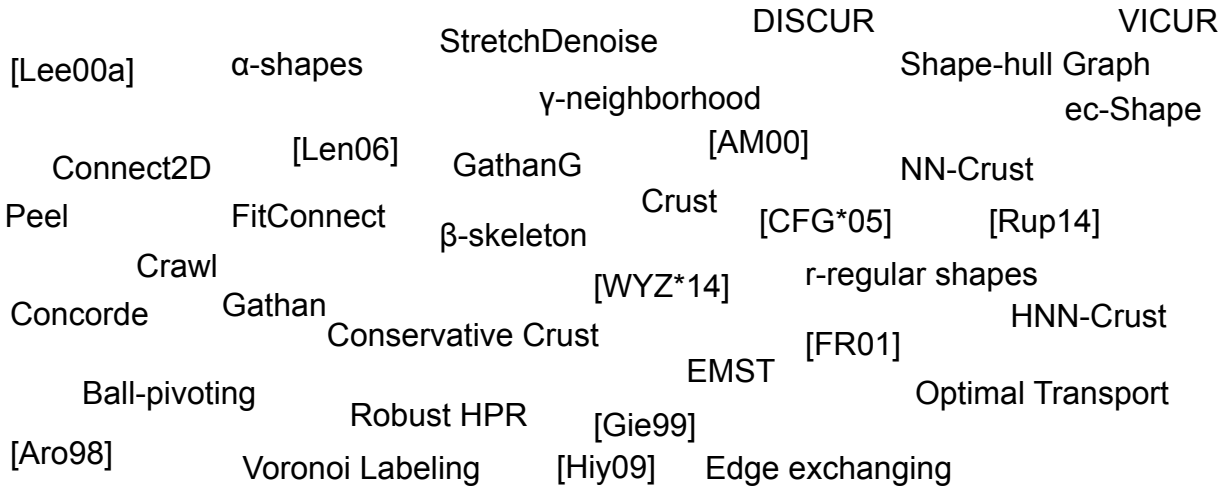
Points + Noise

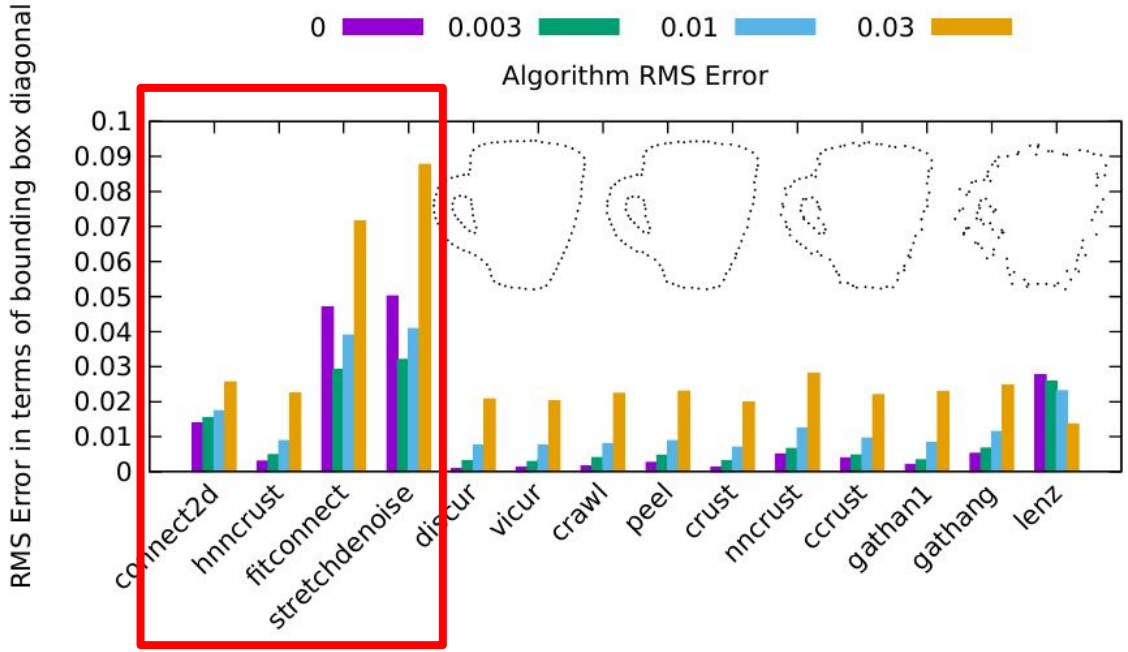
Connectivity

Denoised result

Stefan Ohrhallinger, Michael Wimmer: StretchDenoise: Parametric Curve Reconstruction with Guarantees by Separating Connectivity from Residual Uncertainty of Samples, PG 2018

Curve Reconstruction: Survey and Benchmark

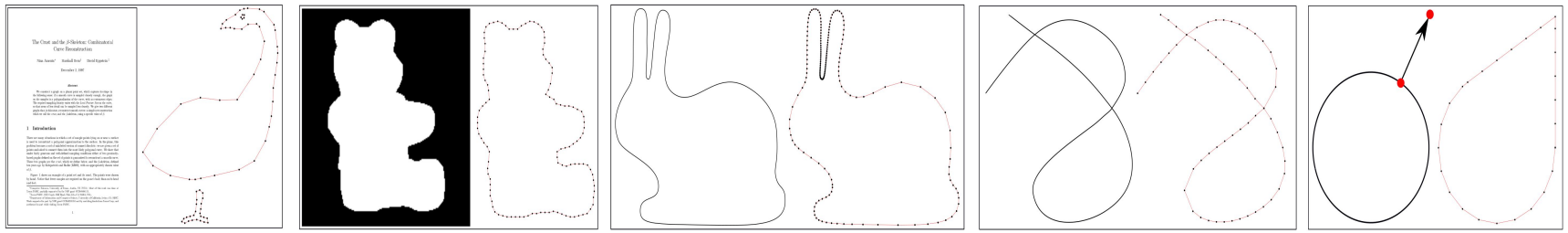




36 curve reconstruction algorithms

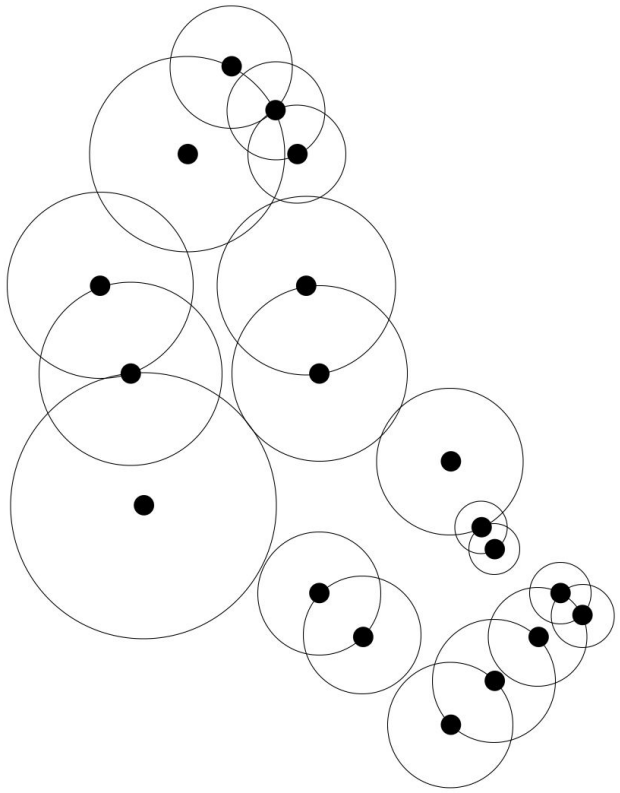
14 algorithms evaluated

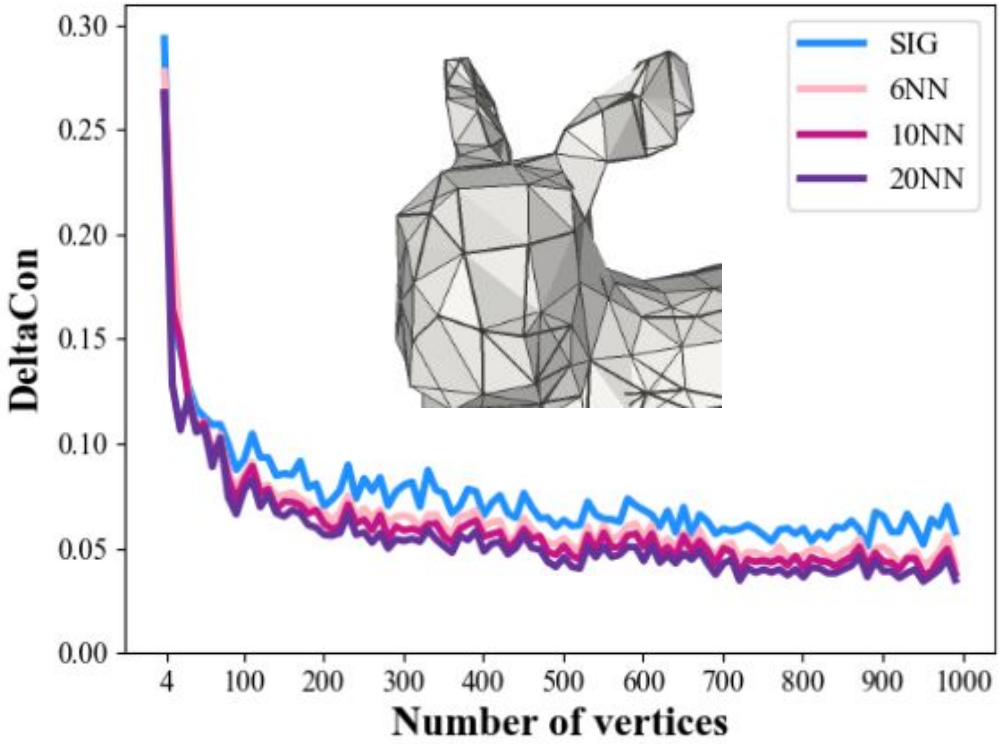
2500 points sets:



Stefan Ohrhallinger, Jiju Peethambaran, Amal Dev Parakkat, Tamal K Dey, Ramanathan Muthuganapathy: 2D Points Curve Reconstruction Survey and Benchmark, EG STAR 2021.

Curve Reconstruction: Spheres-of-Influence Graph (SIG)





edge (v_0, v_1) if $\|v_0, v_1\| \leq nn(v_0) + nn(v_1)$

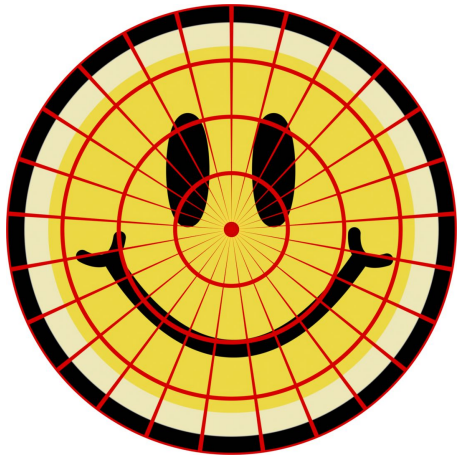
Compare mesh connectivity

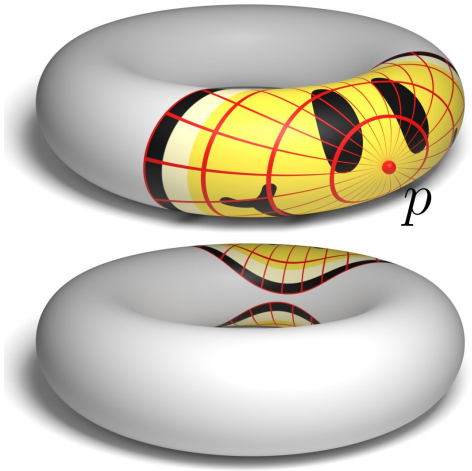
Improved curve reconstruction

SIG > kNN, no parameter!

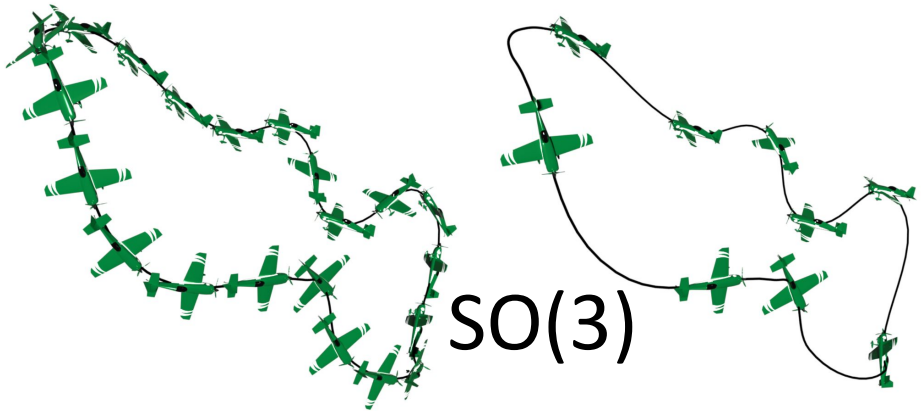
Diana Marin, **Stefan Ohrhallinger**, Michael Wimmer, SIGDT: 2D curve reconstruction, Computer Graphics Forum. Vol. 41. No. 7. 2022.
Diana Marin, **Stefan Ohrhallinger**, Michael Wimmer, Parameter-Free Connectivity for Point Clouds, GRAPP 2024.

Curve Reconstruction: On Riemannian Manifolds





Handles: use injectivity radius

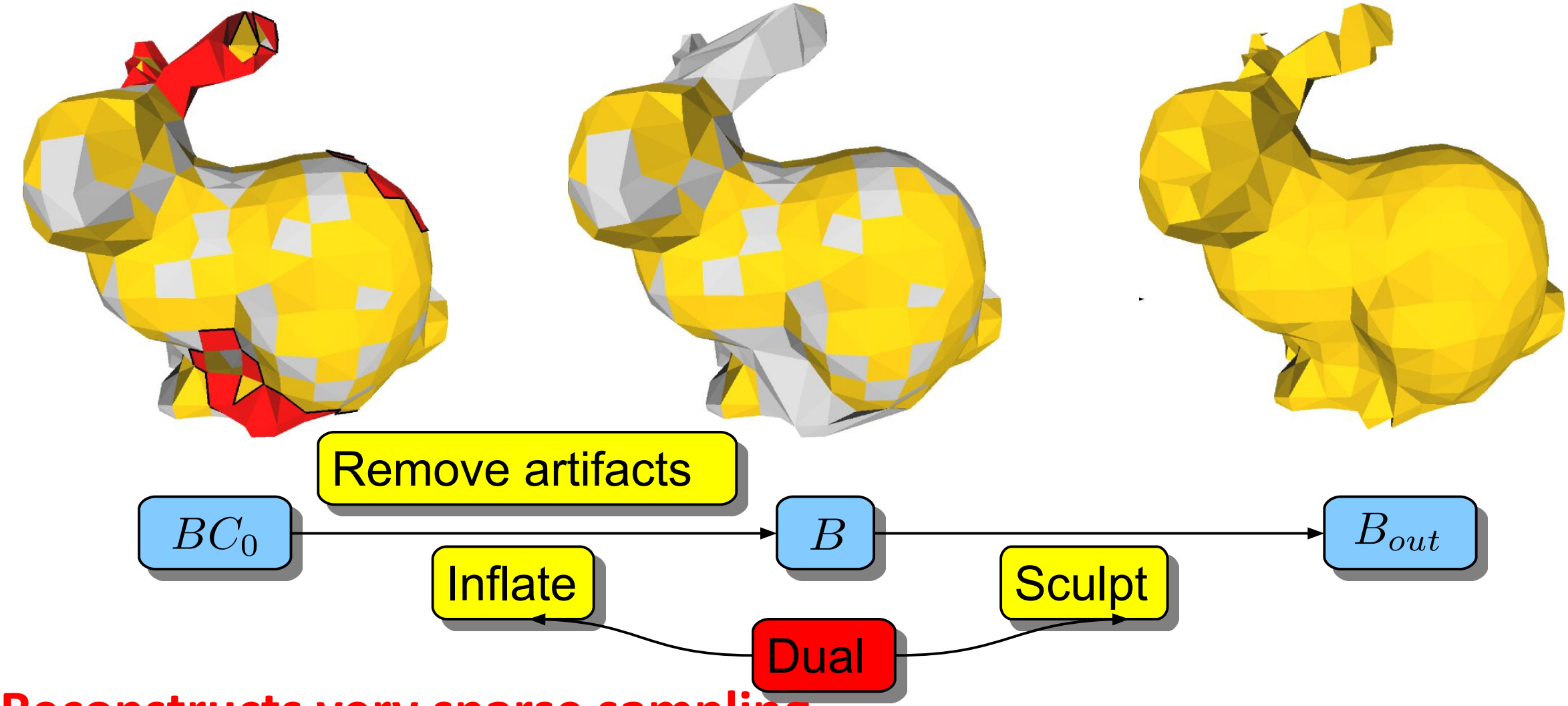


Theorem: Sampling condition

Culture Heritage Results

Diana Marin, Filippo Maggioli, Simone Melzi, **Stefan Ohrhallinger**, Michael Wimmer: Reconstructing Curves from Sparse Samples on Riemannian Manifolds, CGF 2024.

Surface Reconstruction: ~Min. Boundary 2D -> 3D



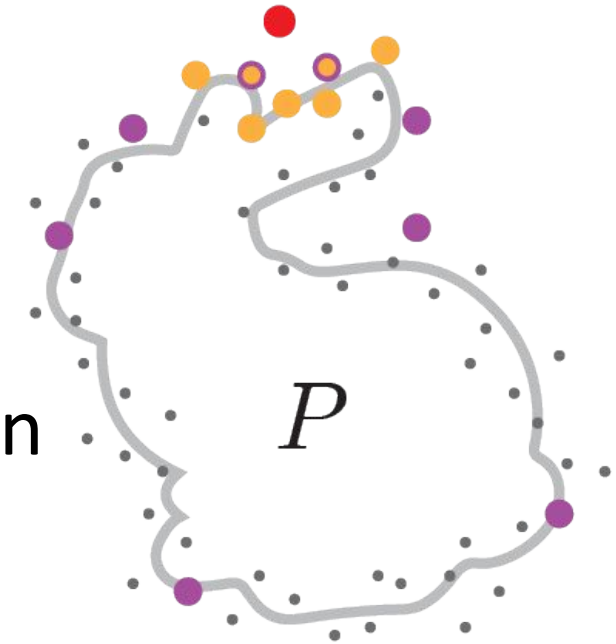
Reconstructs very sparse sampling

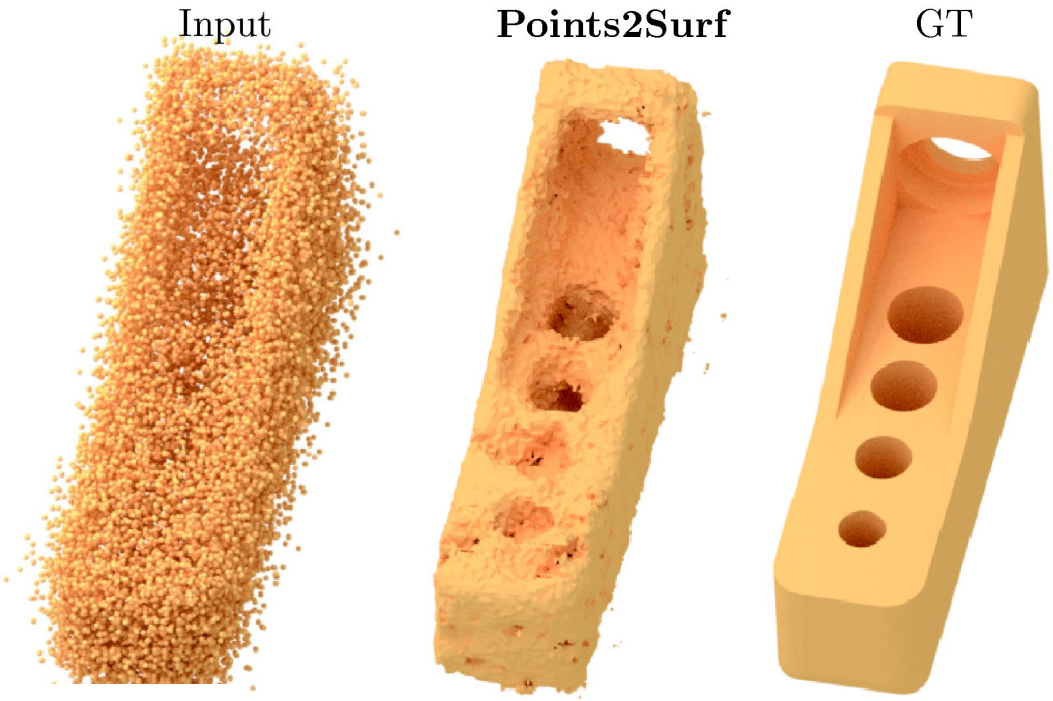
Philipp Erler, Paul Guerrero, **Stefan Ohrhallinger**, Michael Wimmer, Niloy Mitra: Points2Surf: Learning Implicit Surfaces from Point Clouds, ECCV, 2020

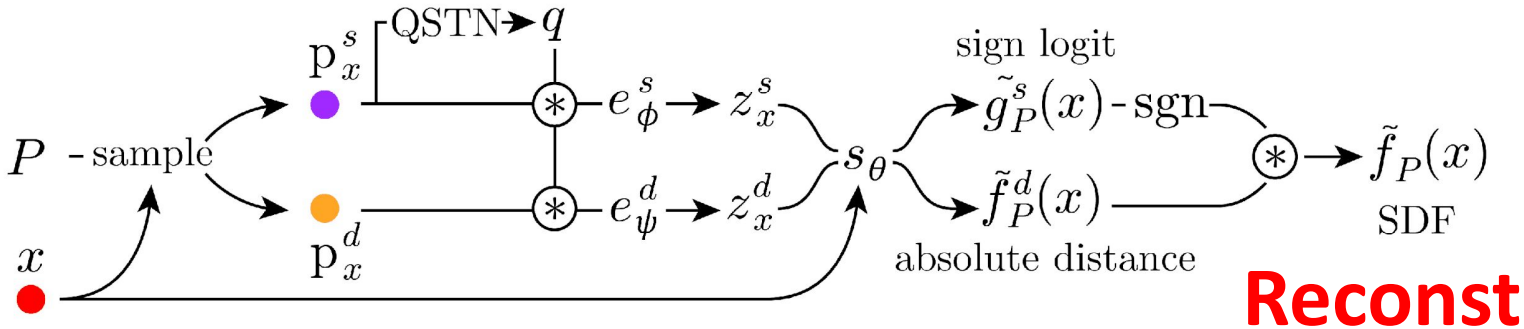
Surface Reconstruction: Local + Global Learning

Train network with:

- Local shape
- Global orientation





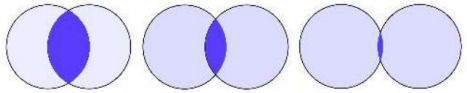


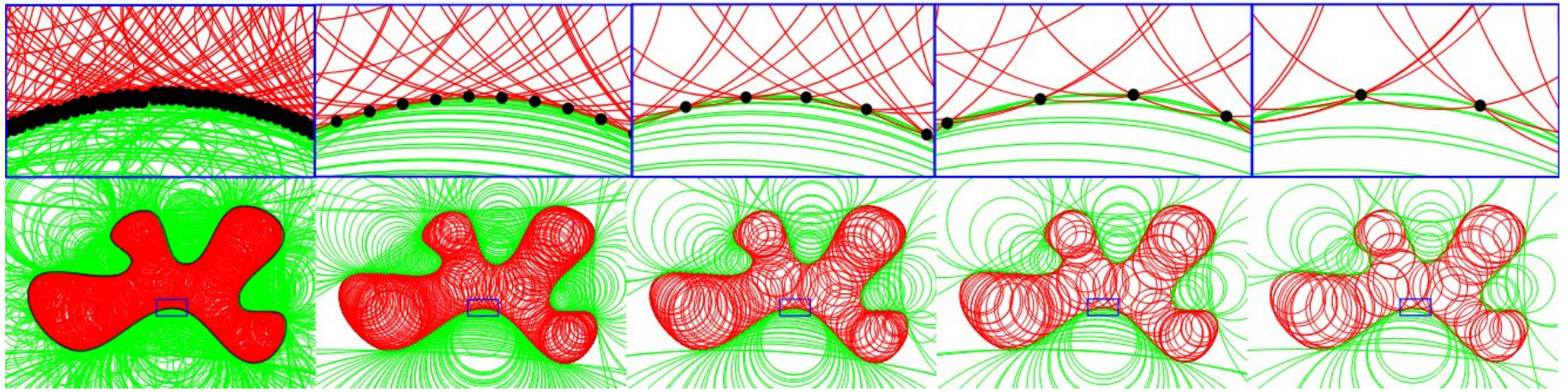
Reconstructs very noisy points

Philipp Erler, Paul Guerrero, **Stefan Ohrhallinger**, Niloy J Mitra, Michael Wimmer: Points2surf learning implicit surfaces from point clouds, EVVC 2020.

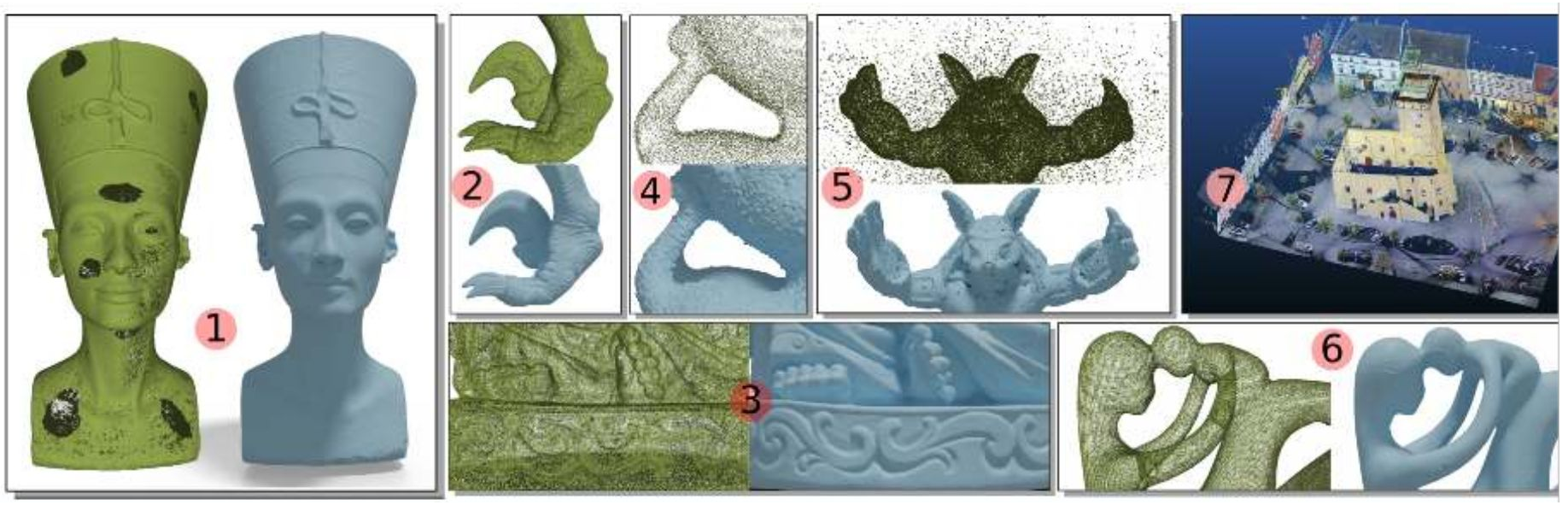
Surface Reconstruction: Fast and Parallel

Voronoi balls overlap ratio:





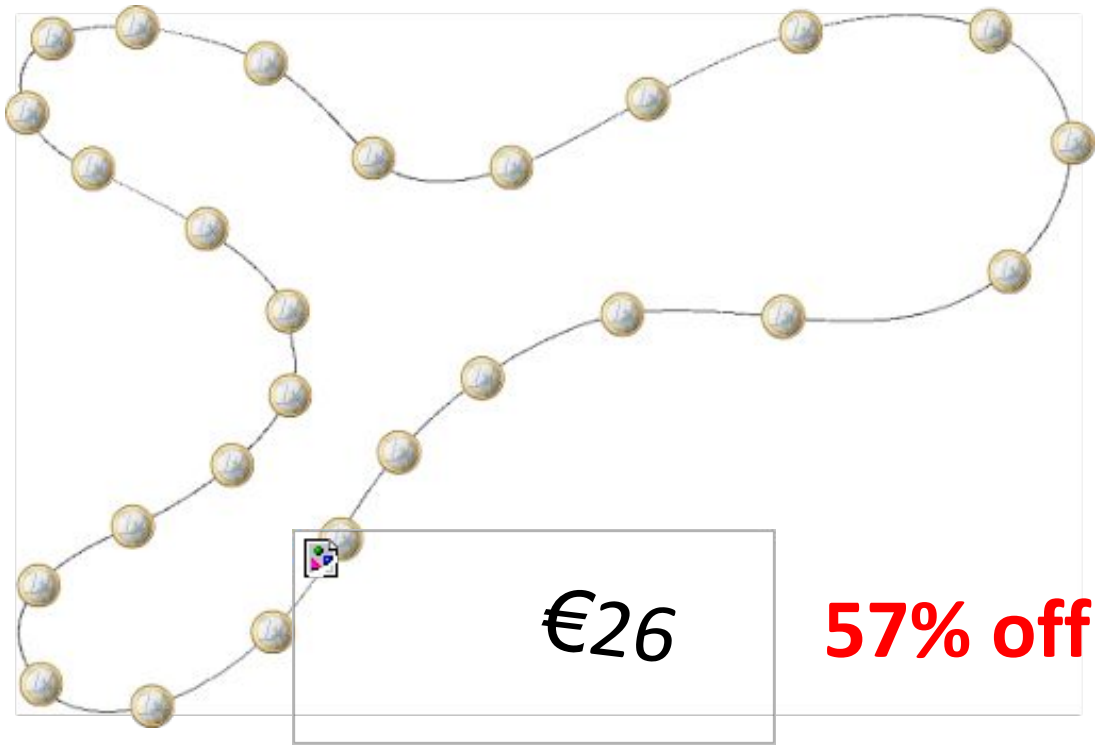
Handles scan artifacts well, 10x speed-up, runs on cluster



Amal Dev Parakkat, **Stefan Ohrhallinger**, Elmar Eisemann, Pooran Memari: BallMerge: High-quality Fast Surface Reconstruction via Voronoi Balls, Eurographics, 2024

Why Sample Boundaries with Fewer Points?

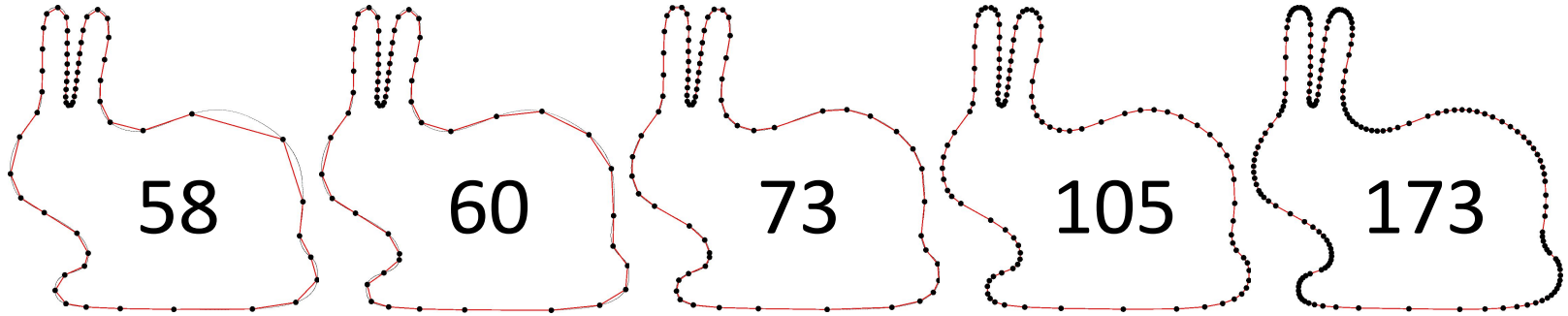
Each sample costs: 

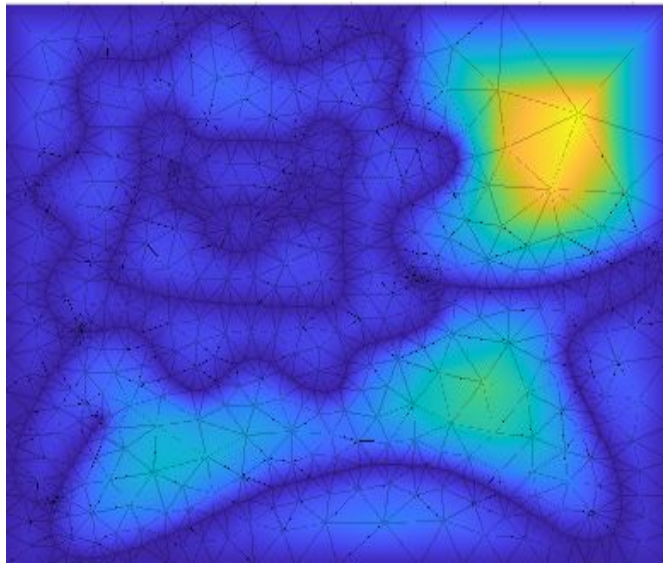


57% off

2D Feature-Sized Sampling: Applications

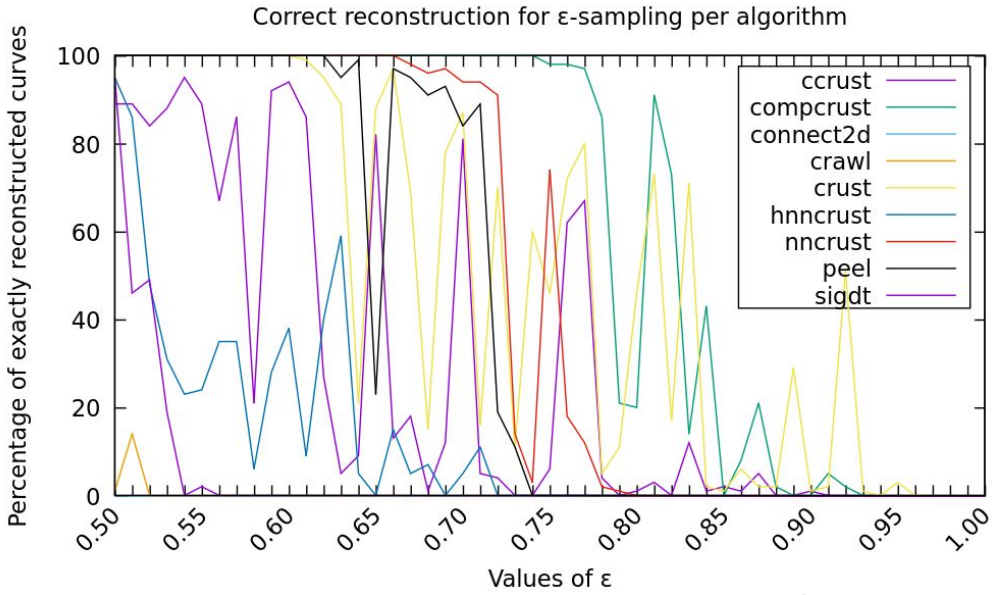
Feature-sized sampled 2D curve:





Faster simulations

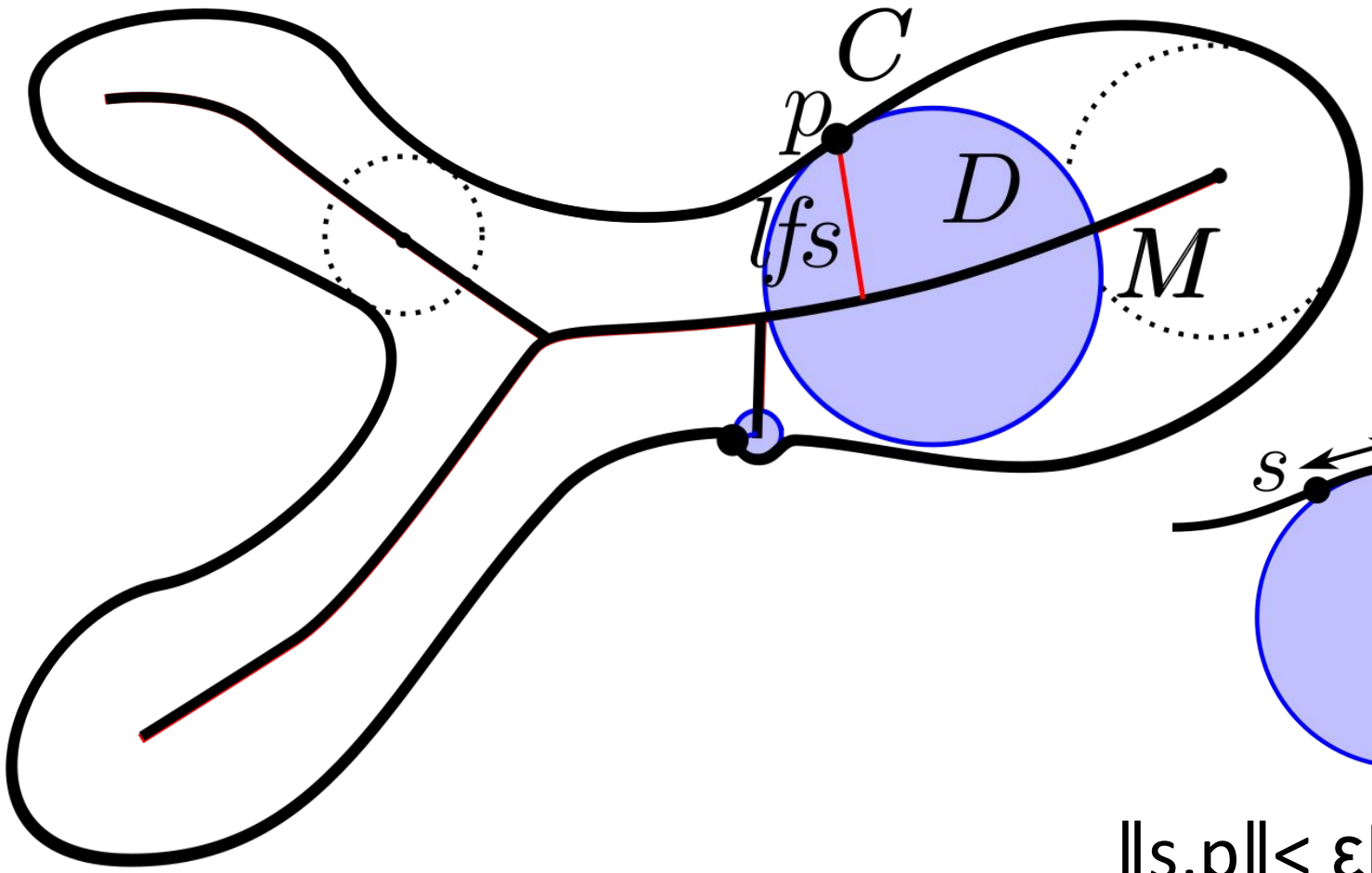
Coloring with gaps



Reconstruction quality

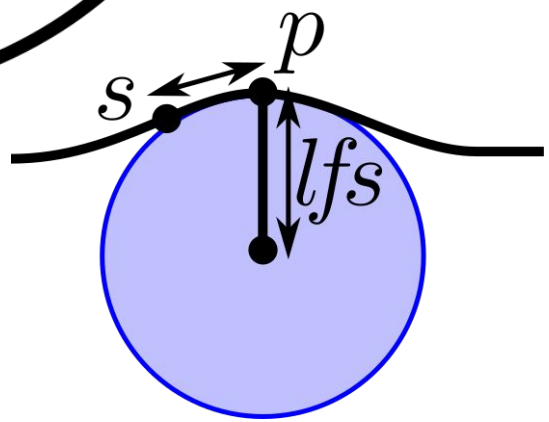
Stefan Ohrhallinger, Amal Dev Parakkat, Pooran Memari: Feature-Sized Sampling for Vector Line Art, Pacific Graphics 2023.

Sampling Theory: Local Feature Size (lfs)



M = medial axis [1]

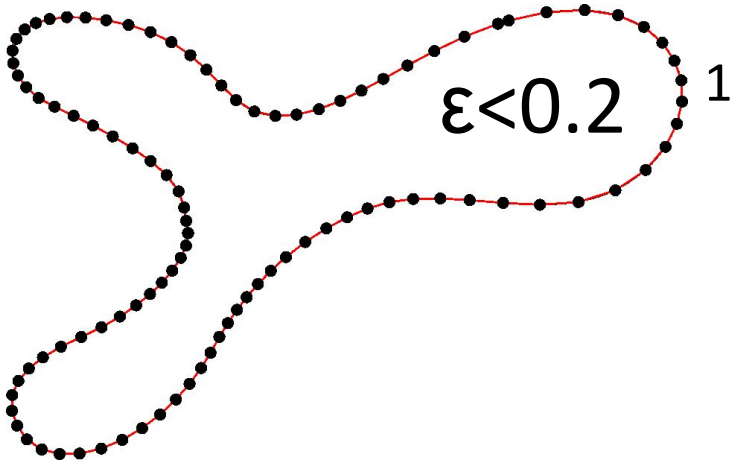
lfs = local feature size [2]



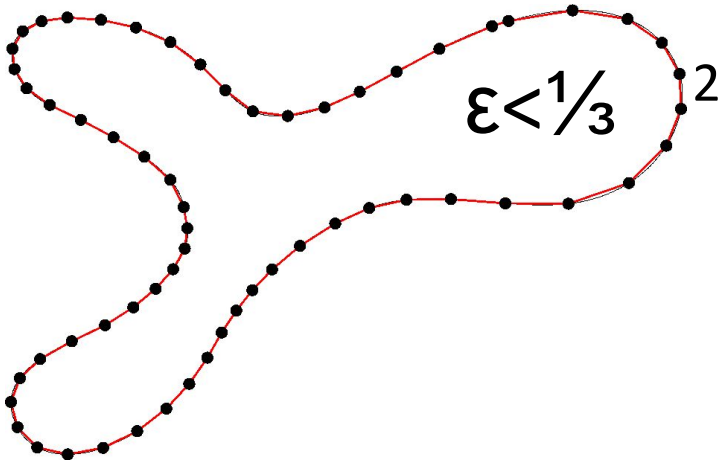
$\|s,p\| < \varepsilon \mathrm{lfs}(p)$ [3]

1 Harry Blum: A Transformation for Extracting New Descriptors of Shape. In Models for the Perception of Speech and Visual Form, MIT Press, Cambridge, 1967, pp. 362–380.
2 Jim Ruppert: A new and simple algorithm for quality 2-dimensional mesh generation. SODA '93, Soc. for Industr. and Appl. Math., pp. 83–92.
3 Nina Amenta, Marshall Bern, David Eppstein: The crust and the beta-skeleton: Combinatorial curve reconstruction. Graphical Models and Image Processing 60, 2 (1998), 125–135.

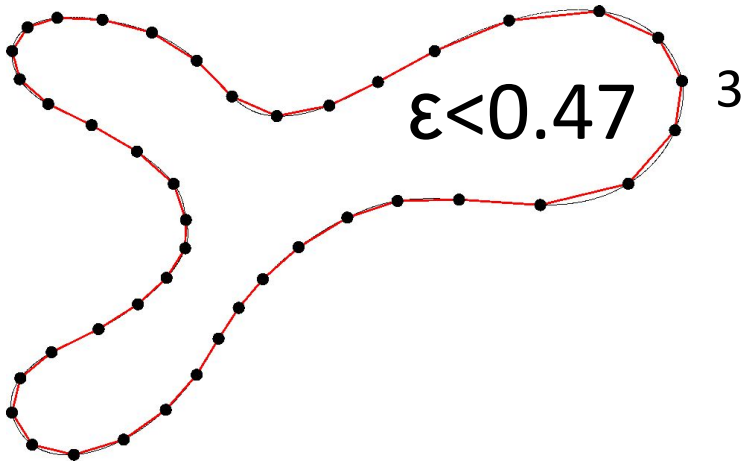
Sampling Conditions for Reconstruction Algorithms



1



2

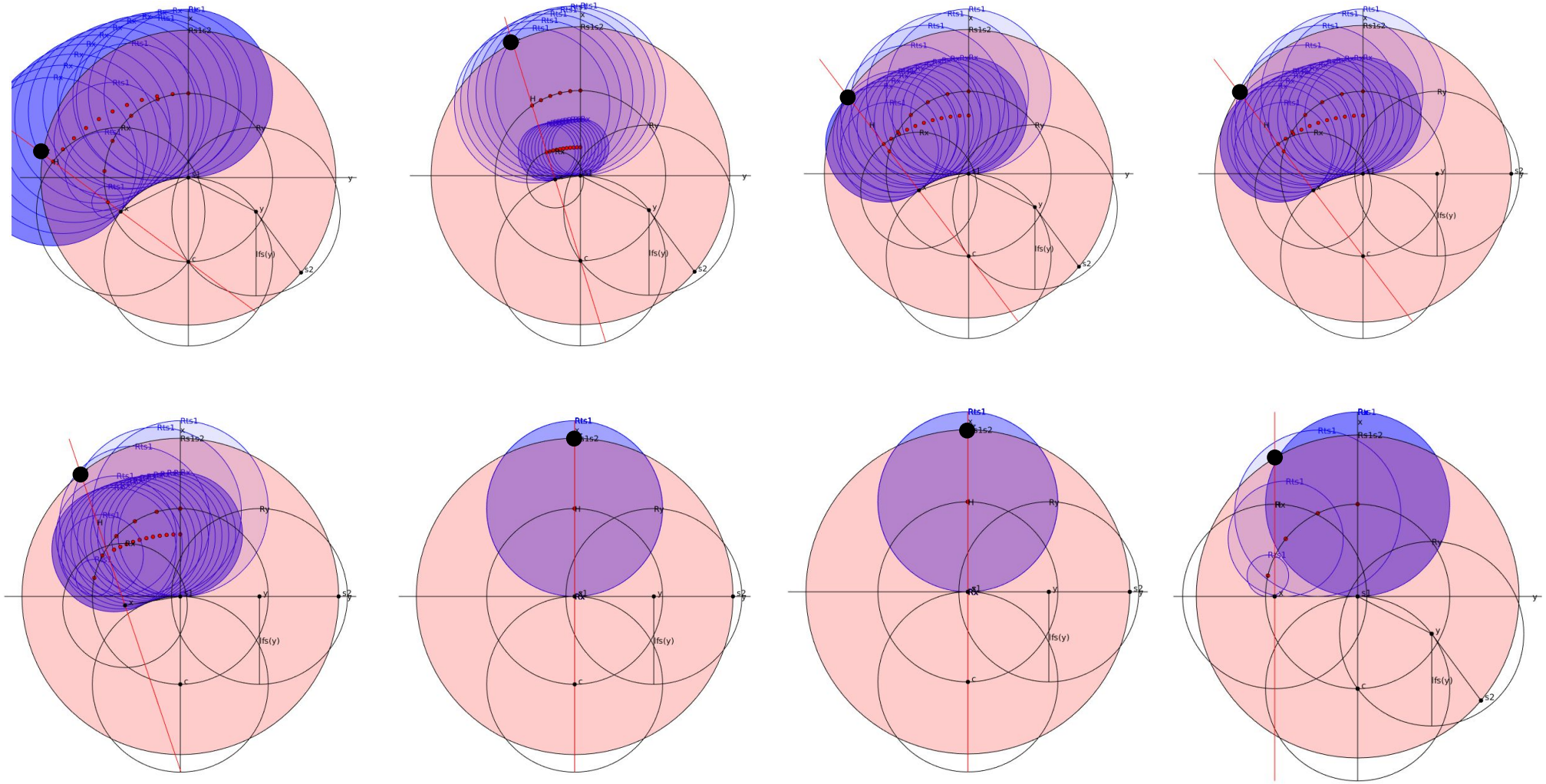


3

Theorem: Reconstructs for ε<0.47

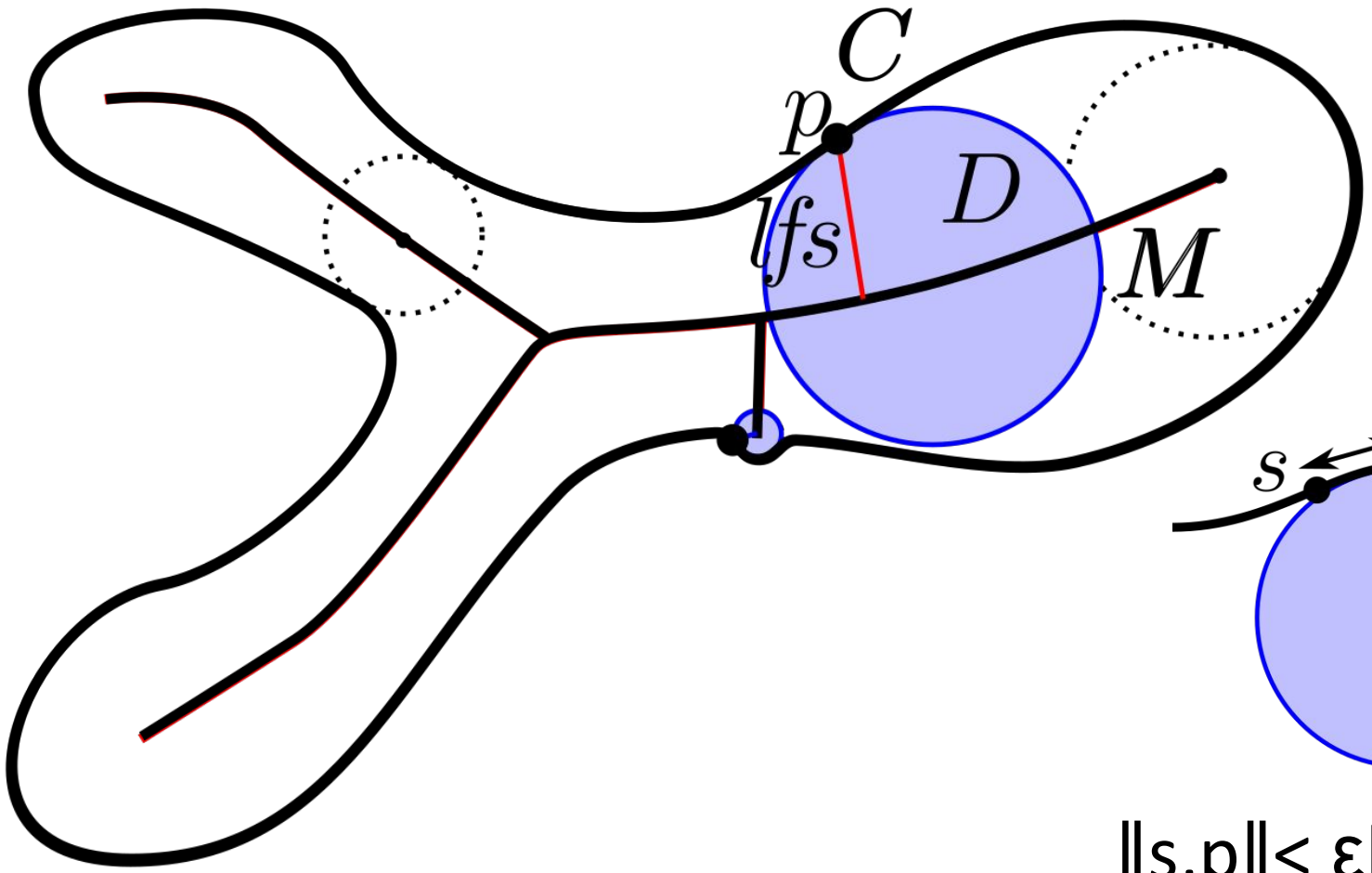
1 Nina Amenta, Marshall Bern, David Eppstein: The crust and the beta-skeleton: Combinatorial curve reconstruction. Graphical Models and Image Processing 60, 2 (1998), 125–135.
2 Tamal K. Dey, Piyush Kumar. "A Simple Provable Algorithm for Curve Reconstruction." *SODA*. Vol. 99. 1999.
3 **Stefan Ohrhallinger**, Scott A. Mitchell, Michael Wimmer: Curve Reconstruction with Many Fewer Samples, Computer Graphics Forum, 35(5):176-176, 2016.

2D Sampling Reconstruction Proof (computer-assisted)



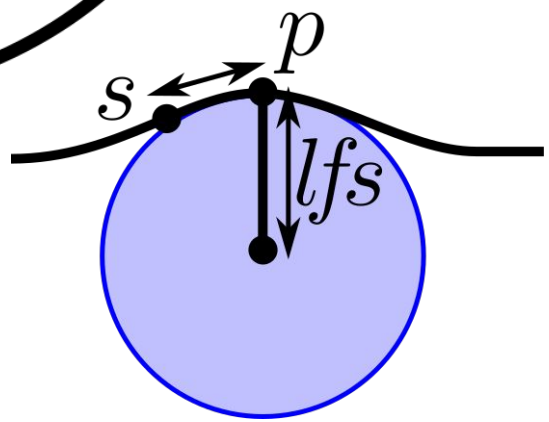
Stefan Ohrhallinger, Scott A. Mitchell, Michael Wimmer: Curve Reconstruction with Many Fewer Samples, Computer Graphics Forum, 35(5):176-176, 2016.

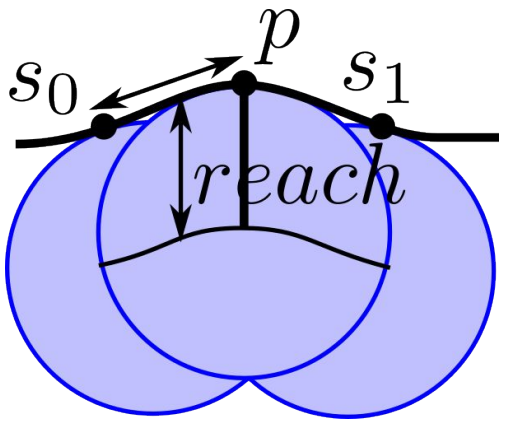
Sampling Theory: Local Feature Size (lfs)



M = medial axis [1]

lfs = local feature size [2]



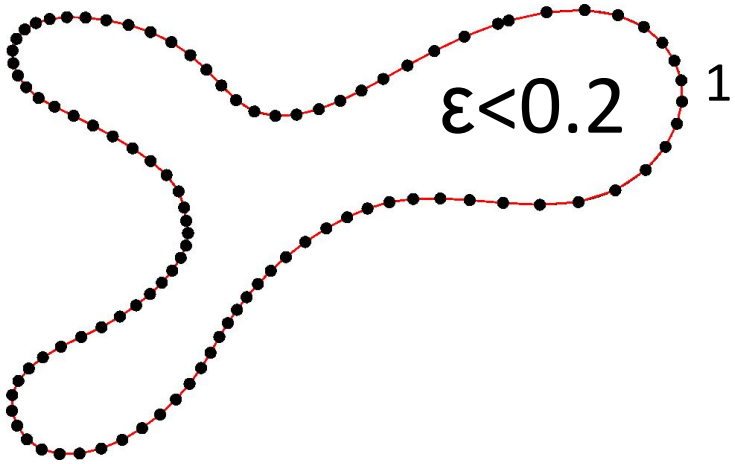


$\|s,p\| < \varepsilon lfs(p)$ [3]

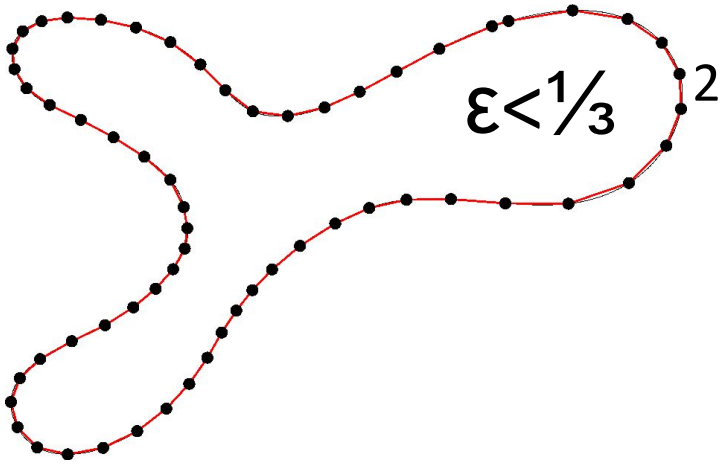
$\|s,p\| < \rho reach(l)$ [4]

1 Harry Blum: A Transformation for Extracting New Descriptors of Shape. In Models for the Perception of Speech and Visual Form, MIT Press, Cambridge, 1967, pp. 362–380.
2 Jim Ruppert: A new and simple algorithm for quality 2-dimensional mesh generation. SODA '93, Soc. for Industr. and Appl. Math., pp. 83–92.
3 Nina Amenta, Marshall Bern, David Eppstein: The crust and the beta-skeleton: Combinatorial curve reconstruction. Graphical Models and Image Processing 60, 2 (1998), 125–135.
4 **Stefan Ohrhallinger**, Scott A. Mitchell, Michael Wimmer: Curve Reconstruction with Many Fewer Samples, Computer Graphics Forum, 35(5):176-176, 2016.

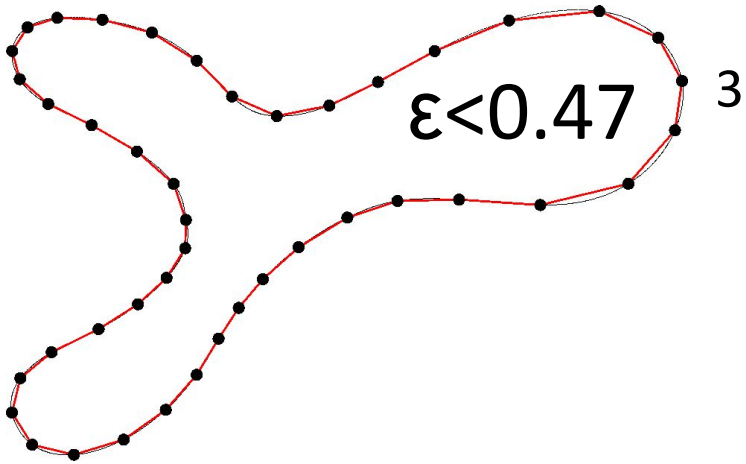
Sampling Conditions for Reconstruction Algorithms



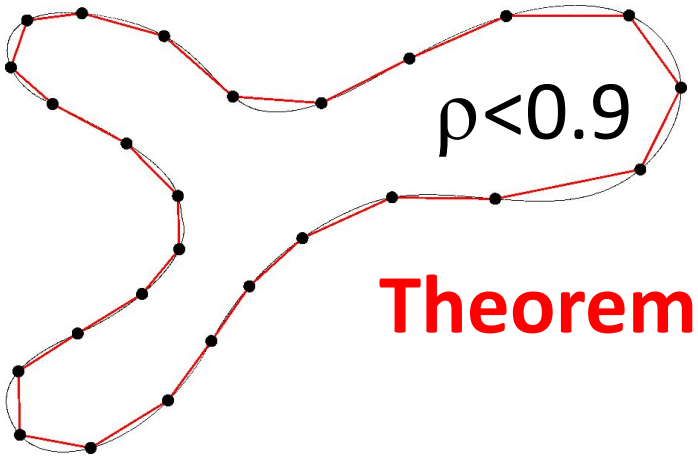
1



2



3

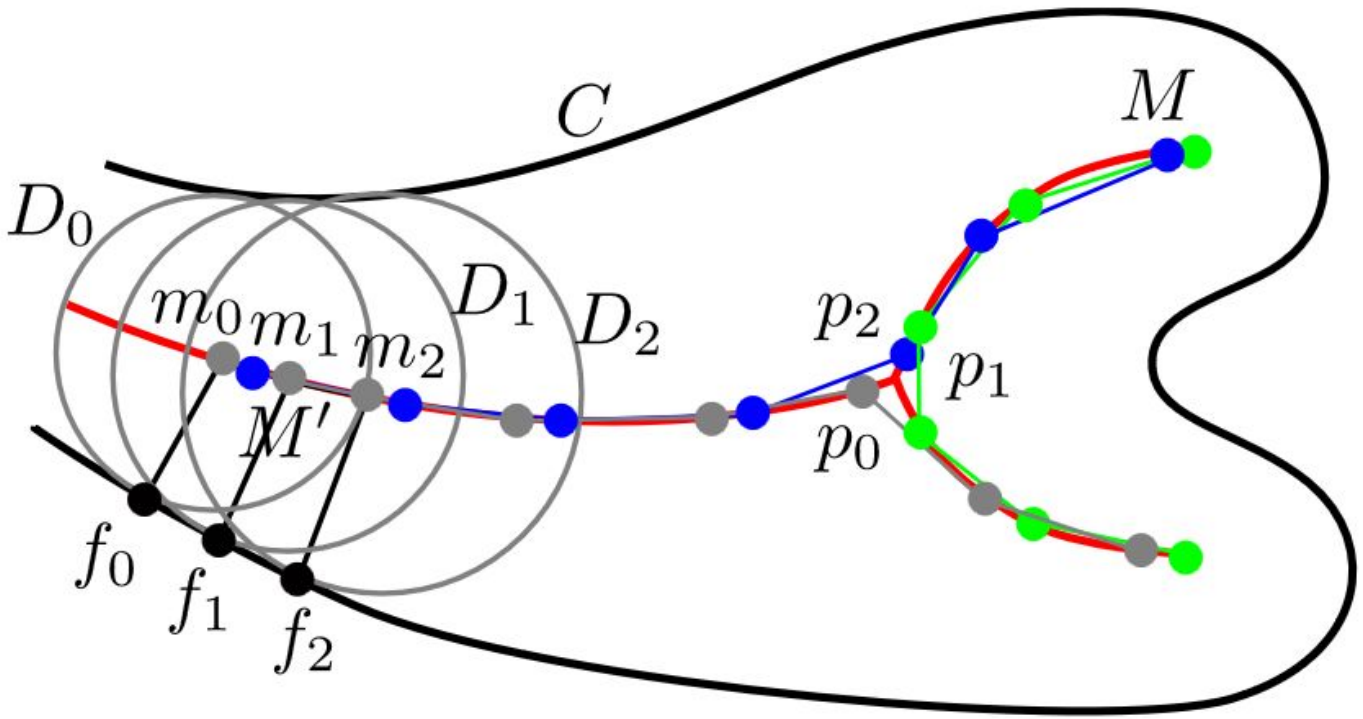


3

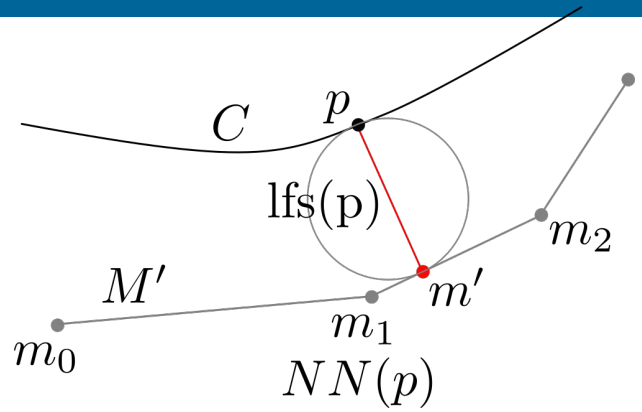
Theorem: ρ<ε/(1-ε)

1 Nina Amenta, Marshall Bern, David Eppstein: The crust and the beta-skeleton: Combinatorial curve reconstruction. Graphical Models and Image Processing 60, 2 (1998), 125–135.
2 Tamal K. Dey, Piyush Kumar. "A Simple Provable Algorithm for Curve Reconstruction." *SODA*. Vol. 99. 1999.
3 **Stefan Ohrhallinger**, Scott A. Mitchell, Michael Wimmer: Curve Reconstruction with Many Fewer Samples, Computer Graphics Forum, 35(5):176-176, 2016.

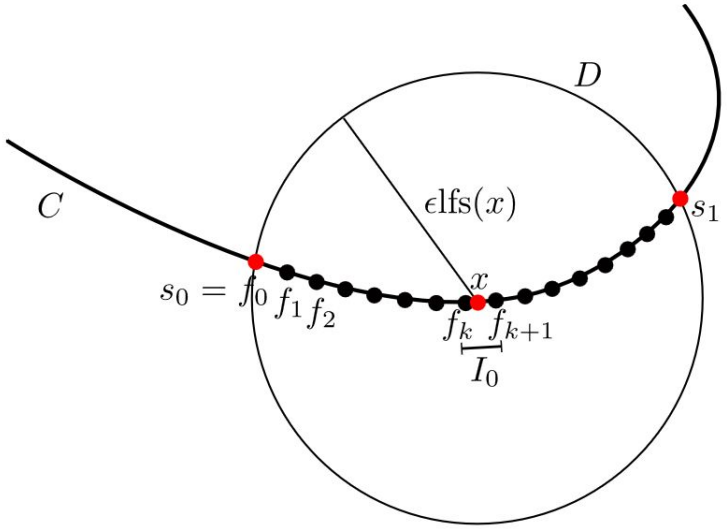
2D Sampling Method



approximate medial axis m_i from f_i



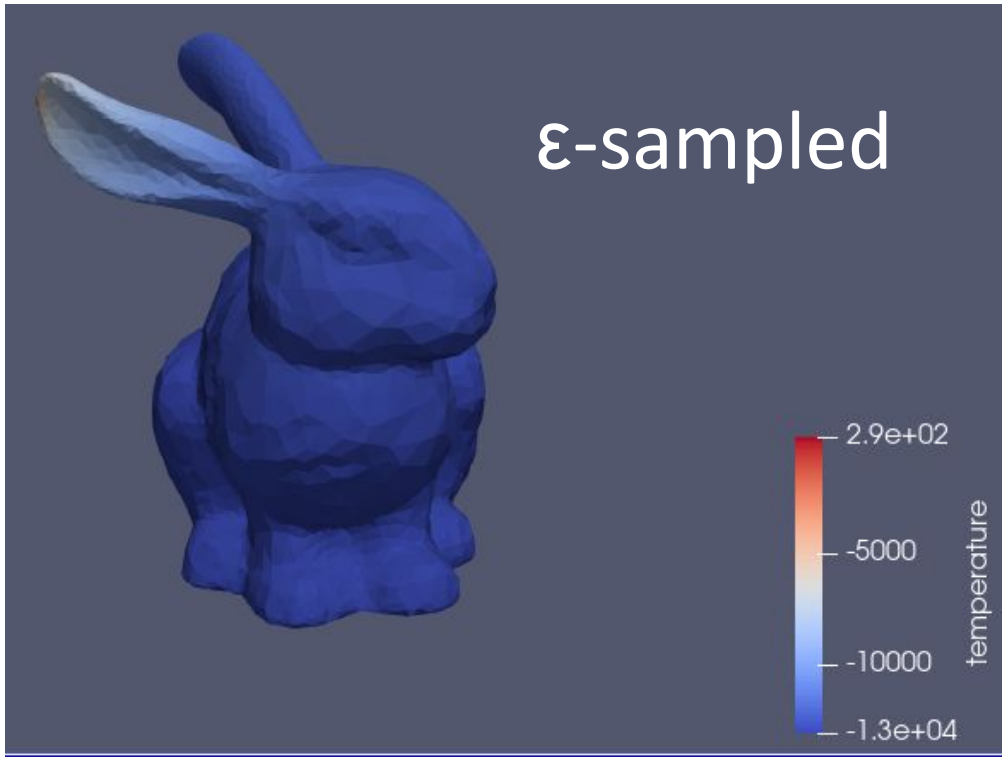
interpolate lfs from m_i

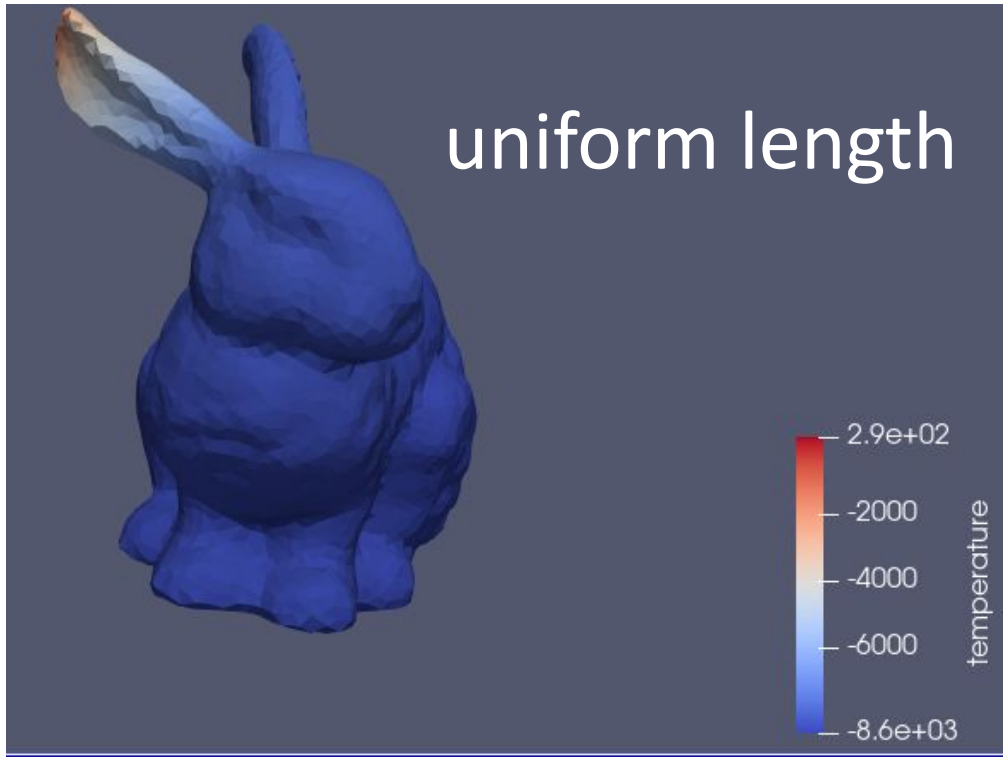


Sample s_1: $D(x, \varepsilon lfs(x)) \cap C$

Stefan Ohrhallinger, Amal Dev Parakkat, Pooran Memari: Feature-Sized Sampling for Vector Line Art, Pacific Graphics 2023.

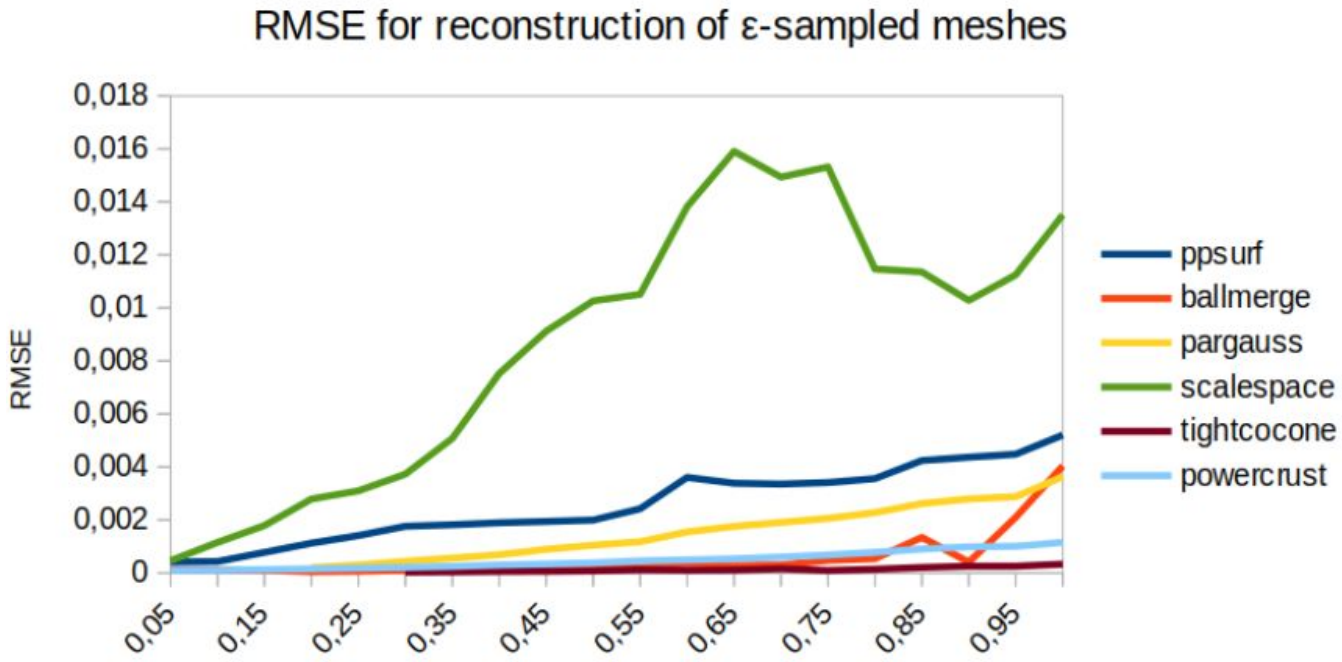
3D Feature-Sized Sampling: Application FEM Simulation





Criterion	cells	seconds	RMSE	max err
$\epsilon = 0.5$ (min l=0.002)	18833	0.32	49.73	9226
l=0.0215	18793	0.30	66.08	12365
l=0.002	2581199	59.69	baseline	baseline

3D Feature-Sized Sampling: Determine ε-Bounds

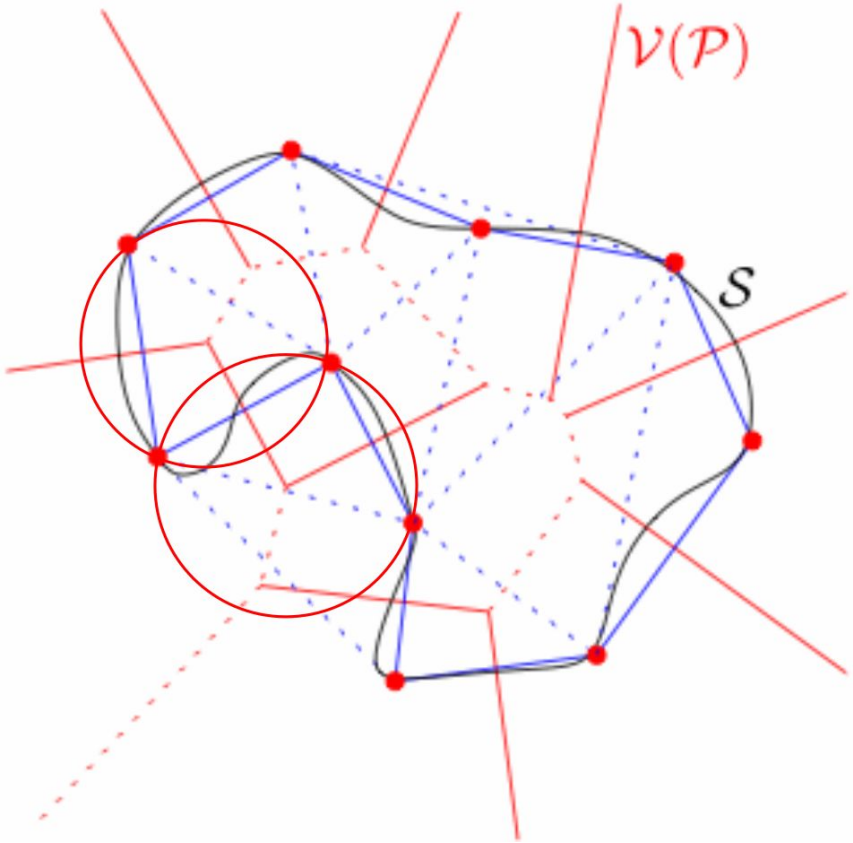


Algorithm	triangles	RMSE
MESHED, $\epsilon = 1$	4246	baseline
TIGHTCOCONE	4246	0.333
POWERCRUST	32556	1.155
PARGAUSS	21252	3.636
BALLMERGE*	3850	4.401
PPSURF	768889	5.207
SCALESPACE	6424	13.510

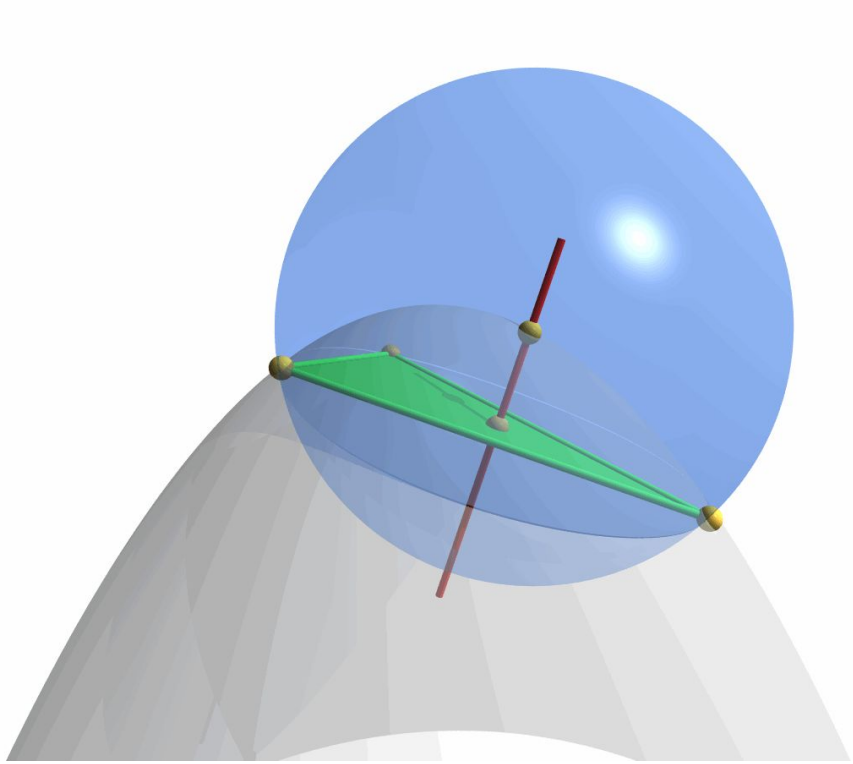
Original TCocone PowCrust ParGauss BallMerge PPSurf ScaleSpace

Amal Dev Parakkat, **Stefan Ohrhallinger**, Elmar Eisemann, Pooran Memari: BallMerge: High-quality Fast Surface Reconstruction via Voronoi Balls, Eurographics, 2024

3D Sampling Method



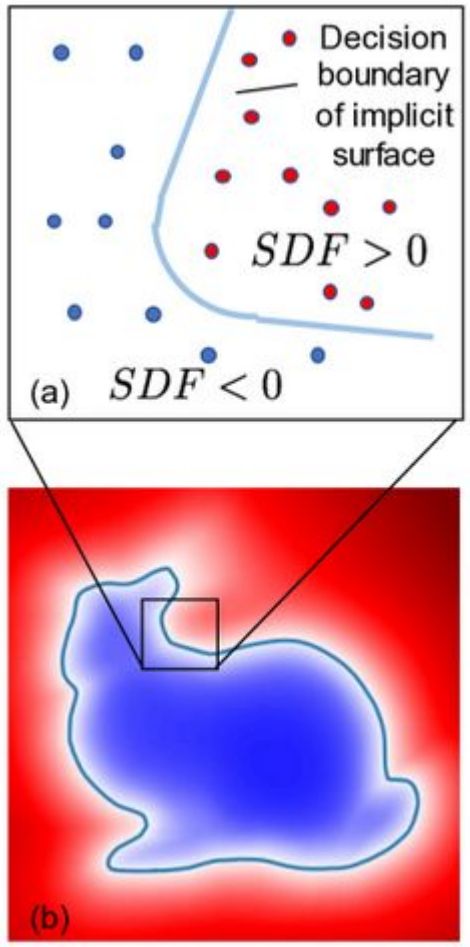
Meshing in 2D[1]



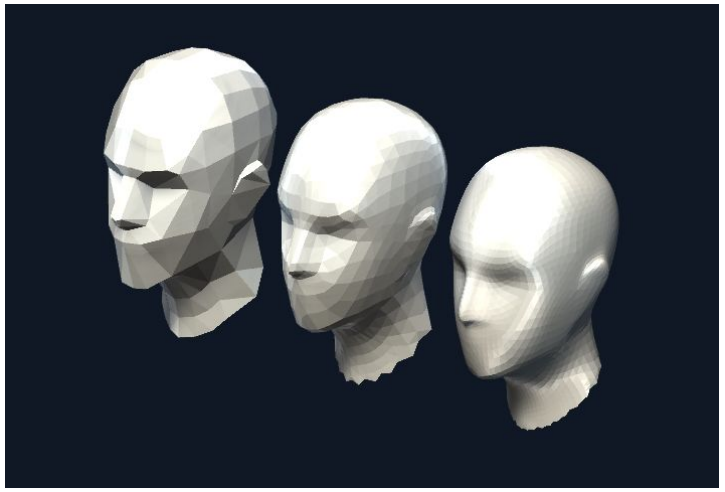
Meshing in 3D[2]

1 L Paul Chew: Guaranteed-quality mesh generation for curved surfaces, SoCG, 1993
2 Jonathan Richard Shewchuk: Tetrahedral mesh generation by Delaunay refinement, SoCG, 1998

Sampling requires a Smooth Boundary

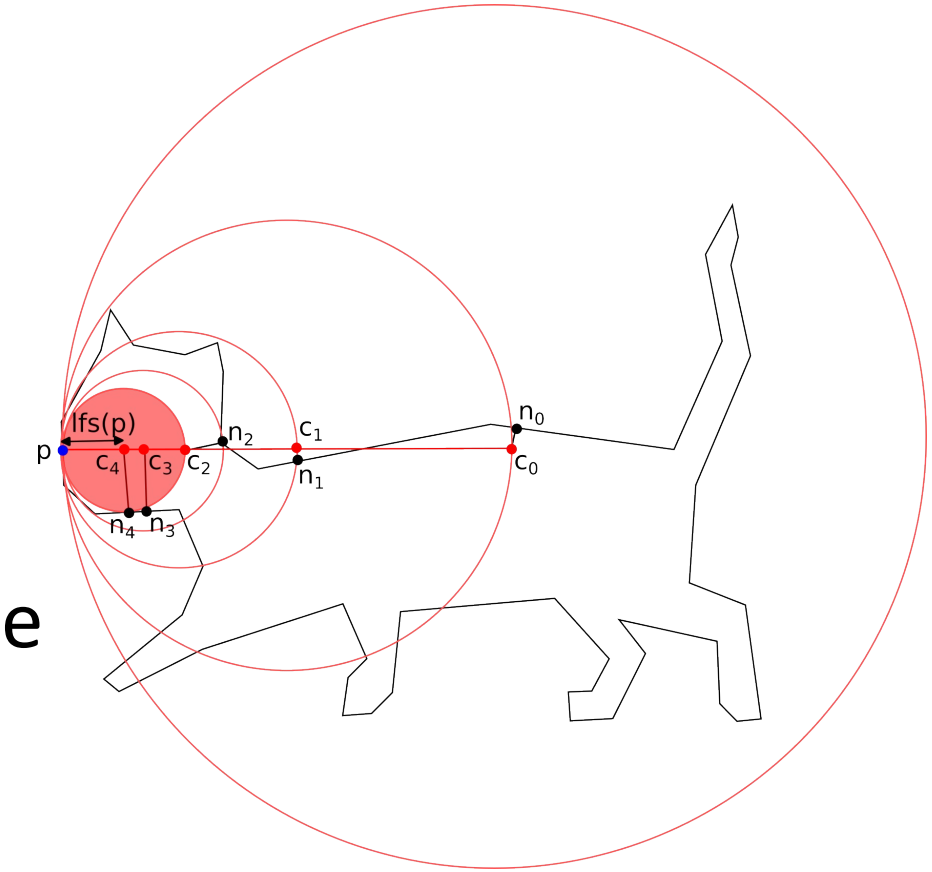


Signed distance function



https://github.com/mattatz/unity-subdivision-surface

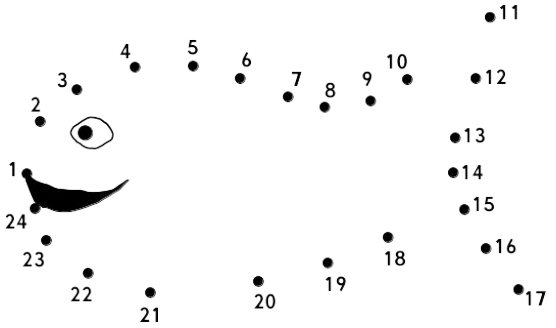
Subdivision limit surface



Proposed lfs computation

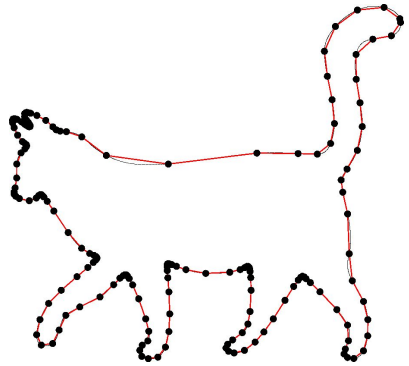
Jeong Joon Park, Peter Florence, Julian Straub, Richard Newcombe, Steven Lovegrove: Deepsdf: Learning continuous signed distance functions for shape representation, CVPR 2019.

Conclusion and Outlook

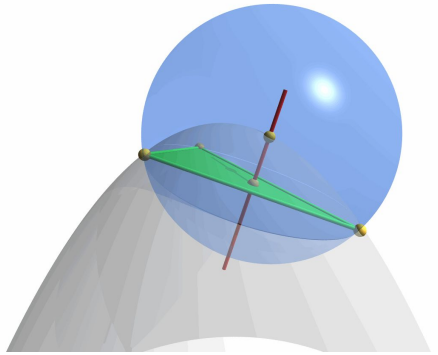


2D Reconstruct

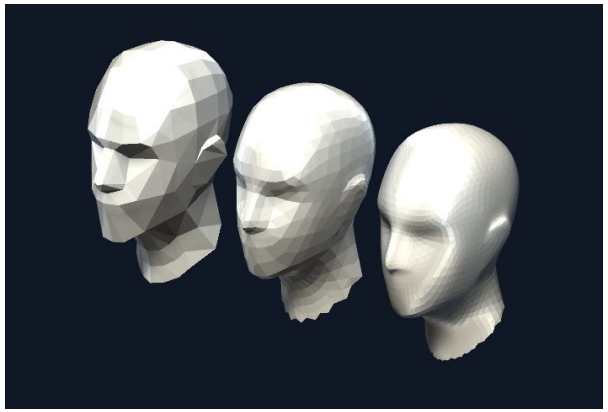
3D Reconstruct



2D Sampling

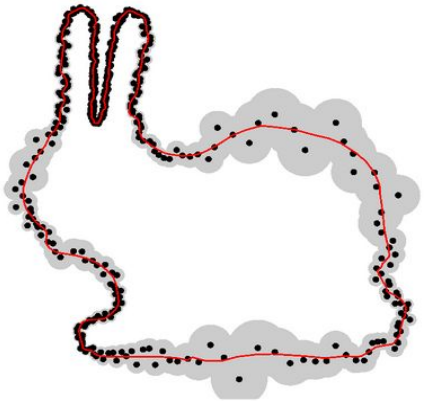


3D Sampling

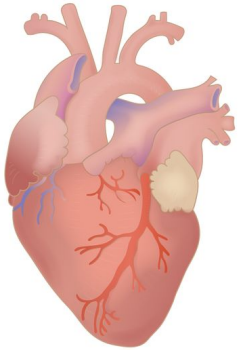


Smooth Surfaces

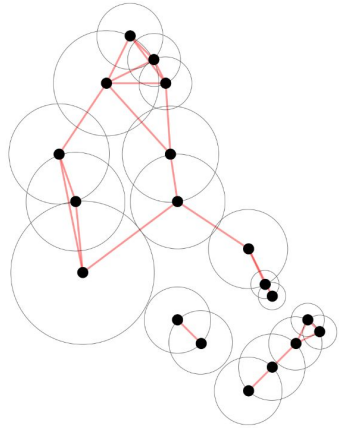
Huge Scans



Probability



4D



SIG>kNN

https://commons.wikimedia.org/wiki/File:Human_Heart.png

Thank you and Thanks to my Collaborators


Sudhir P Mudur


Michael Wimmer


Diana Marin


Philipp Erler


Scott A Mitchell

All code is Open Source:


Amal D Parakkat


Pooran Memari

Tamal K Dey


R Muthuganapathy


Jiju Peethambaran