Exceptional field theory and exotic supergravity

Henning Samtleben
ENS de Lyon

ESI — Higher Structures and Field Theory
09/2020

Lab PHYS ENS de LYON

plan

Kaluza-Klein theory and exceptional field theory

- Kaluza-Klein formulation of higher-dimensional gravity
- gauge structures and tensor hierarchy
- p-forms

exotic supergravities & selfduality equations

- maximal supersymmetry in 6D
- selfdual exotic tensor gauge fields

novel action functionals for (free) exotic supergravities

- 5+1 split
- actions for selfdual two forms (review)
- actions for selfdual exotic tensor fields

Olaf Hohm, HS   [1903.02821, ...]

Yannick Bertrand, Stefan Hohenegger, Olaf Hohm, HS   [2007.11644]

Kaluza-Klein theory and exceptional field theory

Kaluza-Klein reduction of gravity

- D-dimensional gravity $$S = \int d^D X \sqrt{|\det G|}\, R(G)$$
  - coordinate split $X^{\hat\mu} = (x^\mu, y^m)$ $\mu = 1,\dots,n$ $m = 1,\dots,d$

- Kaluza-Klein parametrization

$$\Big\{G_{mn},\ G_{\mu m} = G_{mn}\,A_\mu{}^n,\ G_{\mu\nu} = \phi^{-\frac{1}{n-2}} g_{\mu\nu} + G_{mn}\,A_\mu{}^m A_\nu{}^n\Big\} \qquad \phi = |\det G_{mn}|$$

  - in terms of n-dimensional metric $g_{\mu\nu}$, vectors $A_\mu{}^m$, scalar fields G_{mn}

- reduction to n dimensions: $\partial_m \to 0$

$$S = \int d^n x \sqrt{g}\,\Big(R(g) - \tfrac14 \phi^{\frac{1}{n-2}} G_{mn}\, F^{\mu\nu m} F_{\mu\nu}{}^n + \tfrac14 \partial^\mu G^{mn} \partial_\mu G_{mn} - \tfrac{1}{4(n-2)} \big(\phi^{-1}\partial_\mu \phi\big)^2\Big)$$

  - Einstein-Maxwell theory coupled to scalar fields: coset σ-space model $\mathrm{GL}(d)/\mathrm{SO}(d)$

Kaluza-Klein formulation of gravity

- D-dimensional gravity $$S = \int d^D X \sqrt{|\det G|}\, R(G)$$
  - coordinate split $X^{\hat\mu} = (x^\mu, y^m)$ $\mu = 1, \dots, n$ $m = 1, \dots, d$

- Kaluza-Klein parametrization

$$\left\{ G_{mn},\; G_{\mu m} = G_{mn}\, A_\mu{}^n,\; G_{\mu\nu} = \phi^{-\frac{1}{n-2}} g_{\mu\nu} + G_{mn}\, A_\mu{}^m A_\nu{}^n \right\} \qquad \phi = |\det G_{mn}|$$

  - in terms of n-dimensional metric $g_{\mu\nu}$, vectors $A_\mu{}^m$, scalar fields G_{mn}

- reduction to n dimensions: $\partial_m \to 0$

$$S = \int d^n x \sqrt{g} \left(R(g) - \tfrac{1}{4} \phi^{\frac{1}{n-2}} G_{mn}\, F^{\mu\nu m} F_{\mu\nu}{}^n + \tfrac{1}{4} \partial^\mu G^{mn} \partial_\mu G_{mn} - \tfrac{1}{4(n-2)} \left(\phi^{-1} \partial_\mu \phi\right)^2 \right)$$

- same rewriting without reduction $\partial_m \neq 0$

$$S = \int d^n x\, d^d y \sqrt{g} \left(\widehat{R}(g) - \tfrac{1}{4} \phi^{\frac{1}{n-2}} G_{mn}\, \mathcal{F}^{\mu\nu m} \mathcal{F}_{\mu\nu}{}^n + \tfrac{1}{4} D^\mu G^{mn} D_\mu G_{mn} - \tfrac{1}{4(n-2)} \left(\phi^{-1} D_\mu \phi\right)^2 - V(G, g) \right)$$

  - exhibits an infinite-dimensional gauge structure (internal diffeomorphisms)

Kaluza-Klein formulation of gravity

- D-dimensional gravity $$S = \int d^D X \sqrt{|\det G|}\, R(G)$$

$\Longleftrightarrow$

$$S = \int d^n x d^d y \sqrt{g} \left(\widehat{R}(g) - \tfrac{1}{4} \phi^{\frac{1}{n-2}} G_{mn} \mathcal{F}^{\mu\nu m} \mathcal{F}_{\mu\nu}{}^n + \tfrac{1}{4} D^\mu G^{mn} D_\mu G_{mn} - \tfrac{1}{4(n-2)} (\phi^{-1} D_\mu \phi)^2 - V(G,g) \right)$$

  - exhibits an infinite-dimensional gauge structure (internal diffeomorphisms)

$$D_\mu G_{mn} = \partial_\mu G_{mn} - A_\mu{}^k \partial_k G_{mn} - G_{km} \partial_n A_\mu{}^k - G_{kn} \partial_m A_\mu{}^k$$

$$\mathcal{F}_{\mu\nu}{}^m = \partial_\mu A_\nu{}^m - \partial_\nu A_\mu{}^m - A_\mu{}^n \partial_n A_\nu{}^m + A_\nu{}^n \partial_n A_\mu{}^m$$

- the same can be extended to matter-coupled (super-)gravity theories
  - the resulting infinite-dimensional gauge structure combines internal diffeomorphisms with the (internal) tensor gauge symmetries
  - covariant under the global (exceptional) duality groups

$\longrightarrow$ exceptional field theories (ExFT)

Kaluza-Klein formulation of gravity

- D-dimensional gravity $$S = \int \mathrm{d}^D X \sqrt{|\det \mathrm{G}|}\, R(G)$$

$\Longleftrightarrow$

$$S = \int \mathrm{d}^n x \mathrm{d}^d y \sqrt{g} \left(\widehat{R}(g) - \tfrac{1}{4}\phi^{\frac{1}{n-2}} G_{mn}\, \mathcal{F}^{\mu\nu m} \mathcal{F}_{\mu\nu}{}^{n} + \tfrac{1}{4} D^{\mu} G^{mn} D_{\mu} G_{mn} - \tfrac{1}{4(n-2)} \left(\phi^{-1} D_{\mu}\phi\right)^2 - V(G,g) \right)$$

- the same can be extended to matter-coupled (super-)gravity theories
  - the resulting infinite-dimensional gauge structure combines internal diffeomorphisms with the (internal) tensor gauge symmetries
  - covariant under the global (exceptional) duality groups

$\longrightarrow$ exceptional field theories (ExFT)

- Kaluza-Klein formulation without Kaluza-Klein reduction
  - exhibits interesting structures within the higher-dimensional theory
  - powerful tool in order to study compactifications to lower dimensions

p-forms: an example

- two-form gauge field $B_{\hat\mu\hat\nu}$ coupled to gravity

$$\mathcal{L} \;=\; -\frac{1}{12}\sqrt{|\det G|}\,H_{\hat\mu\hat\nu\hat\rho}H^{\hat\mu\hat\nu\hat\rho} \qquad H_{\hat\mu\hat\nu\hat\rho} = 3\,\partial_{[\hat\mu}B_{\hat\nu\hat\rho]}$$

  - Kaluza-Klein split $X^{\hat\mu} = (x^\mu, y^m)$

$$\{B_{\hat\mu\hat\nu}\} = \;\{B_{mn},\; B_{\mu m} = A_{\mu m} + A_\mu{}^k B_{km},\; B_{\mu\nu} = b_{\mu\nu} - 2A_{[\mu}{}^k A_{\nu]k} + A_\mu{}^m A_\nu{}^n B_{mn}\}$$

- reduction to n dimensions: $\partial_m \to 0$
  - additional Maxwell fields $A_{\mu\,m}$ and 2-form tensor $b_{\mu\nu}$,
    coupled to Einstein-Maxwell and scalar fields ⟶ enhancement of the scalar coset space

p-forms: an example

- two-form gauge field $B_{\hat\mu\hat\nu}$ coupled to gravity

$$\mathcal{L} \;=\; -\frac{1}{12}\sqrt{|\det G|}\,H_{\hat\mu\hat\nu\hat\rho}H^{\hat\mu\hat\nu\hat\rho} \qquad H_{\hat\mu\hat\nu\hat\rho} = 3\,\partial_{[\hat\mu}B_{\hat\nu\hat\rho]}$$

  - Kaluza-Klein split $X^{\hat\mu} = (x^\mu, y^m)$

$$\{B_{\hat\mu\hat\nu}\} = \; \{B_{mn},\; B_{\mu m} = A_{\mu m} + A_\mu{}^k B_{km},\; B_{\mu\nu} = b_{\mu\nu} - 2A_{[\mu}{}^k A_{\nu]k} + A_\mu{}^m A_\nu{}^n B_{mn}\}$$

- reduction to n dimensions: $\partial_m \to 0$
  - additional Maxwell fields $A_{\mu\, m}$ and 2-form tensor $b_{\mu\nu}$, coupled to Einstein-Maxwell and scalar fields $\longrightarrow$ enhancement of the scalar coset space

- Kaluza-Klein reformulation of the full theory $\partial_m \neq 0$
  - gives rise to non-abelian field strengths

non-abelian internal diffeomorphisms

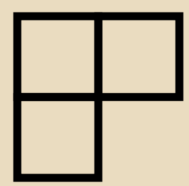
$$\mathcal{H}_{\mu\nu\rho} \;=\; 3\,D_{[\mu}b_{\nu\rho]} - 3\,F_{[\mu\nu}{}^k A_{\rho]k} \qquad \mathcal{F}_{\mu\nu\, m} \;=\; 2\,D_{[\mu}A_{\nu]\,m} + \partial_m b_{\mu\nu}$$

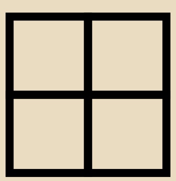
Chern-Simons term

Stückelberg type coupling (tensor hierarchy)

exotic supergravities

maximally supersymmetric 6D theories featuring exotic tensor fields





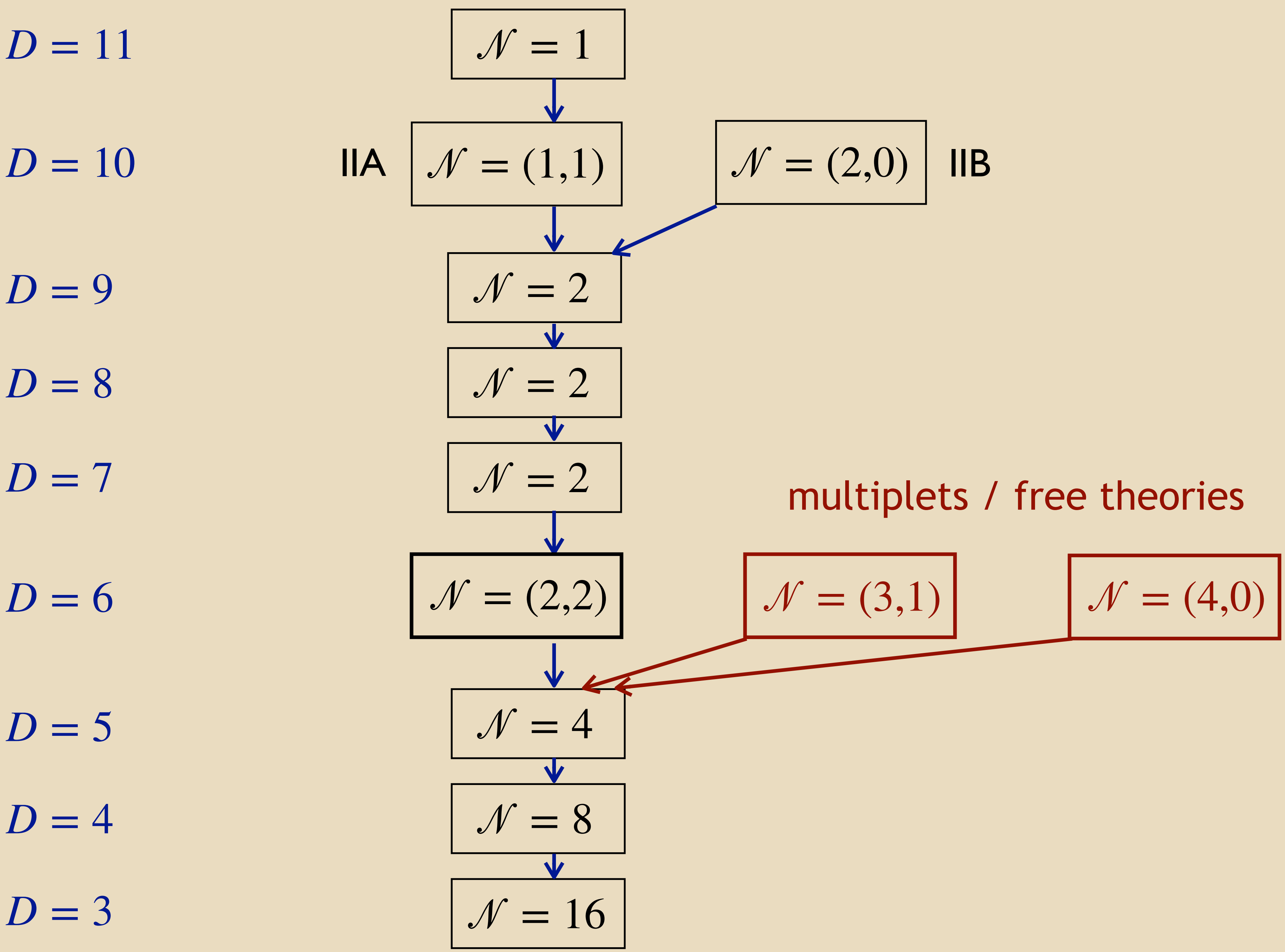
maximal supersymmetry (32 real supercharges)

standard supergravities

$D = 11$	$\mathcal{N} = 1$	
$D = 10$	IIA $\mathcal{N} = (1,1)$	$\mathcal{N} = (2,0)$ IIB
$D = 9$	$\mathcal{N} = 2$	
$D = 8$	$\mathcal{N} = 2$	
$D = 7$	$\mathcal{N} = 2$	
$D = 6$	$\mathcal{N} = (2,2)$	
$D = 5$	$\mathcal{N} = 4$	
$D = 4$	$\mathcal{N} = 8$	
$D = 3$	$\mathcal{N} = 16$	

maximal supersymmetry (32 real supercharges)

standard supergravities



6D exotic supergravities: multiplets

$D = 6$

$\mathcal{N} = (2,2)$	$\mathcal{N} = (3,1)$	$\mathcal{N} = (4,0)$
$g_{\mu\nu}$ (1) A_μ (16) $B_{\mu\nu}$ (5) ϕ (25)	$C^{+}_{\mu\nu,\rho}$ (1) A_μ (14) $B^{+}_{\mu\nu}$ (12) ϕ (28)	$T^{+}_{\mu\nu,\rho\sigma}$ (1) $B^{+}_{\mu\nu}$ (27) ϕ (42)
& fermions	& fermions	& fermions

- exotic tensor fields ("exotic gravitons")
- selfdual in 6D

6D exotic supergravities: multiplets

$D = 6$

$\mathcal{N} = (2,2)$ | $\mathcal{N} = (3,1)$ | $\mathcal{N} = (4,0)$

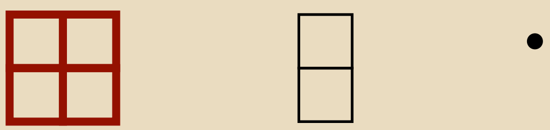
$g_{\mu\nu}$ (1) A_μ (16) $B_{\mu\nu}$ (5) ϕ (25)

& fermions

$C^{+}_{\mu\nu,\rho}$ (1) A_μ (14) $B^{+}_{\mu\nu}$ (12) ϕ (28)

& fermions

$T^{+}_{\mu\nu,\rho\sigma}$ (1) $B^{+}_{\mu\nu}$ (27) ϕ (42)



& fermions

- exotic tensor fields ("exotic gravitons")
- selfdual in 6D
- reduction to 5D yields the unique maximal supergravity multiplet [Hull]

$D = 5$

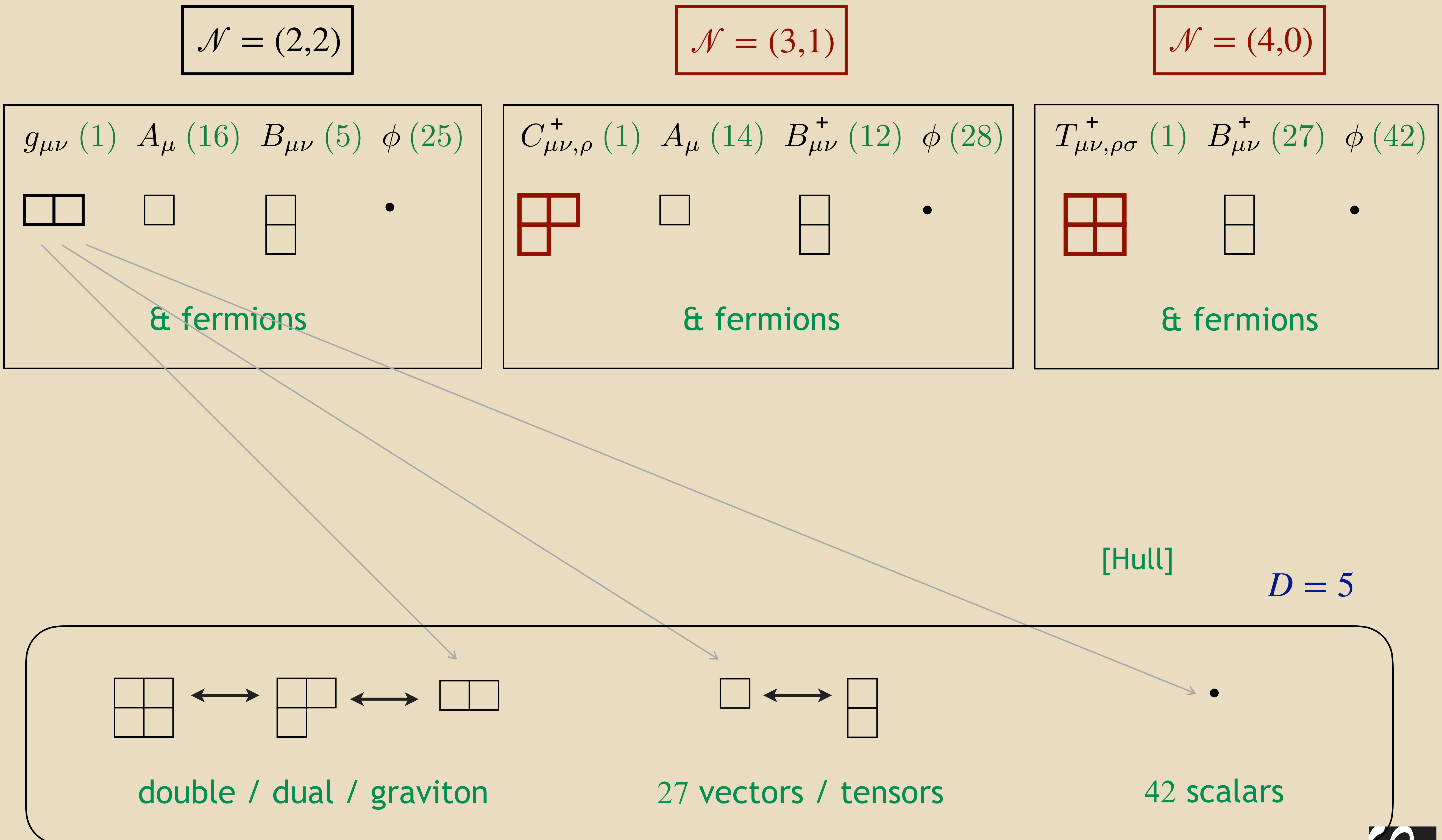
double / dual / graviton

27 vectors / tensors

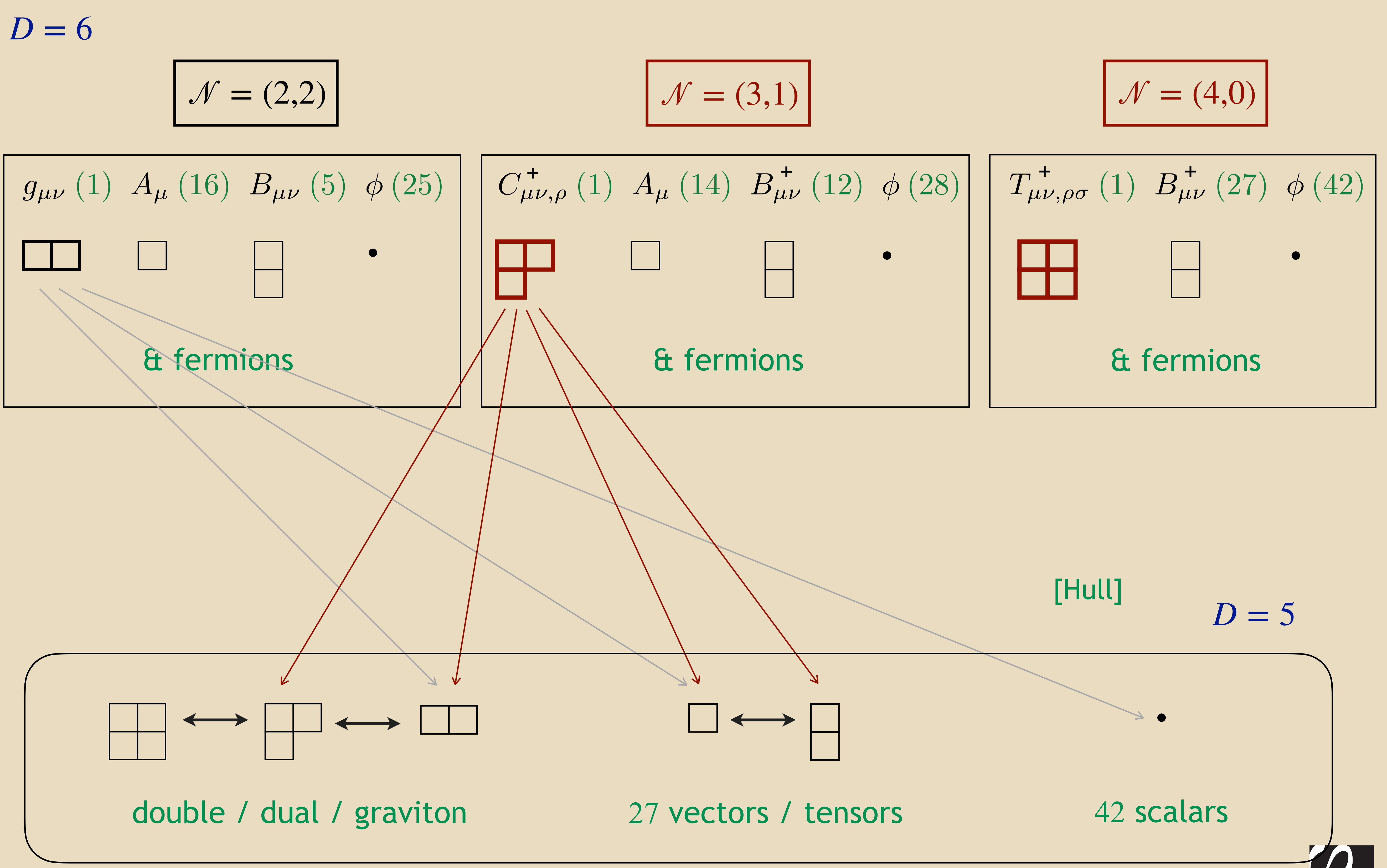
42 scalars

6D exotic supergravities: multiplets

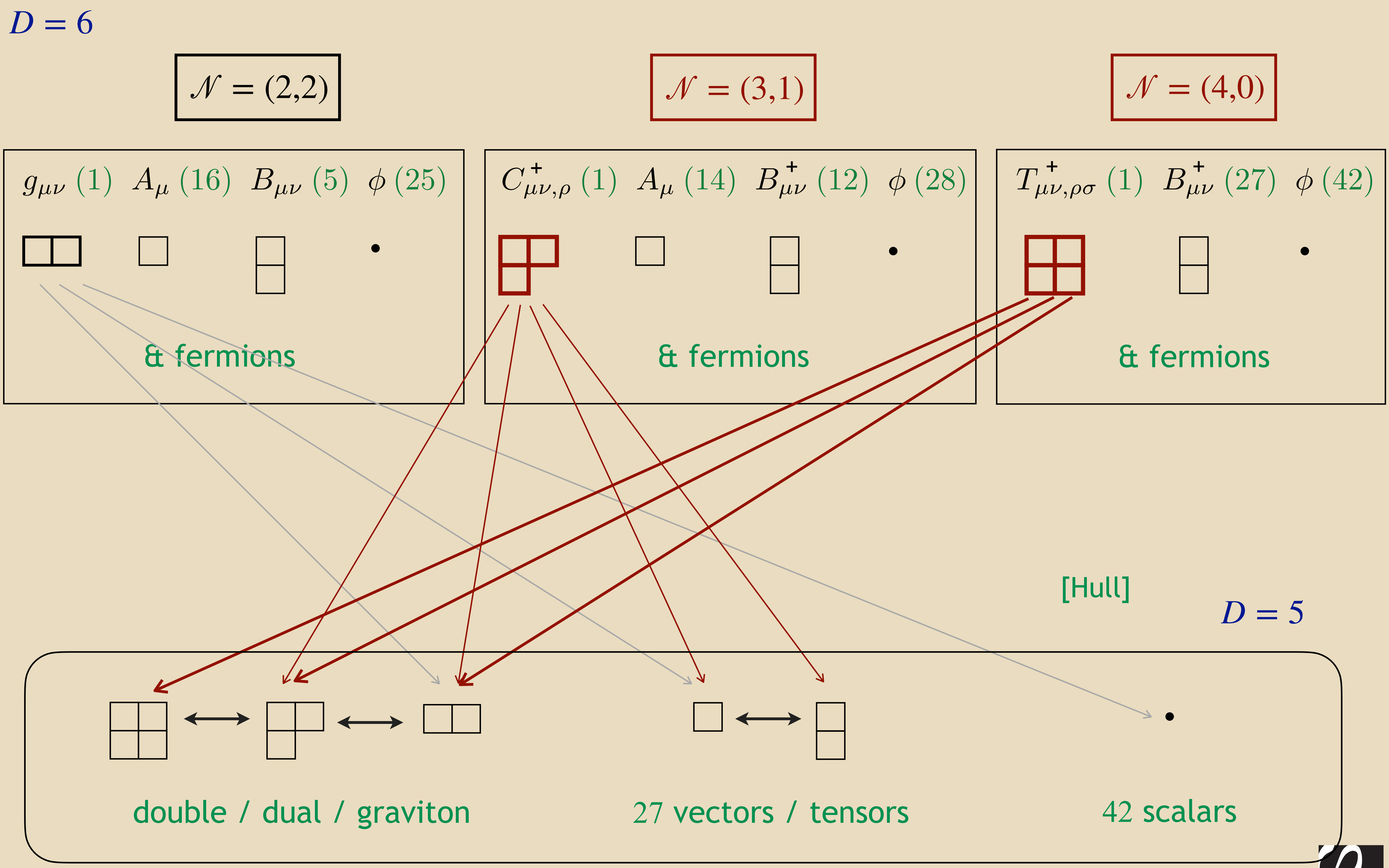
$D = 6$



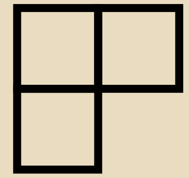
6D exotic supergravities: multiplets



6D exotic supergravities: multiplets

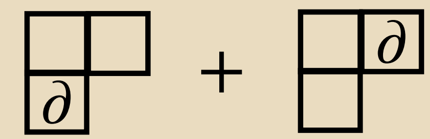


the selfdual Curtright field

- exotic tensor field (6D) $C_{\hat\mu\hat\nu,\hat\rho}$ 

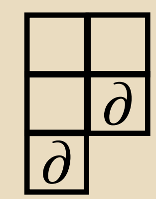
[Curtright, 1980]
[Hull, 2000]

  - gauge freedom $\delta C_{\hat\mu\hat\nu,\hat\rho} = 2\,\partial_{[\hat\mu}\alpha_{\hat\nu]\hat\rho} + \partial_{\hat\rho}\beta_{\hat\mu\hat\nu} - \partial_{[\hat\rho}\beta_{\hat\mu\hat\nu]}$

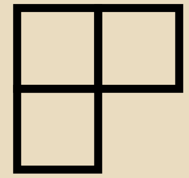


  - gauge invariant second order curvature

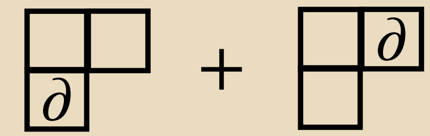
$$S_{\hat\mu\hat\nu\hat\rho,\hat\sigma\hat\tau} = 3\,\partial_{\hat\sigma}\partial_{[\hat\mu}C_{\hat\nu\hat\rho],\hat\tau} - 3\,\partial_{\hat\tau}\partial_{[\hat\mu}C_{\hat\nu\hat\rho],\hat\sigma}$$



the selfdual Curtright field

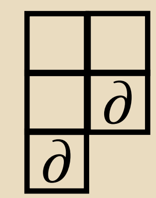
- exotic tensor field (6D) $C_{\hat{\mu}\hat{\nu},\hat{\rho}}$ 

[Curtright, 1980]
[Hull, 2000]

  - gauge freedom $\delta C_{\hat{\mu}\hat{\nu},\hat{\rho}} = 2\,\partial_{[\hat{\mu}}\alpha_{\hat{\nu}]\hat{\rho}} + \partial_{\hat{\rho}}\beta_{\hat{\mu}\hat{\nu}} - \partial_{[\hat{\rho}}\beta_{\hat{\mu}\hat{\nu}]}$ 

  - gauge invariant second order curvature

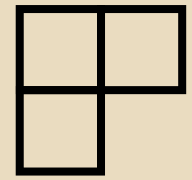
$$S_{\hat{\mu}\hat{\nu}\hat{\rho},\hat{\sigma}\hat{\tau}} = 3\,\partial_{\hat{\sigma}}\partial_{[\hat{\mu}}C_{\hat{\nu}\hat{\rho}],\hat{\tau}} - 3\,\partial_{\hat{\tau}}\partial_{[\hat{\mu}}C_{\hat{\nu}\hat{\rho}],\hat{\sigma}}$$



  - selfduality equation (2$^{\text{nd}}$ order)

$$S_{\hat{\mu}\hat{\nu}\hat{\rho},\hat{\sigma}\hat{\tau}} = \frac{1}{6}\,\varepsilon_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\eta}\hat{\kappa}\hat{\lambda}}\,S^{\hat{\eta}\hat{\kappa}\hat{\lambda}}{}_{\hat{\sigma}\hat{\tau}}$$

the selfdual Curtright field

- exotic tensor field (6D) $C_{\hat\mu\hat\nu,\hat\rho}$ 

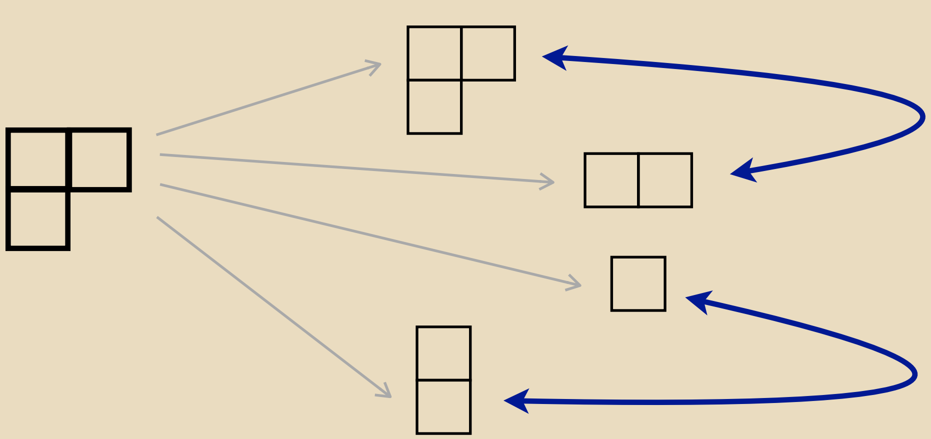
[Curtright, 1980]
[Hull, 2000]

  - gauge invariant second order curvature $S_{\hat\mu\hat\nu\hat\rho,\hat\sigma\hat\tau} = 3\,\partial_{\hat\sigma}\partial_{[\hat\mu}C_{\hat\nu\hat\rho],\hat\tau} - 3\,\partial_{\hat\tau}\partial_{[\hat\mu}C_{\hat\nu\hat\rho],\hat\sigma}$
  - selfduality equation (2nd order)

$$S_{\hat\mu\hat\nu\hat\rho,\hat\sigma\hat\tau} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\rho\hat\eta\hat\kappa\hat\lambda}\,S^{\hat\eta\hat\kappa\hat\lambda}{}_{\hat\sigma\hat\tau}$$

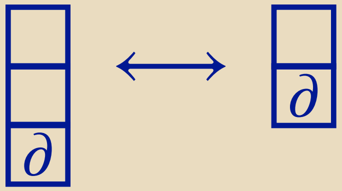
  - Kaluza-Klein reduction to 5D

$$\left\{C_{\hat\mu\hat\nu,\hat\rho}\right\} = \left\{C_{\mu\nu,\rho} - 2\,A_{[\mu}\eta_{\nu]\rho}\,;\; C_{\mu 6,\nu} = h_{\mu\nu} + B_{\mu\nu}\,;\; C_{\mu 6,6} = 2\,A_\mu\right\}$$



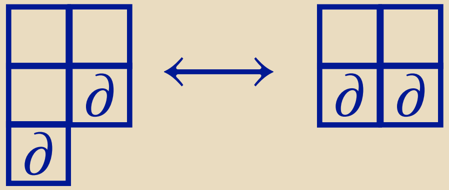
$$\partial_\lambda\left(F_{\mu\nu} + \tfrac{1}{6}\,\varepsilon_{\mu\nu\rho\sigma\tau}\,H^{\rho\sigma\tau}\right) \;=\; 0$$

vector-tensor duality



$$R_{\mu\nu,\rho\sigma} - \tfrac{1}{2}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial_{[\rho}\partial^{\kappa}C^{\lambda\tau}{}_{\sigma]} \;=\; 0$$

spin-2 duality

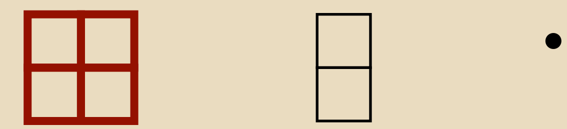


  - Exotic tensor calculus / generalized Poincaré Lemma [de Medeiros, Hull] [Bekaert, Boulanger]

6D exotic supergravities: multiplets

$D = 6$

$\mathcal{N} = (2,2)$	$\mathcal{N} = (3,1)$	$\mathcal{N} = (4,0)$
$g_{\mu\nu}$ (1) A_μ (16) $B_{\mu\nu}$ (5) ϕ (25)	$C^{+}_{\mu\nu,\rho}$ (1) A_μ (14) $B^{+}_{\mu\nu}$ (12) ϕ (28)	$T^{+}_{\mu\nu,\rho\sigma}$ (1) $B^{+}_{\mu\nu}$ (27) ϕ (42)

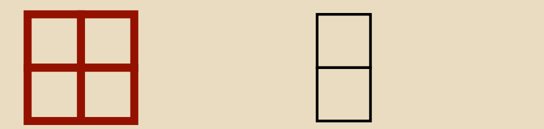


- $\mathcal{N} = (3,1)$ and $\mathcal{N} = (4,0)$ only known as free theories
  - interacting theories conjectured to describe strong coupling limits of maximal D=5 theories [Hull, 2000]
- selfduality equations (for 2nd order field strengths) [Hull] ⟶ no action principle

6D exotic supergravities: multiplets

$D = 6$

$\mathcal{N} = (2,2)$	$\mathcal{N} = (3,1)$	$\mathcal{N} = (4,0)$
$g_{\mu\nu}$ (1) A_μ (16) $B_{\mu\nu}$ (5) ϕ (25)	$C^{+}_{\mu\nu,\rho}$ (1) A_μ (14) $B^{+}_{\mu\nu}$ (12) ϕ (28)	$T^{+}_{\mu\nu,\rho\sigma}$ (1) $B^{+}_{\mu\nu}$ (27) ϕ (42)



- $\mathcal{N} = (3,1)$ and $\mathcal{N} = (4,0)$ only known as free theories
  - interacting theories conjectured to describe strong coupling limits of maximal D=5 theories [Hull, 2000]

- selfduality equations (for 2nd order field strengths) [Hull] $\longrightarrow$ no action principle
  until [Henneaux, Lekeu, Leonard, Matulich, Prohazka, 2018]
  free actions in prepotential formalism
  - 5+1 split
  - prepotentials for gauge fields, action $\left\{ \begin{array}{l} \text{linear in time derivatives} \\ 4^{\text{th}} \text{ order in spatial derivatives} \end{array} \right.$

6D exotic supergravities: multiplets

$D = 6$

$\mathcal{N} = (2,2)$	$\mathcal{N} = (3,1)$	$\mathcal{N} = (4,0)$
$g_{\mu\nu}$ (1) A_μ (16) $B_{\mu\nu}$ (5) ϕ (25)	$C^{+}_{\mu\nu,\rho}$ (1) A_μ (14) $B^{+}_{\mu\nu}$ (12) ϕ (28)	$T^{+}_{\mu\nu,\rho\sigma}$ (1) $B^{+}_{\mu\nu}$ (27) ϕ (42)

- $\mathcal{N} = (3,1)$ and $\mathcal{N} = (4,0)$ only known as free theories
  - interacting theories conjectured to describe strong coupling limits of maximal D=5 theories [Hull, 2000]

- selfduality equations (for 2nd order field strengths) [Hull] ⟶ no action principle
  until [Henneaux, Lekeu, Leonard, Matulich, Prohazka, 2018]
  free actions in prepotential formalism
  - 5+1 split
  - prepotentials for gauge fields, action { linear in time derivatives; 4th order in spatial derivatives

here: ⟶ two-derivative actions (ExFT inspired)

6D exotic supergravities: multiplets

$D = 6$

$\mathcal{N} = (2,2)$

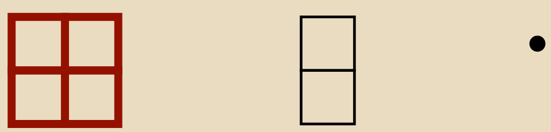
$g_{\mu\nu}$ (1) A_μ (16) $B_{\mu\nu}$ (5) ϕ (25)

$\mathcal{N} = (3,1)$

$C^{+}_{\mu\nu,\rho}$ (1) A_μ (14) $B^{+}_{\mu\nu}$ (12) ϕ (28)

$\mathcal{N} = (4,0)$

$T^{+}_{\mu\nu,\rho\sigma}$ (1) $B^{+}_{\mu\nu}$ (27) ϕ (42)



here: $\longrightarrow$ two-derivative actions (ExFT inspired)

- 5+1 split
- after reduction to 5D: standard action of (the free limit of) maximal supergravity
  - after KK-decomposition, redefinition, and dualization of fields
- keeping the 6th dimension: 'deformation' of the 5D action in ∂_6
- for supergravity the analogous construction yields ExFT
  - upon embedding the internal coordinates into a larger covariant structure

novel action functionals

$\longrightarrow$ selfdual two forms (review)

$\longrightarrow$ selfdual exotic tensors

review: actions for selfdual two forms

- $\mathcal{N} = (3,1)$ and $\mathcal{N} = (4,0)$ in particular carry 6D selfdual two-forms

$$H_{\hat\mu\hat\nu\hat\rho} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\kappa\hat\lambda}\,H^{\hat\sigma\hat\kappa\hat\lambda} \qquad\qquad H_{\hat\mu\hat\nu\hat\rho} = 3\,\partial_{[\hat\mu}B_{\hat\nu\hat\rho]}$$

- various mechanisms for constructing actions
  - 5+1 split [Henneaux, Teitelboim]
  - covariant, extra scalar [Pasti, Sorokin, Tonin]
  - ExFT style, 5+1 split, (dual to HT)
  - extra fields [Sen] [Mkrtchyan]

review: actions for selfdual two forms

- $\mathcal{N} = (3{,}1)$ and $\mathcal{N} = (4{,}0)$ in particular carry 6D selfdual two-forms

$$H_{\hat\mu\hat\nu\hat\rho} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\kappa\hat\lambda}\,H^{\hat\sigma\hat\kappa\hat\lambda} \qquad\qquad H_{\hat\mu\hat\nu\hat\rho} = 3\,\partial_{[\hat\mu}B_{\hat\nu\hat\rho]}$$

- various mechanisms for constructing actions
  - 5+1 split [Henneaux, Teitelboim]
  - covariant, extra scalar [Pasti, Sorokin, Tonin]
  - ExFT style, 5+1 split, (dual to HT)
  - extra fields [Sen] [Mkrtchyan]

- 5+1 split $\{x^{\hat\mu}\} \longrightarrow \{x^\mu, y\}$ but no reduction

$$\{B_{\hat\mu\hat\nu}\} = \{B_{\mu\nu}, B_{\mu 6} \equiv A_\mu\}$$

  - selfduality ⟶ duality equation (vector/tensor)

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + \partial_6 B_{\mu\nu}$$

$$F_{\mu\nu} = 2\,\partial_{[\mu}A_{\nu]}$$

$$H_{\mu\nu\rho} = 3\,\partial_{[\mu}B_{\nu\rho]}$$

$$\boxed{\mathcal{F}_{\mu\nu} + \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\,H^{\rho\sigma\tau} = 0}$$

- action principle for this equation

review: actions for selfdual two forms

$$\mathcal{F}_{\mu\nu} + \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau} = 0$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + \partial_6 B_{\mu\nu}$$

$$\Longrightarrow \quad \partial^\mu \mathcal{F}_{\mu\nu} = 0 \qquad \text{(divergence)}$$

$$\partial_\lambda H^{\lambda\mu\nu} = \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda\sigma\tau}\partial_6 H_{\lambda\sigma\tau} \qquad \text{(curl)}$$

review: actions for selfdual two forms

$$\mathcal{F}_{\mu\nu} + \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau} = 0$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + \partial_6 B_{\mu\nu}$$

$$\Longrightarrow \quad \partial^\mu \mathcal{F}_{\mu\nu} = 0 \qquad \text{(divergence)}$$

$$\partial_\lambda H^{\lambda\mu\nu} = \tfrac{1}{6}\, \varepsilon^{\mu\nu\lambda\sigma\tau} \partial_6 H_{\lambda\sigma\tau} \qquad \text{(curl)}$$

- Henneaux-Teitelboim action [1988]

$$\mathcal{L} = -\frac{1}{12}\, H_{\mu\nu\rho}\, H^{\mu\nu\rho} + \frac{1}{24}\, \varepsilon^{\mu\nu\rho\sigma\tau}\, \partial_6 B_{\mu\nu}\, H_{\rho\sigma\tau}$$

  - does not feature the vector field

review: actions for selfdual two forms

$$\mathcal{F}_{\mu\nu} + \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau} = 0$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + \partial_6 B_{\mu\nu}$$

$$\Longrightarrow \quad \partial^\mu \mathcal{F}_{\mu\nu} = 0 \qquad \text{(divergence)}$$

$$\partial_\lambda H^{\lambda\mu\nu} = \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda\sigma\tau}\partial_6 H_{\lambda\sigma\tau} \qquad \text{(curl)}$$

- Henneaux-Teitelboim action [1988]

$$\mathcal{L} = -\frac{1}{12}\, H_{\mu\nu\rho}\, H^{\mu\nu\rho} + \frac{1}{24}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\, H_{\rho\sigma\tau}$$

  - does not feature the vector field

e.o.m.: $\partial_\lambda H^{\lambda\mu\nu} = \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda\sigma\tau}\partial_6 H_{\lambda\sigma\tau}$

review: actions for selfdual two forms

$$\mathcal{F}_{\mu\nu} + \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau} = 0$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + \partial_6 B_{\mu\nu}$$

$$\Longrightarrow \quad \partial^\mu \mathcal{F}_{\mu\nu} = 0 \qquad \text{(divergence)}$$

$$\partial_\lambda H^{\lambda\mu\nu} = \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda\sigma\tau}\partial_6 H_{\lambda\sigma\tau} \qquad \text{(curl)}$$

- Henneaux-Teitelboim action [1988]

$$\mathcal{L} = -\frac{1}{12}\, H_{\mu\nu\rho}\, H^{\mu\nu\rho} + \frac{1}{24}\,\varepsilon^{\mu\nu\rho\sigma\tau}\, \partial_6 B_{\mu\nu}\, H_{\rho\sigma\tau}$$

  - does not feature the vector field

e.o.m.: $\partial_\lambda H^{\lambda\mu\nu} = \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda\sigma\tau}\partial_6 H_{\lambda\sigma\tau}$

  - integrability relation for defining the vector field via

$$2\,\partial_{[\mu} A_{\nu]} = -\partial_6 B_{\mu\nu} - \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau}$$

review: actions for selfdual two forms

$$\mathcal{F}_{\mu\nu} + \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau} = 0$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + \partial_6 B_{\mu\nu}$$

$$\Longrightarrow \quad \partial^\mu \mathcal{F}_{\mu\nu} = 0 \qquad \text{(divergence)}$$

$$\partial_\lambda H^{\lambda\mu\nu} = \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda\sigma\tau}\partial_6 H_{\lambda\sigma\tau} \qquad \text{(curl)}$$

- ExFT type action

$$\mathcal{L} = -\frac{1}{4}\,\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{24}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\, H_{\rho\sigma\tau}$$

review: actions for selfdual two forms

$$\mathcal{F}_{\mu\nu} + \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau} = 0$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + \partial_6 B_{\mu\nu}$$

$$\Longrightarrow \quad \partial^\mu \mathcal{F}_{\mu\nu} = 0 \qquad \text{(divergence)}$$

$$\partial_\lambda H^{\lambda\mu\nu} = \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda\sigma\tau}\partial_6 H_{\lambda\sigma\tau} \qquad \text{(curl)}$$

- ExFT type action

$$\mathcal{L} = -\frac{1}{4}\,\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{24}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\, H_{\rho\sigma\tau}$$

e.o.m.: $\partial^\mu \mathcal{F}_{\mu\nu} = 0$

$$0 = \partial_6\Big(\mathcal{F}_{\mu\nu} + \tfrac{1}{6}\,\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau}\Big)$$

review: actions for selfdual two forms

$$\mathcal{F}_{\mu\nu} + \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau} = 0$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + \partial_6 B_{\mu\nu}$$

$$\Longrightarrow \quad \partial^\mu \mathcal{F}_{\mu\nu} = 0 \qquad \text{(divergence)}$$

$$\partial_\lambda H^{\lambda\mu\nu} = \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda\sigma\tau}\partial_6 H_{\lambda\sigma\tau} \qquad \text{(curl)}$$

- ExFT type action

$$\mathcal{L} = -\frac{1}{4}\,\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{24}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\, H_{\rho\sigma\tau}$$

e.o.m.: $\partial^\mu \mathcal{F}_{\mu\nu} = 0$

$$0 = \partial_6\Big(\mathcal{F}_{\mu\nu} + \tfrac{1}{6}\,\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau}\Big)$$

$$\Longrightarrow \qquad \mathcal{F}_{\mu\nu} + \tfrac{1}{6}\,\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau} = \chi_{\mu\nu}(x) = \tfrac{1}{2}\,\varepsilon_{\mu\nu\rho\sigma\tau}\,\partial^\rho b^{\sigma\tau}$$

- yields the original duality equation upon redefining $B_{\mu\nu} \longrightarrow B_{\mu\nu} - b_{\mu\nu}$
- fixing a local gauge symmetry of the ExFT action

review: actions for selfdual two forms

$$\mathcal{F}_{\mu\nu} + \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma\tau}\, H^{\rho\sigma\tau} = 0$$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} + \partial_6 B_{\mu\nu}$$

$$\Longrightarrow \quad \partial^\mu \mathcal{F}_{\mu\nu} = 0 \qquad \text{(divergence)}$$

$$\partial_\lambda H^{\lambda\mu\nu} = \tfrac{1}{6}\,\varepsilon^{\mu\nu\lambda\sigma\tau}\partial_6 H_{\lambda\sigma\tau} \qquad \text{(curl)}$$

- Henneaux-Teitelboim action

$$\mathcal{L} = -\frac{1}{12}\, H_{\mu\nu\rho}\, H^{\mu\nu\rho} + \frac{1}{24}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\, H_{\rho\sigma\tau}$$

  - 'deformation' of the free tensor field Lagrangian

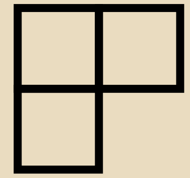
- ExFT type action

$$\mathcal{L} = -\frac{1}{4}\,\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{24}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\, H_{\rho\sigma\tau}$$

  - 'deformation' of the free vector field Lagrangian

⟶ generalize this to selfdual exotic tensor fields

actions for selfdual exotic tensors: the $\mathcal{N} = (3,1)$ model

- exotic tensor field $C_{\hat\mu\hat\nu,\hat\rho}$ 

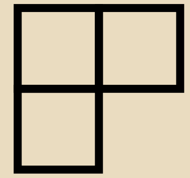
$$S_{\hat\mu\hat\nu\hat\rho,\hat\sigma\hat\tau} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\rho\hat\eta\hat\kappa\hat\lambda}\,S^{\hat\eta\hat\kappa\hat\lambda}{}_{\hat\sigma\hat\tau}$$

  - gauge invariant second order curvature $S_{\hat\mu\hat\nu\hat\rho,\hat\sigma\hat\tau} = 3\,\partial_{\hat\sigma}\partial_{[\hat\mu}C_{\hat\nu\hat\rho],\hat\tau} - 3\,\partial_{\hat\tau}\partial_{[\hat\mu}C_{\hat\nu\hat\rho],\hat\sigma}$

  - selfduality equation in 5+1 split $\left\{C_{\hat\mu\hat\nu,\hat\rho}\right\} = \left\{C_{\mu\nu,\rho} - 2\,A_{[\mu}\eta_{\nu]\rho}\,;\; C_{\mu6,\nu} = h_{\mu\nu} + B_{\mu\nu}\,;\; C_{\mu6,6} = 2\,A_\mu\right\}$

    no reduction!

actions for selfdual exotic tensors: the $\mathcal{N}=(3,1)$ model

- exotic tensor field $C_{\hat\mu\hat\nu,\hat\rho}$ 

$$S_{\hat\mu\hat\nu\hat\rho,\hat\sigma\hat\tau} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\rho\hat\eta\hat\kappa\hat\lambda}\,S^{\hat\eta\hat\kappa\hat\lambda}{}_{\hat\sigma\hat\tau}$$

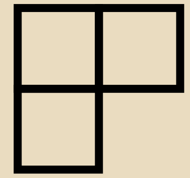
  - gauge invariant second order curvature $\quad S_{\hat\mu\hat\nu\hat\rho,\hat\sigma\hat\tau} = 3\,\partial_{\hat\sigma}\partial_{[\hat\mu}C_{\hat\nu\hat\rho],\hat\tau} - 3\,\partial_{\hat\tau}\partial_{[\hat\mu}C_{\hat\nu\hat\rho],\hat\sigma}$
  - selfduality equation in 5+1 split $\quad \left\{C_{\hat\mu\hat\nu,\hat\rho}\right\} = \left\{C_{\mu\nu,\rho} - 2\,A_{[\mu}\eta_{\nu]\rho}\,;\; C_{\mu6,\nu} = h_{\mu\nu} + B_{\mu\nu}\,;\; C_{\mu6,6} = 2\,A_\mu\right\}$

    no reduction!

- $$\underbrace{\partial_\rho\left(F_{\mu\nu} + \frac{1}{6}\,\varepsilon_{\mu\nu\lambda\sigma\tau}\,H^{\lambda\sigma\tau}\right)}_{\text{vector/tensor duality}} = \partial_6\partial_{[\mu}h_{\nu]\rho} + \tfrac14\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial_6\partial^\kappa C^{\lambda\tau}{}_\rho + \tfrac12\,\partial_6\partial_6 C_{\mu\nu,\rho} - \partial_6\partial_6 A_{[\mu}\eta_{\nu]\rho} - \tfrac14\,\varepsilon_{\rho\mu\nu\sigma\tau}\,\partial_6 F^{\sigma\tau} + \partial_6\partial_{[\mu}B_{\nu]\rho} - \partial_6\partial_\rho B_{\mu\nu}$$

- $$\underbrace{R_{\mu\nu,\rho\sigma} - \tfrac12\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial^\kappa\partial_{[\rho}C^{\lambda\tau}{}_{\sigma]}}_{\text{spin-2 duality}} = \tfrac12\,\partial_\rho\left(H_{\mu\nu\sigma} - \tfrac12\,\varepsilon_{\mu\nu\sigma\kappa\lambda}\,F^{\kappa\lambda}\right) - \tfrac12\,\partial_\sigma\left(H_{\mu\nu\rho} - \tfrac12\,\varepsilon_{\mu\nu\rho\kappa\lambda}\,F^{\kappa\lambda}\right) + \tfrac12\,\partial_6\partial_\rho C_{\mu\nu,\sigma} - \tfrac12\,\partial_6\partial_\sigma C_{\mu\nu,\rho} - \partial_6\partial_\rho A_{[\mu}\eta_{\nu]\sigma} + \partial_6\partial_\sigma A_{[\mu}\eta_{\nu]\rho}$$

actions for selfdual exotic tensors: the $\mathcal{N} = (3,1)$ model

- exotic tensor field $C_{\hat\mu\hat\nu,\hat\rho}$ 

$$S_{\hat\mu\hat\nu\hat\rho,\hat\sigma\hat\tau} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\rho\hat\eta\hat\kappa\hat\lambda}\, S^{\hat\eta\hat\kappa\hat\lambda}{}_{\hat\sigma\hat\tau}$$

- $$\partial_\rho\Big(F_{\mu\nu} + \frac{1}{6}\,\varepsilon_{\mu\nu\lambda\sigma\tau}\,H^{\lambda\sigma\tau}\Big) = \partial_6\partial_{[\mu}h_{\nu]\rho} + \tfrac{1}{4}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial_6\partial^\kappa C^{\lambda\tau}{}_\rho + \tfrac{1}{2}\,\partial_6\partial_6 C_{\mu\nu,\rho} - \partial_6\partial_6 A_{[\mu}\eta_{\nu]\rho} - \tfrac{1}{4}\,\varepsilon_{\rho\mu\nu\sigma\tau}\,\partial_6 F^{\sigma\tau} + \partial_6\partial_{[\mu}B_{\nu]\rho} - \partial_6\partial_\rho B_{\mu\nu}$$

- $$R_{\mu\nu,\rho\sigma} - \tfrac{1}{2}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial^\kappa\partial_{[\rho}C^{\lambda\tau}{}_{\sigma]} = \tfrac{1}{2}\,\partial_\rho\Big(H_{\mu\nu\sigma} - \tfrac{1}{2}\,\varepsilon_{\mu\nu\sigma\kappa\lambda}\,F^{\kappa\lambda}\Big) - \tfrac{1}{2}\,\partial_\sigma\Big(H_{\mu\nu\rho} - \tfrac{1}{2}\,\varepsilon_{\mu\nu\rho\kappa\lambda}\,F^{\kappa\lambda}\Big) + \tfrac{1}{2}\,\partial_6\partial_\rho C_{\mu\nu,\sigma} - \tfrac{1}{2}\,\partial_6\partial_\sigma C_{\mu\nu,\rho} - \partial_6\partial_\rho A_{[\mu}\eta_{\nu]\sigma} + \partial_6\partial_\sigma A_{[\mu}\eta_{\nu]\rho}$$

> integrate to first-order duality equations

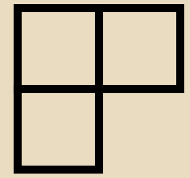
- $$\partial_{[\mu}h_{\nu]\rho} + \tfrac{1}{4}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial^\kappa C^{\lambda\tau}{}_\rho - \partial_\rho u_{\mu\nu} = \tfrac{1}{4}\,\varepsilon_{\mu\nu\rho\kappa\lambda}\,\partial_6\Big(u^{\kappa\lambda} - \tfrac{3}{2}\,B^{\kappa\lambda}\Big) - \tfrac{1}{2}\,\partial_6 C_{\mu\nu,\rho} + \partial_6 A_{[\mu}\eta_{\nu]\rho}$$

- $$F_{\mu\nu} + \tfrac{1}{6}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,H^{\kappa\lambda\tau} = \partial_6 u_{\mu\nu} - \tfrac{3}{2}\,\partial_6 B_{\mu\nu}$$

up to an antisymmetric tensor $u_{\mu\nu}$

- construct an action as a '∂_6-deformation' of the standard 5D action

actions for selfdual exotic tensors: the $\mathcal{N} = (3,1)$ model

- exotic tensor field $C_{\hat\mu\hat\nu,\hat\rho}$ 

$$S_{\hat\mu\hat\nu\hat\rho,\hat\sigma\hat\tau} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\rho\hat\eta\hat\kappa\hat\lambda}\,S^{\hat\eta\hat\kappa\hat\lambda}{}_{\hat\sigma\hat\tau}$$

- $$\partial_\rho\Big(F_{\mu\nu} + \frac{1}{6}\,\varepsilon_{\mu\nu\lambda\sigma\tau}\,H^{\lambda\sigma\tau}\Big) = \partial_6\partial_{[\mu}h_{\nu]\rho} + \tfrac14\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial_6\partial^\kappa C^{\lambda\tau}{}_\rho + \tfrac12\,\partial_6\partial_6 C_{\mu\nu,\rho} - \partial_6\partial_6 A_{[\mu}\eta_{\nu]\rho}$$
$$-\tfrac14\,\varepsilon_{\rho\mu\nu\sigma\tau}\,\partial_6 F^{\sigma\tau} + \partial_6\partial_{[\mu}B_{\nu]\rho} - \partial_6\partial_\rho B_{\mu\nu}$$

- $$R_{\mu\nu,\rho\sigma} - \tfrac12\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial^\kappa\partial_{[\rho}C^{\lambda\tau}{}_{\sigma]} = \tfrac12\,\partial_\rho\Big(H_{\mu\nu\sigma} - \tfrac12\,\varepsilon_{\mu\nu\sigma\kappa\lambda}\,F^{\kappa\lambda}\Big) - \tfrac12\,\partial_\sigma\Big(H_{\mu\nu\rho} - \tfrac12\,\varepsilon_{\mu\nu\rho\kappa\lambda}\,F^{\kappa\lambda}\Big)$$
$$+\tfrac12\,\partial_6\partial_\rho C_{\mu\nu,\sigma} - \tfrac12\,\partial_6\partial_\sigma C_{\mu\nu,\rho} - \partial_6\partial_\rho A_{[\mu}\eta_{\nu]\sigma} + \partial_6\partial_\sigma A_{[\mu}\eta_{\nu]\rho}$$

> integrate to first-order duality equations

- $$\partial_{[\mu}h_{\nu]\rho} + \tfrac14\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial^\kappa C^{\lambda\tau}{}_\rho - \partial_\rho u_{\mu\nu} = \tfrac14\,\varepsilon_{\mu\nu\rho\kappa\lambda}\,\partial_6\Big(u^{\kappa\lambda} - \tfrac32\,B^{\kappa\lambda}\Big) - \tfrac12\,\partial_6 C_{\mu\nu,\rho} + \partial_6 A_{[\mu}\eta_{\nu]\rho}$$

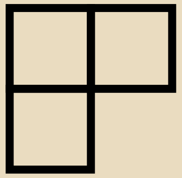
- $$F_{\mu\nu} + \tfrac16\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,H^{\kappa\lambda\tau} = \partial_6 u_{\mu\nu} - \tfrac32\,\partial_6 B_{\mu\nu}$$

up to an antisymmetric tensor $u_{\mu\nu}$

- construct an action as a '∂_6-deformation' of the standard 5D action

actions for selfdual exotic tensors: the $\mathcal{N} = (3,1)$ model

- exotic tensor field $C_{\hat\mu\hat\nu,\hat\rho}$



- $$\partial_{[\mu}h_{\nu]\rho} + \tfrac{1}{4}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial^\kappa C^{\lambda\tau}{}_\rho - \partial_\rho u_{\mu\nu} \;=\; \tfrac{1}{4}\,\varepsilon_{\mu\nu\rho\kappa\lambda}\,\partial_6\Big(u^{\kappa\lambda} - \tfrac{3}{2}\,B^{\kappa\lambda}\Big) - \tfrac{1}{2}\,\partial_6 C_{\mu\nu,\rho} + \partial_6 A_{[\mu}\eta_{\nu]\rho}$$
- $$F_{\mu\nu} + \tfrac{1}{6}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,H^{\kappa\lambda\tau} \;=\; \partial_6 u_{\mu\nu} - \tfrac{3}{2}\,\partial_6 B_{\mu\nu}$$

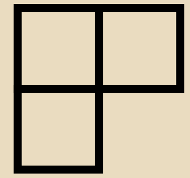
- construct an action as a '∂_6-deformation' of the standard 5D action

$$\mathcal{L} \;=\; \underbrace{-\tfrac{1}{4}\,\Omega^{\mu\nu\rho}\Omega_{\mu\nu\rho} + \tfrac{1}{2}\,\Omega^{\mu\nu\rho}\Omega_{\nu\rho\mu} + \Omega^\mu\Omega_\mu}_{\text{Pauli-Fierz}} - \underbrace{\tfrac{3}{4}\,F^{\mu\nu}F_{\mu\nu}}_{\text{Maxwell}}$$

$$\Omega_{\mu\nu\rho} = \partial_{[\mu}h_{\nu]\rho}$$
$$\Omega_\mu = \Omega_{\mu\nu}{}^\nu$$
$$F_{\mu\nu} = 2\,\partial_{[\mu}A_{\nu]}$$

actions for selfdual exotic tensors: the $\mathcal{N}=(3,1)$ model

- exotic tensor field $C_{\hat{\mu}\hat{\nu},\hat{\rho}}$



- $$\partial_{[\mu}h_{\nu]\rho} + \tfrac{1}{4}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial^\kappa C^{\lambda\tau}{}_\rho - \partial_\rho u_{\mu\nu} \;=\; \tfrac{1}{4}\,\varepsilon_{\mu\nu\rho\kappa\lambda}\,\partial_6\Big(u^{\kappa\lambda} - \tfrac{3}{2}\,B^{\kappa\lambda}\Big) - \tfrac{1}{2}\,\partial_6 C_{\mu\nu,\rho} + \partial_6 A_{[\mu}\eta_{\nu]\rho}$$
- $$F_{\mu\nu} + \tfrac{1}{6}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,H^{\kappa\lambda\tau} \;=\; \partial_6 u_{\mu\nu} - \tfrac{3}{2}\,\partial_6 B_{\mu\nu}$$

- construct an action as a '∂_6-deformation' of the standard 5D action

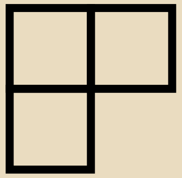
$$\mathcal{L} \;=\; -\tfrac{1}{4}\,\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\mu\nu\rho} + \tfrac{1}{2}\,\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\nu\rho\mu} + \widehat{\Omega}^{\mu}\widehat{\Omega}_{\mu} - \tfrac{3}{4}\,\widehat{F}^{\mu\nu}\widehat{F}_{\mu\nu}$$

$$\widehat{\Omega}_{\mu\nu\rho} = \partial_{[\mu}h_{\nu]\rho} - \partial_6 A_{[\mu}\eta_{\nu]\rho} + \tfrac{1}{2}\,\partial_6 C_{\mu\nu,\rho} + \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma\tau}\,\partial_6 u^{\sigma\tau}$$

$$\Omega_\mu = \Omega_{\mu\nu}{}^{\nu}$$

$$\widehat{F}_{\mu\nu} = 2\,\partial_{[\mu}A_{\nu]} + \tfrac{3}{2}\,\partial_6 B_{\mu\nu}$$

actions for selfdual exotic tensors: the $\mathcal{N} = (3,1)$ model

- exotic tensor field $C_{\hat\mu\hat\nu,\hat\rho}$ 

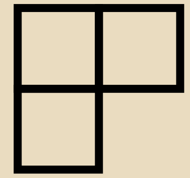
- $$\partial_{[\mu}h_{\nu]\rho} + \tfrac{1}{4}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial^\kappa C^{\lambda\tau}{}_\rho - \partial_\rho u_{\mu\nu} \;=\; \tfrac{1}{4}\,\varepsilon_{\mu\nu\rho\kappa\lambda}\,\partial_6\Big(u^{\kappa\lambda} - \tfrac{3}{2}\,B^{\kappa\lambda}\Big) - \tfrac{1}{2}\,\partial_6 C_{\mu\nu,\rho} + \partial_6 A_{[\mu}\eta_{\nu]\rho}$$
- $$F_{\mu\nu} + \tfrac{1}{6}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,H^{\kappa\lambda\tau} \;=\; \partial_6 u_{\mu\nu} - \tfrac{3}{2}\,\partial_6 B_{\mu\nu}$$

- construct an action as a '∂_6-deformation' of the standard 5D action

$$\mathcal{L}_{\boxplus} = -\tfrac{1}{4}\,\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\mu\nu\rho} + \tfrac{1}{2}\,\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\nu\rho\mu} + \widehat\Omega^{\mu}\widehat\Omega_{\mu} - \tfrac{3}{4}\,\widehat F^{\mu\nu}\widehat F_{\mu\nu}$$
$$-\tfrac{1}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6\widehat C_{\mu\nu,}{}^{\lambda}\partial_\rho\widehat C_{\sigma\tau,\lambda} - \tfrac{9}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\partial_\rho B_{\sigma\tau} - \tfrac{3}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\,\partial_6\widehat C_{\rho\sigma,\tau}$$

$$\widehat\Omega_{\mu\nu\rho} = \partial_{[\mu}h_{\nu]\rho} - \partial_6 A_{[\mu}\eta_{\nu]\rho} + \tfrac{1}{2}\,\partial_6 C_{\mu\nu,\rho} + \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma\tau}\,\partial_6 u^{\sigma\tau}$$
$$\Omega_\mu = \Omega_{\mu\nu}{}^{\nu}$$
$$\widehat F_{\mu\nu} = 2\,\partial_{[\mu}A_{\nu]} + \tfrac{3}{2}\,\partial_6 B_{\mu\nu}$$
$$\widehat C_{\mu\nu,\rho} = C_{\mu\nu,\rho} + \varepsilon_{\mu\nu\rho\sigma\tau}\,u^{\sigma\tau}$$

actions for selfdual exotic tensors: the $\mathcal{N} = (3,1)$ model

- exotic tensor field $C_{\hat\mu\hat\nu,\hat\rho}$ 

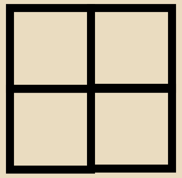
- $\partial_{[\mu}h_{\nu]\rho} + \frac{1}{4}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,\partial^\kappa C^{\lambda\tau}{}_\rho - \partial_\rho u_{\mu\nu} \;=\; \frac{1}{4}\,\varepsilon_{\mu\nu\rho\kappa\lambda}\,\partial_6\left(u^{\kappa\lambda} - \frac{3}{2}\,B^{\kappa\lambda}\right) - \frac{1}{2}\,\partial_6 C_{\mu\nu,\rho} + \partial_6 A_{[\mu}\eta_{\nu]\rho}$
- $F_{\mu\nu} + \frac{1}{6}\,\varepsilon_{\mu\nu\kappa\lambda\tau}\,H^{\kappa\lambda\tau} \;=\; \partial_6 u_{\mu\nu} - \frac{3}{2}\,\partial_6 B_{\mu\nu}$

- construct an action as a '∂_6-deformation' of the standard 5D action

$$
\begin{aligned}
\mathcal{L}_{\boxplus} = {} & -\tfrac{1}{4}\,\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\mu\nu\rho} + \tfrac{1}{2}\,\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\nu\rho\mu} + \widehat\Omega^{\mu}\widehat\Omega_{\mu} - \tfrac{3}{4}\,\widehat F^{\mu\nu}\widehat F_{\mu\nu} \\
& -\tfrac{1}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6\widehat C_{\mu\nu,}{}^{\lambda}\partial_\rho\widehat C_{\sigma\tau,\lambda} - \tfrac{9}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\partial_\rho B_{\sigma\tau} - \tfrac{3}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\,\partial_6\widehat C_{\rho\sigma,\tau}
\end{aligned}
$$

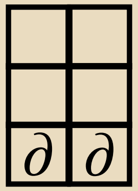
  - variation w.r.t. $h_{\mu\nu}, A_\mu$ yields (deformed) Pauli-Fierz and Maxwell equations
  - variation w.r.t. $C_{\mu\nu,\rho}, B_{\mu\nu}, u_{\mu\nu}$ yields first-order duality equations (under ∂_6)
  - (locally) equivalent reformulation of the 6D selfduality equations

actions for selfdual exotic tensors: the $\mathcal{N} = (4{,}0)$ model

- exotic tensor field $T_{\hat\mu\hat\nu,\hat\rho\hat\sigma}$  [Hull, 2000]

  - gauge invariant second order curvature

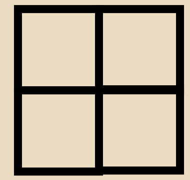
$$G_{\hat\mu\hat\nu\hat\lambda,\hat\rho\hat\sigma\hat\tau} = 3\,\partial_{\hat\rho}\partial_{[\hat\mu}T_{\hat\nu\hat\lambda],\hat\sigma\hat\tau} + 3\,\partial_{\hat\sigma}\partial_{[\hat\mu}T_{\hat\nu\hat\lambda],\hat\tau\hat\rho} + 3\,\partial_{\hat\tau}\partial_{[\hat\mu}T_{\hat\nu\hat\lambda],\hat\rho\hat\sigma}$$



  - selfduality equation (2nd order)

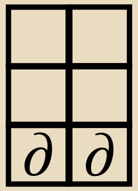
$$G_{\hat\mu\hat\nu\hat\lambda,\hat\rho\hat\sigma\hat\tau} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\lambda\hat\alpha\hat\beta\hat\gamma}\,G^{\hat\alpha\hat\beta\hat\gamma}{}_{\hat\rho\hat\sigma\hat\tau}$$

actions for selfdual exotic tensors: the $\mathcal{N} = (4{,}0)$ model

- exotic tensor field $T_{\hat\mu\hat\nu,\hat\rho\hat\sigma}$  [Hull, 2000]

  - gauge invariant second order curvature

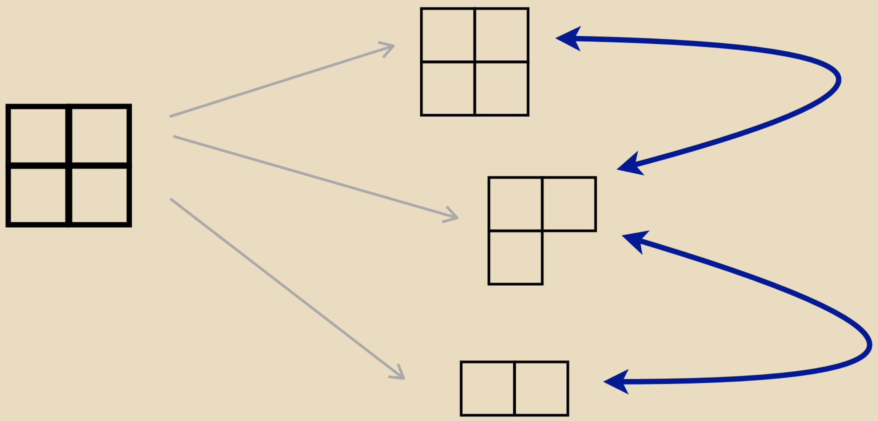
$$G_{\hat\mu\hat\nu\hat\lambda,\hat\rho\hat\sigma\hat\tau} = 3\,\partial_{\hat\rho}\partial_{[\hat\mu}T_{\hat\nu\hat\lambda],\hat\sigma\hat\tau} + 3\,\partial_{\hat\sigma}\partial_{[\hat\mu}T_{\hat\nu\hat\lambda],\hat\tau\hat\rho} + 3\,\partial_{\hat\tau}\partial_{[\hat\mu}T_{\hat\nu\hat\lambda],\hat\rho\hat\sigma}$$



  - selfduality equation (2nd order)

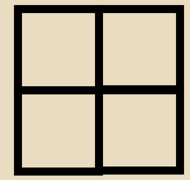
$$G_{\hat\mu\hat\nu\hat\lambda,\hat\rho\hat\sigma\hat\tau} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\lambda\hat\alpha\hat\beta\hat\gamma}\,G^{\hat\alpha\hat\beta\hat\gamma}{}_{\hat\rho\hat\sigma\hat\tau}$$

  - Kaluza-Klein 5+1 split $\quad \{T_{\hat\mu\hat\nu,\hat\rho\hat\sigma}\} \;=\; \{T_{\mu\nu,\rho\sigma}\,;\, T_{\mu\nu,\rho 6} = C_{\mu\nu,\rho}\,;\, T_{\mu 6,\nu 6} = h_{\mu\nu}\}$



actions for selfdual exotic tensors: the $\mathcal{N} = (4,0)$ model

- exotic tensor field $T_{\hat\mu\hat\nu,\hat\rho\hat\sigma}$



$$G_{\hat\mu\hat\nu\hat\lambda,\hat\rho\hat\sigma\hat\tau} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\lambda\hat\alpha\hat\beta\hat\gamma}\,G^{\hat\alpha\hat\beta\hat\gamma}{}_{\hat\rho\hat\sigma\hat\tau}$$

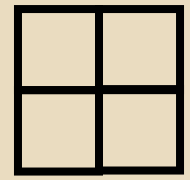
> integrate to first-order duality equations

- $\partial_{[\mu}h_{\nu]\rho} + \frac{1}{4}\,\varepsilon_{\mu\nu\lambda\sigma\tau}\,\partial^\lambda C^{\sigma\tau}{}_\rho - \partial_\rho u_{\mu\nu} \;=\; -\,\frac{1}{2}\,\partial_6 C_{\mu\nu,\rho} - \frac{1}{2}\,\partial_6 v_{\rho,\mu\nu}$
- $\frac{1}{2}\,\varepsilon_{\mu\nu\alpha\beta\gamma}\,\partial^\alpha T_{\sigma\tau}{}^{\beta\gamma} + \partial_\mu C_{\sigma\tau,\nu} - \partial_\nu C_{\sigma\tau,\mu} - 2\,\partial_{[\sigma}v_{\tau],\mu\nu} \;=\; -\,\partial_6 T_{\sigma\tau,\mu\nu}$

up to tensors $u_{\mu\nu}, v_{\rho\mu\nu}$

actions for selfdual exotic tensors: the $\mathcal{N} = (4{,}0)$ model

- exotic tensor field $T_{\hat\mu\hat\nu,\hat\rho\hat\sigma}$



$$G_{\hat\mu\hat\nu\hat\lambda,\hat\rho\hat\sigma\hat\tau} = \frac{1}{6}\,\varepsilon_{\hat\mu\hat\nu\hat\lambda\hat\alpha\hat\beta\hat\gamma}\,G^{\hat\alpha\hat\beta\hat\gamma}{}_{\hat\rho\hat\sigma\hat\tau}$$

> integrate to first-order duality equations

- $\partial_{[\mu}h_{\nu]\rho} + \frac14\,\varepsilon_{\mu\nu\lambda\sigma\tau}\,\partial^\lambda C^{\sigma\tau}{}_\rho - \partial_\rho u_{\mu\nu} \;=\; -\frac12\,\partial_6 C_{\mu\nu,\rho} - \frac12\,\partial_6 v_{\rho,\mu\nu}$
- $\frac12\,\varepsilon_{\mu\nu\alpha\beta\gamma}\,\partial^\alpha T_{\sigma\tau}{}^{\beta\gamma} + \partial_\mu C_{\sigma\tau,\nu} - \partial_\nu C_{\sigma\tau,\mu} - 2\,\partial_{[\sigma}v_{\tau],\mu\nu} \;=\; -\,\partial_6 T_{\sigma\tau,\mu\nu}$

up to tensors $u_{\mu\nu}, v_{\rho\mu\nu}$

- construct an action as a '∂_6-deformation' of the standard 5D action

$$\mathcal{L}_{\boxplus} = -\tfrac14\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\mu\nu\rho} + \tfrac12\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\nu\rho\mu} + \widehat\Omega^{\mu}\widehat\Omega_{\mu} - \tfrac18\varepsilon_{\mu\nu\sigma\kappa\lambda}\,\partial^\mu\widehat C^{\nu\sigma}{}_\rho\,\partial_6\widehat C^{\kappa\lambda,\rho} + \tfrac1{32}\varepsilon_{\mu\nu\sigma\kappa\lambda}\,\partial^\mu\mathcal{T}^{\nu\sigma}{}_\rho\,\partial_6\mathcal{T}^{\kappa\lambda,\rho}$$
$$-\tfrac18\partial_6\mathcal{T}_{\sigma\tau,\nu}\,\partial_\mu T^{\mu\nu,\sigma\tau} + \tfrac14\partial_6\mathcal{T}_{\kappa\lambda,\tau}\,\partial^\kappa T^{\lambda\sigma,\tau}{}_\sigma + \tfrac14\partial_\nu\mathcal{T}_{\sigma\mu}{}^{\mu}\,\partial_6 T^{\sigma\tau,\nu}{}_\tau - \tfrac18\partial_6\mathcal{T}_{\sigma\mu}{}^{\mu}\,\partial^\sigma T_{\tau\nu}{}^{\tau\nu}$$
$$-\tfrac1{64}\varepsilon_{\mu\nu\alpha\beta\gamma}\,\partial^\alpha T_{\sigma\tau}{}^{\beta\gamma}\,\partial_6 T^{\mu\nu,\sigma\tau} - \tfrac1{32}\partial_6 T_{\sigma\tau,\mu\nu}\,\partial_6 T^{\mu\nu,\sigma\tau} + \tfrac18\partial_6 T_{\sigma\mu,\nu}{}^{\mu}\,\partial_6 T^{\sigma\tau,\nu}{}_\tau - \tfrac1{32}\partial_6 T_{\mu\nu}{}^{\mu\nu}\,\partial_6 T_{\sigma\tau}{}^{\sigma\tau}$$

$$\widehat\Omega_{\mu\nu\rho} = \partial_{[\mu}h_{\nu]\rho} + \partial_6\widehat C_{\mu\nu,\rho} - \tfrac12\partial_6\mathcal{T}_{\mu\nu,\rho}$$

$$\widehat C_{\mu\nu,\rho} = C_{\mu\nu,\rho} + \varepsilon_{\mu\nu\rho\sigma\tau}\,u^{\sigma\tau}$$

$$\mathcal{T}_{\mu\nu,\rho} = C_{\mu\nu,\rho} - v_{\rho,\mu\nu} + 2\,v_{[\rho,\mu\nu]} + 2\,\varepsilon_{\mu\nu\rho\sigma\tau}\,u^{\sigma\tau}$$

actions for exotic supergravities

$\mathcal{N} = (2,2)$

$g_{\mu\nu}$ (1) A_μ (16) $B_{\mu\nu}$ (5) ϕ (25)

$\mathcal{N} = (3,1)$

$C^{+}_{\mu\nu,\rho}$ (1) A_μ (14) $B^{+}_{\mu\nu}$ (12) ϕ (28)

$\mathcal{N} = (4,0)$

$T^{+}_{\mu\nu,\rho\sigma}$ (1) $B^{+}_{\mu\nu}$ (27) ϕ (42)

$$\mathcal{L}_{\boxplus} = -\tfrac{1}{4}\,\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\mu\nu\rho} + \tfrac{1}{2}\,\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\nu\rho\mu} + \widehat{\Omega}^{\mu}\widehat{\Omega}_{\mu} - \tfrac{3}{4}\,\widehat{F}^{\mu\nu}\widehat{F}_{\mu\nu}$$
$$-\tfrac{1}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\partial_6\widehat{C}_{\mu\nu,}{}^{\lambda}\partial_\rho\widehat{C}_{\sigma\tau,\lambda} - \tfrac{9}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\partial_6 B_{\mu\nu}\partial_\rho B_{\sigma\tau} - \tfrac{3}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\,\partial_6\widehat{C}_{\rho\sigma,\tau}$$

$$\mathcal{L}_{\boxplus} = -\tfrac{1}{4}\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\mu\nu\rho} + \tfrac{1}{2}\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\nu\rho\mu} + \widehat{\Omega}^{\mu}\widehat{\Omega}_{\mu} - \tfrac{1}{8}\varepsilon_{\mu\nu\sigma\kappa\lambda}\,\partial^\mu\widehat{C}^{\nu\sigma}{}_{\rho}\,\partial_6\widehat{C}^{\kappa\lambda,\rho} + \tfrac{1}{32}\varepsilon_{\mu\nu\sigma\kappa\lambda}\,\partial^\mu\mathcal{T}^{\nu\sigma}{}_{\rho}\,\partial_6\mathcal{T}^{\kappa\lambda,\rho}$$
$$-\tfrac{1}{8}\partial_6\mathcal{T}_{\sigma\tau,\nu}\,\partial_\mu T^{\mu\nu,\sigma\tau} + \tfrac{1}{4}\partial_6\mathcal{T}_{\kappa\lambda,\tau}\,\partial^\kappa T^{\lambda\sigma,\tau}{}_{\sigma} + \tfrac{1}{4}\partial_\nu\mathcal{T}_{\sigma\mu}{}^{\mu}\,\partial_6 T^{\sigma\tau,\nu}{}_{\tau} - \tfrac{1}{8}\partial_6\mathcal{T}_{\sigma\mu}{}^{\mu}\,\partial^\sigma T_{\tau\nu}{}^{\tau\nu}$$
$$-\tfrac{1}{64}\varepsilon_{\mu\nu\alpha\beta\gamma}\,\partial^\alpha T_{\sigma\tau}{}^{\beta\gamma}\,\partial_6 T^{\mu\nu,\sigma\tau} - \tfrac{1}{32}\partial_6 T_{\sigma\tau,\mu\nu}\,\partial_6 T^{\mu\nu,\sigma\tau} + \tfrac{1}{8}\partial_6 T_{\sigma\mu,\nu}{}^{\mu}\,\partial_6 T^{\sigma\tau,\nu}{}_{\tau} - \tfrac{1}{32}\partial_6 T_{\mu\nu}{}^{\mu\nu}\,\partial_6 T_{\sigma\tau}{}^{\sigma\tau}$$

actions for exotic supergravities

$\mathcal{N} = (2,2)$	$\mathcal{N} = (3,1)$	$\mathcal{N} = (4,0)$
$g_{\mu\nu}$ (1) A_μ (16) $B_{\mu\nu}$ (5) ϕ (25)	$C^{+}_{\mu\nu,\rho}$ (1) A_μ (14) $B^{+}_{\mu\nu}$ (12) ϕ (28)	$T^{+}_{\mu\nu,\rho\sigma}$ (1) $B^{+}_{\mu\nu}$ (27) ϕ (42)

$$\mathcal{L}_{\boxplus} = -\tfrac{1}{4}\,\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\mu\nu\rho} + \tfrac{1}{2}\,\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\nu\rho\mu} + \widehat{\Omega}^{\mu}\widehat{\Omega}_{\mu} - \tfrac{3}{4}\,\widehat{F}^{\mu\nu}\widehat{F}_{\mu\nu}$$
$$-\tfrac{1}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\partial_6\widehat{C}_{\mu\nu,}{}^{\lambda}\partial_\rho\widehat{C}_{\sigma\tau,\lambda} - \tfrac{9}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\partial_6 B_{\mu\nu}\partial_\rho B_{\sigma\tau} - \tfrac{3}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\,\partial_6\widehat{C}_{\rho\sigma,\tau}$$

$$\mathcal{L}_{\boxplus\!\!\boxplus} = -\tfrac{1}{4}\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\mu\nu\rho} + \tfrac{1}{2}\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\nu\rho\mu} + \widehat{\Omega}^{\mu}\widehat{\Omega}_{\mu} - \tfrac{1}{8}\varepsilon_{\mu\nu\sigma\kappa\lambda}\,\partial^\mu\widehat{C}^{\nu\sigma}{}_{\rho}\,\partial_6\widehat{C}^{\kappa\lambda,\rho} + \tfrac{1}{32}\varepsilon_{\mu\nu\sigma\kappa\lambda}\,\partial^\mu\mathcal{T}^{\nu\sigma}{}_{\rho}\,\partial_6\mathcal{T}^{\kappa\lambda,\rho}$$
$$-\tfrac{1}{8}\partial_6\mathcal{T}_{\sigma\tau,\nu}\,\partial_\mu T^{\mu\nu,\sigma\tau} + \tfrac{1}{4}\partial_6\mathcal{T}_{\kappa\lambda,\tau}\,\partial^\kappa T^{\lambda\sigma,\tau}{}_{\sigma} + \tfrac{1}{4}\partial_\nu\mathcal{T}_{\sigma\mu}{}^{\mu}\,\partial_6 T^{\sigma\tau,\nu}{}_{\tau} - \tfrac{1}{8}\partial_6\mathcal{T}_{\sigma\mu}{}^{\mu}\,\partial^\sigma T_{\tau\nu}{}^{\tau\nu}$$
$$-\tfrac{1}{64}\varepsilon_{\mu\nu\alpha\beta\gamma}\,\partial^\alpha T_{\sigma\tau}{}^{\beta\gamma}\,\partial_6 T^{\mu\nu,\sigma\tau} - \tfrac{1}{32}\partial_6 T_{\sigma\tau,\mu\nu}\,\partial_6 T^{\mu\nu,\sigma\tau} + \tfrac{1}{8}\partial_6 T_{\sigma\mu,\nu}{}^{\mu}\,\partial_6 T^{\sigma\tau,\nu}{}_{\tau} - \tfrac{1}{32}\partial_6 T_{\mu\nu}{}^{\mu\nu}\,\partial_6 T_{\sigma\tau}{}^{\sigma\tau}$$

- finally: add standard scalar, vector, tensor couplings (ExFT form)

$$\mathcal{L}_{\Box\Box} = -\tfrac{1}{4}\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\mu\nu\rho} + \tfrac{1}{2}\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\nu\rho\mu} + \widehat{\Omega}^{\mu}\widehat{\Omega}_{\mu} - \tfrac{1}{3}(\partial^\mu\phi - 2\,\partial_6 A^\mu)(\partial_\mu\phi - 2\,\partial_6 A_\mu)$$
$$-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{5}{9}\partial_6\phi\partial_6\phi - \tfrac{2}{3}\partial_6 h_\sigma{}^\sigma\partial_6\phi + \tfrac{1}{4}\partial_6 h_\sigma{}^\sigma\partial_6 h_\rho{}^\rho - \tfrac{1}{4}\partial_6 h^{\mu\nu}\partial_6 h_{\mu\nu}$$

$$\mathcal{L}_{\boxminus} = -\tfrac{1}{4}\,(F_{\mu\nu} + \partial_6 B_{\mu\nu})(F^{\mu\nu} + \partial_6 B^{\mu\nu}) - \tfrac{1}{24}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\,H_{\rho\sigma\tau}$$

$$\mathcal{L}_{\Box} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}\,(\partial^\mu\phi - \partial_6 A^\mu)(\partial_\mu\phi - \partial_6 A_\mu)$$

$$\mathcal{L}_{\cdot} = -\tfrac{1}{2}\partial^\mu\phi\partial_\mu\phi - \tfrac{1}{2}\partial_6\phi\partial_6\phi$$

actions for exotic supergravities

$\mathcal{N}=(2,2)$

$g_{\mu\nu}$ (1)	A_μ (16)	$B_{\mu\nu}$ (5)	ϕ (25)
$\Box\Box$	$\Box$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\bullet$

$\mathcal{N}=(3,1)$

$C^{+}_{\mu\nu,\rho}$ (1)	A_μ (14)	$B^{+}_{\mu\nu}$ (12)	ϕ (28)
$\begin{smallmatrix}\Box\Box\\\Box\end{smallmatrix}$	$\Box$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\bullet$

$\mathcal{N}=(4,0)$

$T^{+}_{\mu\nu,\rho\sigma}$ (1)	$B^{+}_{\mu\nu}$ (27)	ϕ (42)
$\boxplus$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\bullet$

$$\mathcal{L}_{\Box\Box} = -\tfrac14\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\mu\nu\rho}+\tfrac12\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\nu\rho\mu}+\widehat\Omega^{\mu}\widehat\Omega_{\mu}-\tfrac13(\partial^\mu\phi-2\,\partial_6 A^\mu)(\partial_\mu\phi-2\,\partial_6 A_\mu)$$
$$-\tfrac14 F^{\mu\nu}F_{\mu\nu}+\tfrac59\partial_6\phi\partial_6\phi-\tfrac23\partial_6 h_\sigma{}^\sigma\partial_6\phi+\tfrac14\partial_6 h_\sigma{}^\sigma\partial_6 h_\rho{}^\rho-\tfrac14\partial_6 h^{\mu\nu}\partial_6 h_{\mu\nu}$$

$$\mathcal{L}_{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}} = -\tfrac14\,(F_{\mu\nu}+\partial_6 B_{\mu\nu})(F^{\mu\nu}+\partial_6 B^{\mu\nu})-\tfrac{1}{24}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\,H_{\rho\sigma\tau}$$

$$\mathcal{L}_{\Box} = -\tfrac14 F^{\mu\nu}F_{\mu\nu}-\tfrac12\,(\partial^\mu\phi-\partial_6 A^\mu)(\partial_\mu\phi-\partial_6 A_\mu)$$

$$\mathcal{L}_{\cdot} = -\tfrac12\partial^\mu\phi\partial_\mu\phi-\tfrac12\partial_6\phi\partial_6\phi$$

$$\mathcal{L}_{\begin{smallmatrix}\Box\Box\\\Box\end{smallmatrix}} = -\tfrac14\,\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\mu\nu\rho}+\tfrac12\,\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\nu\rho\mu}+\widehat\Omega^{\mu}\widehat\Omega_{\mu}-\tfrac34\,\widehat F^{\mu\nu}\widehat F_{\mu\nu}$$
$$-\tfrac{1}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\partial_6\widehat C_{\mu\nu,}{}^{\lambda}\partial_\rho\widehat C_{\sigma\tau,\lambda}-\tfrac{9}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\partial_6 B_{\mu\nu}\partial_\rho B_{\sigma\tau}-\tfrac{3}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\,\partial_6\widehat C_{\rho\sigma,\tau}$$

$$\mathcal{L}_{\boxplus} = -\tfrac14\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\mu\nu\rho}+\tfrac12\widehat\Omega^{\mu\nu\rho}\widehat\Omega_{\nu\rho\mu}+\widehat\Omega^{\mu}\widehat\Omega_{\mu}-\tfrac18\varepsilon_{\mu\nu\sigma\kappa\lambda}\,\partial^\mu\widehat C^{\nu\sigma}{}_\rho\,\partial_6\widehat C^{\kappa\lambda,\rho}+\tfrac{1}{32}\varepsilon_{\mu\nu\sigma\kappa\lambda}\,\partial^\mu\mathcal{T}^{\nu\sigma}{}_\rho\,\partial_6\mathcal{T}^{\kappa\lambda,\rho}$$
$$-\tfrac18\partial_6\mathcal{T}_{\sigma\tau,\nu}\,\partial_\mu T^{\mu\nu,\sigma\tau}+\tfrac14\partial_6\mathcal{T}_{\kappa\lambda,\tau}\,\partial^\kappa T^{\lambda\sigma,\tau}{}_\sigma+\tfrac14\partial_\nu\mathcal{T}_{\sigma\mu}{}^{\mu}\,\partial_6 T^{\sigma\tau,\nu}{}_\tau-\tfrac18\partial_6\mathcal{T}_{\sigma\mu}{}^{\mu}\,\partial^\sigma T_{\tau\nu}{}^{\tau\nu}$$
$$-\tfrac{1}{64}\varepsilon_{\mu\nu\alpha\beta\gamma}\,\partial^\alpha T_{\sigma\tau}{}^{\beta\gamma}\,\partial_6 T^{\mu\nu,\sigma\tau}-\tfrac{1}{32}\partial_6 T_{\sigma\tau,\mu\nu}\,\partial_6 T^{\mu\nu,\sigma\tau}+\tfrac18\partial_6 T_{\sigma\mu,\nu}{}^{\mu}\,\partial_6 T^{\sigma\tau,\nu}{}_\tau-\tfrac{1}{32}\partial_6 T_{\mu\nu}{}^{\mu\nu}\,\partial_6 T_{\sigma\tau}{}^{\sigma\tau}$$

- uniform action ('deformation' of free 5D supergravity)

$$\mathcal{L}_0 = -\tfrac14\Omega^{\mu\nu\rho}\Omega_{\mu\nu\rho}+\tfrac12\Omega^{\mu\nu\rho}\Omega_{\nu\rho\mu}+\Omega^\mu\Omega_\mu-\tfrac12\partial^\mu\phi^A\partial_\mu\phi^A-\tfrac14 F^{\mu\nu\,M}F_{\mu\nu}{}^M$$
$$+\,\mathcal{O}(\partial_6)$$

$A=1,\ldots,42$

$M=1,\ldots,27$

  - where $\mathcal{O}(\partial_6)$ carries the dual fields $B_{\mu\nu\,M}, C_{\mu\nu,\rho}, T_{\mu\nu,\rho\sigma}, u_{\mu\nu}, v_{\rho,\mu\nu}$

- master formulation (extension of exceptional field theory)

conclusions

- novel two-derivative action functionals for (free) $\mathcal{N} = (3{,}1)$, $\mathcal{N} = (4{,}0)$ exotic tensors

$$\mathcal{L}_{\boxplus} = -\tfrac{1}{4}\,\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\mu\nu\rho} + \tfrac{1}{2}\,\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\nu\rho\mu} + \widehat{\Omega}^{\mu}\widehat{\Omega}_{\mu} - \tfrac{3}{4}\,\widehat{F}^{\mu\nu}\widehat{F}_{\mu\nu} - \tfrac{1}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\partial_6\widehat{C}_{\mu\nu,}{}^{\lambda}\partial_\rho\widehat{C}_{\sigma\tau,\lambda} - \tfrac{9}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\partial_6 B_{\mu\nu}\partial_\rho B_{\sigma\tau} - \tfrac{3}{16}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,\partial_6 B_{\mu\nu}\,\partial_6\widehat{C}_{\rho\sigma,\tau}$$

$$\mathcal{L}_{\boxplus} = -\tfrac{1}{4}\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\mu\nu\rho} + \tfrac{1}{2}\widehat{\Omega}^{\mu\nu\rho}\widehat{\Omega}_{\nu\rho\mu} + \widehat{\Omega}^{\mu}\widehat{\Omega}_{\mu} - \tfrac{1}{8}\varepsilon_{\mu\nu\sigma\kappa\lambda}\,\partial^\mu\widehat{C}^{\nu\sigma}{}_{\rho}\,\partial_6\widehat{C}^{\kappa\lambda,\rho} + \tfrac{1}{32}\varepsilon_{\mu\nu\sigma\kappa\lambda}\,\partial^\mu\mathcal{T}^{\nu\sigma}{}_{\rho}\,\partial_6\mathcal{T}^{\kappa\lambda,\rho}$$
$$-\tfrac{1}{8}\partial_6\mathcal{T}_{\sigma\tau,\nu}\,\partial_\mu T^{\mu\nu,\sigma\tau} + \tfrac{1}{4}\partial_6\mathcal{T}_{\kappa\lambda,\tau}\,\partial^\kappa T^{\lambda\sigma,\tau}{}_{\sigma} + \tfrac{1}{4}\partial_\nu\mathcal{T}_{\sigma\mu}{}^{\mu}\,\partial_6 T^{\sigma\tau,\nu}{}_{\tau} - \tfrac{1}{8}\partial_6\mathcal{T}_{\sigma\mu}{}^{\mu}\,\partial^\sigma T_{\tau\nu}{}^{\tau\nu}$$
$$-\tfrac{1}{64}\varepsilon_{\mu\nu\alpha\beta\gamma}\,\partial^\alpha T_{\sigma\tau}{}^{\beta\gamma}\,\partial_6 T^{\mu\nu,\sigma\tau} - \tfrac{1}{32}\partial_6 T_{\sigma\tau,\mu\nu}\,\partial_6 T^{\mu\nu,\sigma\tau} + \tfrac{1}{8}\partial_6 T_{\sigma\mu,\nu}{}^{\mu}\,\partial_6 T^{\sigma\tau,\nu}{}_{\tau} - \tfrac{1}{32}\partial_6 T_{\mu\nu}{}^{\mu\nu}\,\partial_6 T_{\sigma\tau}{}^{\sigma\tau}$$

  - 5+1 split
  - additional non-physical fields $u_{\mu\nu}, v_{\rho,\mu\nu}$
  - uniform actions, (‘deformation’ of free 5D supergravity)

$$\mathcal{L}_0 \;=\; -\tfrac{1}{4}\Omega^{\mu\nu\rho}\Omega_{\mu\nu\rho} + \tfrac{1}{2}\Omega^{\mu\nu\rho}\Omega_{\nu\rho\mu} + \Omega^{\mu}\Omega_{\mu} - \tfrac{1}{2}\partial^\mu\phi^A\partial_\mu\phi^A - \tfrac{1}{4}F^{\mu\nu\,M}F_{\mu\nu}{}^{M}$$

→ $+\,\mathcal{O}(\partial_6)$ $\quad\mathcal{N} = (2{,}2)$

→ $+\,\mathcal{O}(\partial_6)$ $\quad\mathcal{N} = (3{,}1)$

→ $+\,\mathcal{O}(\partial_6)$ $\quad\mathcal{N} = (4{,}0)$

- master formulation extending (free) E_6 exceptional field theory
  - extra coordinates $\{\partial_M, \partial_\bullet\}$ and ‘generalized’ section constraint

$$d^{MNK}\partial_N \otimes \partial_K - \frac{1}{\sqrt{10}}\,\Delta^{MN}(\partial_N \otimes \partial_\bullet + \partial_\bullet \otimes \partial_N) = 0$$

  - induces correct couplings and tensor hierarchy

outlook

- fermions & supersymmetry
  - similar extensions of supersymmetric ExFT, exotic fermions?
- relating to higher-rank exceptional field theories
  - embedding into suitable extensions of $E_{7(7)}$, $E_{8(8)}$ ExFT
- interacting theory ?

$$\mathcal{L}_0 = \underbrace{-\tfrac{1}{4}\Omega^{\mu\nu\rho}\Omega_{\mu\nu\rho} + \tfrac{1}{2}\Omega^{\mu\nu\rho}\Omega_{\nu\rho\mu} + \Omega^{\mu}\Omega_{\mu} - \tfrac{1}{2}\partial^{\mu}\phi^{A}\partial_{\mu}\phi^{A} - \tfrac{1}{4}F^{\mu\nu\,M}F_{\mu\nu}{}^{M}}_{\mathcal{L}_{5D\,\text{sugra}}} + \mathcal{O}(\partial_6)$$

$$\mathcal{L}_{5D\,\text{sugra}} \quad + \quad \boxed{\;?\;}$$

  - non-abelian gauge structures with extra coordinates?
  - extension of the Leibniz structure of ExFT?
  - generalized section constraint?

$$\boxed{d^{MNK}\partial_N \otimes \partial_K - \tfrac{1}{\sqrt{10}}\Delta^{MN}(\partial_N \otimes \partial_\bullet + \partial_\bullet \otimes \partial_N) = 0} \longrightarrow \;?$$