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Workshop on

"Geometric Transport Equations in General Relativity"

February 20 – 24, 2017

organized by

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All talks take place at the ESI, Boltzmann Lecture Hall.

- **Monday, February 20, 2017**

  09:00 – 10:00 **Opening & Registration**

  10:00 – 10:15 **Welcome word**

  10:15 – 11:00 **Hans Ringström (KTH, Sweden)**
  On the cosmic no-hair conjecture in the Einstein-Vlasov setting
  The standard starting point in cosmology is the assumption of spatial homogeneity and isotropy. However, it is preferable to prove that solutions generally isotropise and that the spatial variation (as seen by observers) becomes negligible. This is expected to happen in the presence of a positive cosmological constant; in fact, solutions are in that case expected to appear like the de Sitter spacetime to observers at late times. The latter expectation goes under the name of the cosmic no-hair conjecture. In the talk, we present a result (based on joint work with Håkan Andréasson) concerning a class of spacetimes (T^3-Gowdy, in the Einstein-Vlasov setting) whose members are neither spatially homogeneous nor isotropic, but which all satisfy the cosmic no-hair conjecture. Moreover, we demonstrate that the members of this class are future stable under general perturbations (without symmetries), and that the perturbed solutions satisfy the cosmic no-hair conjecture.

  11:00 – 11:30 *Coffee break*

  11:30 –12:15 **Ernesto Nungesser (Instituto de Ciencias Matemáticas, Spain)**
  Self-similarity breaking of cosmological solutions with collisionless matter
  In this talk we consider the Einstein-Vlasov system with Bianchi VII0 symmetry. Under the assumption of small data we show that self-similarity breaking occurs for reflection symmetric solutions. This generalizes previous work concerning the non-tilted fluid case to the Vlasov case. We will discuss physical implications and how this work can be extended. This is joint work with Ho Lee.

  12:15 – 14:45 *Lunch break*

  14:45 – 15:30 **Mahir Hadžić (King's College, U.K.)**
  Nonlinear stability of expanding stars in the mass-critical Euler-Poisson system
  The gravitational Euler-Poisson system is a fundamental astrophysics model of a Newtonian star. We shall first explain an invariant scaling structure which is responsible for the existence of two different types of self-similar expanding star solutions, discovered by Goldreich and Weber in 1980 for the case where the adiabatic exponent equals 4/3. We then show that these solutions are nonlinearly nonradially stable with respect to small perturbations. We thus construct a new class of global-in-time solutions,

which are not homologous and therefore not encompassed by the existing works.
To handle the free boundary we use the Lagrangian coordinates, and we explain how our analysis fits in a wider context of stable Type I singularity formation dynamics. Time permitting, we present some interesting open questions at the end. This is a joint work with Juhi Jang.

15:30 – 16:00 *Coffee break*

16:00 – 16:45 **Carsten Gundlach (University of Southampton, U.K.)**
Static solutions of the spherically symmetric Einstein-Vlasov system and their perturbation spectrum
I will review a formulation of the spherically symmetric Einstein-Vlasov system in which the limit of zero particle mass is explicitly regular and a reduction of the field equations in that limit. I will review static solutions (arXiv:1610.08908) and then talk about ongoing work on their linear perturbations. The motivation is to better understand critical collapse in this system.

- **Tuesday, February 21, 2017**

09:30 – 10:15 **Philippe LeFloch (University Paris 6, France)**
Hyperboloidal foliations and self-gravitating massive fields
For analyzing the global existence problem for self-gravitating massive fields, I have recently developed, in collaboration with Yue Ma (Xian), a new approach we refer to as the Hyperboloidal Foliation Method. This approach relies on a spacetime foliation by asymptotically hyperboloidal hypersurfaces and on a detailed investigation of the algebraic structure of the Einstein equations. In this lecture, I will explain the main ideas which led us to the global nonlinear stability of Minkowski spacetime in this context, and I will discuss the next challenges to be overcome in order to understand other relativistic transport phenomena involving massive fields, which are of major interest in astrophysics.

10:20 – 11:05 **Martin Taylor (Imperial College, U.K.)**
Global nonlinear stability of Minkowski space for the massless Einstein–Vlasov system

11:05 – 11:35 *Coffee break*

11:35 – 12:20 **Jacques Smulevici (University Paris Sud, France)**
Sharp asymptotics for small data solutions of the Vlasov-Nordström system in three dimensions
Joint work with David Fajman and Jérémie Joudioux.
I will present recent results concerning sharp asymptotics for the solutions of the Vlasov-Norstrm system in dimension three. This system can be viewed as a model problem for the Einstein-Vlasov system. Previous results were concerned only with the asymptotics of velocity averages of the distribution function and did not address the question of sharp asymptotics for its derivatives. To address this issue, we use here a variant of the vector-field method that we introduced in previous work. More precisely, we construct modified vector fields, depending on the solutions, to commute the transport equations. The vector fields are tailored made to have good commutation properties with the transport operator and yet to still control sufficiently strongly the solutions to allow for an (almost) sharp Klainerman-Sobolev type inequality.

12:20 – 13:55 *Lunch break*

13:55 – 14:40 **Mihalis Dafermos (Cambridge, U.K .)**
On the Einstein-Vlasov system under surface symmetry

14:45 – 15:30 **Pieter Blue (Maxwell Institute for Mathematics & University of Edinburgh, U.K.)**
Decay of fields outside black holes: Massless Vlasov outside a very slow Kerr
I will discuss energy and Morawetz (or integrated local decay) estimates for fields outside black holes, in particular the Vlasov equation. This builds on earlier work for the wave and Maxwell equation. Much of the work on these problems in the last decade has used the vector-field method and its generalisations. One generalisation has focused on using symmetries, differential operators that take solutions of a PDE to solutions. In this context, a hidden symmetry is a symmetry that does not decompose into first-order symmetries coming from a smooth family of isometries of the underlying manifold. In this talk, I will build on applications of the vector-field method to the Vlasov equation to prove an integrated energy decay for the Vlasov equation outside a very slowly rotating Kerr black hole, and I will discuss some new features of the symmetry algebra for the Vlasov equation, which illustrate the difficulties in passing

to pointwise-decay estimates for the Vlasov equation in this context.
This is joint work with L. Andersson and J. Joudioux.

15:30 – 16:00 *Coffee break*

16:00 – 16:45 **Olivier Sarbach (Universidad Michoacana de San Nicolas de Hidalgo, Mexico)**
Accretion of a relativistic collisionless kinetic gas into a Schwarzschild black hole
In this talk we provide a systematic discussion for the behavior of a collisionless kinetic gas that is accreted by a non-rotating black hole. To this end, we first solve the relativistic Liouville equation on a Schwarzschild background and give an explicit solution representation in terms of appropriate symplectic coordinates on the cotangent bundle. Next, we explore the particular case in which the flow is steady-state, spherically symmetric and is described by an equilibrium distribution function in the asymptotic region. We determine the relevant parameters of the accretion flow as a function of the particle density and the temperature of the gas at infinity and compare the properties of our model to those of the Bondi-Michel accretion model in the hydrodynamic case. Finally, we establish the asymptotic stability of the spherical steady-state flows by proving pointwise convergence results which show that a large class of (possibly nonstationary and nonspherical) initial conditions for the distribution function lead to solutions of the Liouville equation which relax in time to a spherical steady-state configuration.
Based on joint work with P. Rioseco, see arXiv:1611.02389, arXiv:1701.07104.

- **Wednesday, February 22, 2017**

09:30 – 10:15 **Ellery Ames (KTH, Sweden)**
Stationary Solutions to the Einstein-Vlasov System in Axisymmetry
We present numerical solutions of the Einstein-Vlasov equations which are far-from spherically symmetric in the sense that the particle distributions take flattened and toroidal shapes, and the solutions have non-zero net angular momentum. Certain families of regular solutions are found to contain ergoregions. In this talk we will present properties of the solutions obtained as well as the numerical methods.

10:20 – 11:05 **Chulmoon Yoo (University of Nagoya, Japan)**
3D simulation of Spindle Gravitational Collapse of a Collisionless Particle System
We simulate the spindle gravitational collapse of a collisionless particle system in a 3D numerical relativity code and compare the qualitative results with the old work done by Shapiro and Teukolsky(ST). The simulation starts from the prolate-shaped distribution of particles and a spindle collapse is observed. The peak value and its spatial position of curvature invariants are monitored during the time evolution. We find that the peak value of the Kretschmann invariant takes a maximum at some moment, when there is no apparent horizon, and its value is greater for a finer resolution, which is consistent with what is reported in ST. We also find a similar tendency for the Weyl curvature invariant. Therefore, our results lend support to the formation of a naked singularity as a result of the axially symmetric spindle collapse of a collisionless particle system in the limit of infinite resolution. However, unlike in ST, our code does not break down then but go well beyond. We find that the peak values of the curvature invariants start to gradually decrease with time for a certain period of time. Another notable difference from ST is that, in our case, the peak position of the Kretschmann curvature invariant is always inside the matter distribution.

11:05 – 11:35 *Coffee break*

11:35 – 12:20 **Oliver Rinne (HTW Berlin / Albert Einstein Institute, Germany)**
Einstein-Vlasov evolution in axisymmetry
We present a formulation of the axisymmetric Einstein-Vlasov equations based on the (2+1)+1 formalism, including rotation. Preliminary numerical results demonstrate the convergence, mass and angular momentum conservation of our code. Numerical evolutions forming an apparent horizon will be shown.
This is joint work with Håkan Andréasson and Ellery Ames.

12:25 – 13:10 **Juan Velázquez (Hausdorff Center for Mathematics, Germany)**
Veiled singularities for the massless Einstein-Vlasov system
In this talk I will discuss a class of measured valued solutions of the isotropic Einstein-Vlasov system yielding singularity formation without horizon formation and are asymptotically flat at infinity. The con-

struction of the solution is made in two steps. First, a family of self-similar solutions is constructed by means of a detailed analysis of a dynamical system problem.The resulting self-similar solutions are not asymptotically flat at infinity. In order to obtain such asymptotically flat solutions a truncation procedure is used, combined with a solution of the Vlasov-Einstein system outside a given radius. The space-time associated to the corresponding singularity shows that the singularity is reached in finite proper time at the center. On the other hand, the points at the center with arbitrarily large curvature are casually connected with points with arbitrarily large values of r. (Joint work with A.Rendall).

13:10 – 15:45 *Lunch break & free afternoon*

15:45 – 16:30 *Coffee Break*

19:30 **Conference dinner at the restaurant das Kolin**

- **Thursday, February 23, 2017**

10:00 – 10:45 **Paul Tod (Oxford Mathematical Institute, U.K.)**
Isotropic singularities in Einstein-Vlasov and Einstein-Boltzmann

10:45 – 11:15 *Coffee break*

11:15 – 12:00 **Ho Lee (Kyung Hee University, Korea)**
Late time behaviour of Israel particles in a FLRW spacetime with $\Lambda > 0$
In this talk we study the relativistic Boltzmann equation in a spatially flat FLRW spacetime. We consider Israel particles, which are the relativistic counterpart of the Maxwellian particles, and obtain global-in-time existence and the asymptotic behaviour of solutions. The main argument of the paper is to use the energy method, which is a well-known method to study near-equilibrium solutions of the Newtonian Boltzmann equation. We observe that the method can be applied to study small solutions in a cosmological case. It is the first result of this type where a physically well-motivated scattering kernel is considered for the general relativistic Boltzmann equation. This is a jointwork with Ernesto Nungesser.

12:20 – 14:30 *Lunch break*

14:30 – 15:15 **Jonathan Luk (Stanford University, U.S.)**
High frequency limits in general relativity
It is well-known that weak limits of solutions to the Einstein vacuum equations are not necessarily vacuum solutions, but may have a non-trivial stress-energy-momentum tensor, which can be viewed physically as "effective matter fields" arising from back-reaction of high frequency gravitational waves. A conjecture that can be found in the physics literature is that these limits are solutions to the Einstein-massless Vlasov system. In previously known examples of this phenomenon, the limits are solutions to the Einstein-null dust system with at most two families of null dusts (which can be viewed formally as a (singular) limit of the Einstein-massless Vlasov system) under some 2-dimensional surface symmetry assumptions. We will discuss recent constructions removing some of these restrictions and showing that this is a much more general phenomenon. The talk is based on joint works with Igor Rodnianski and Cécile Huneau.

15:20 – 16:05 **Mohammed Lemou (IRMAR, France)**
Extended rearrangement inequality and quantitiative orbital stability for gravitational Vlasov-Poisson and HMF models.
A generalized notion of rearangements and a functional inequality of Hardy-Littlewood type for this kind of rearrangements will be presented. Then, it will be shown how this functional inequality provides quantitative stability results of steady states to evolution systems that essentially preserve the rearrangements and some suitable energy functional. Two applications will be presented: Gravitational Vlasov-Poisson (GVP) and Hamiltonian Mean Field (HMF) equations.
For GVP system, this inequality provides a quantitative stability result of compactly-supported steady state solutions which are decreasing dunctions of their microscopic energy. The stability of this class was already obtained in the past but the proofs heavily relied on compactness arguments, which provides no quantitative information on the perturbation. For the HMF model, the monotonicity of the steady state is no more sufficient to conduct the stability proof, and an additional stability criteria, which can be found in

the physics literature, is needed. Moreover, the stability of non-compactly supported steady states (which are relevent for HMF, and not for GVP) induces additional difficulties which can be covered by our analysis.

16:05 – 16:35 *Coffee break*

17:00 – 18:00 **Mihalis Dafermos (Cambridge, U.K.)**
Erwin Schrödinger Lecture: On falling into black holes
The celebrated "black hole" spacetimes of Schwarzschild and Kerr play a central role in our current understanding of Einstein's general theory of relativity. Are these spacetimes stable, however, as solutions to the Einstein vacuum equations, in their exterior region? And what fate awaits physical observers who enter inside a "generic" black hole? It turns out that these two questions are intimately related and the answer to the second may be more disturbing than previously thought. This talk will try to explain how so.

- **Friday, February 24, 2017**

09:30 – 10:15 **Jean Dolbeault (CEREMADE, France)**
On a result of symmetry based on nonlinear flows Entropy methods have been introduced in the theory of nonlinear diffusion equations with the purpose of studying rates of convergence to equilibrium. In presence of singular weights, these methods can also be used to establish symmetry results for the solutions of nonlinear elliptic equations. This lecture will address the issues of uniqueness, symmetry and symmetry breaking of the optimal functions in weighted inequalities that have been introduced by L. Caffarelli, R. Kohn and L. Nirenberg. It is based on joint work with M.J. Esteban, M. Loss and M. Muratori.

10:20 – 11:05 **Maximillian Thaller (Chalmers, Sweden)**
On static solutions of the Einstein-Vlasov system
The Einstein-Vlasov system describes the motion of an ensemble of collisionless particles in the framework of general relativity. This presentation focuses on static solutions in spherical symmetry. Three different settings will be discussed: Solutions with cosmological constant, massless solutions with matter of compact support, and solutions describing charged particles. Using properties of static solutions in the ßimple setting"(i.e. no cosmological constant, no charge) and other techniques existence of solutions in the three cases is shown. Moreover, properties of massless solutions with compactly supported matter are discussed and analogies to the notion of geons are pointed out. In the case of charged particles, limits that saturate an inequality for the critical stability radius will be addressed. Some of the results are a collaboration with H. Andréasson and D. Fajman.

11:05 – 11:35 *Coffee break*

11:35 – 12:20 **Gerhard Rein (University of Bayreuth, Germany)**
Gravitational collapse - dust versus Vlasov
Important gravitational collapse solutions in general relativity such as Oppenheimer-Snyder use dust as matter model. However, this matter model can lead to violation of cosmic censorship. The dust matter model can be viewed as a singular case of the Vlasov matter model (a collisionless gas), and for the latter there is hope that cosmic censorship holds. In my talk I will explore the relation between dust and Vlasov in a Newtonian collapse situation. This is intended as a blueprint for work in progress together with Håkan Andréasson for the general relativistic case.