Traces and higher structures

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Based on work with
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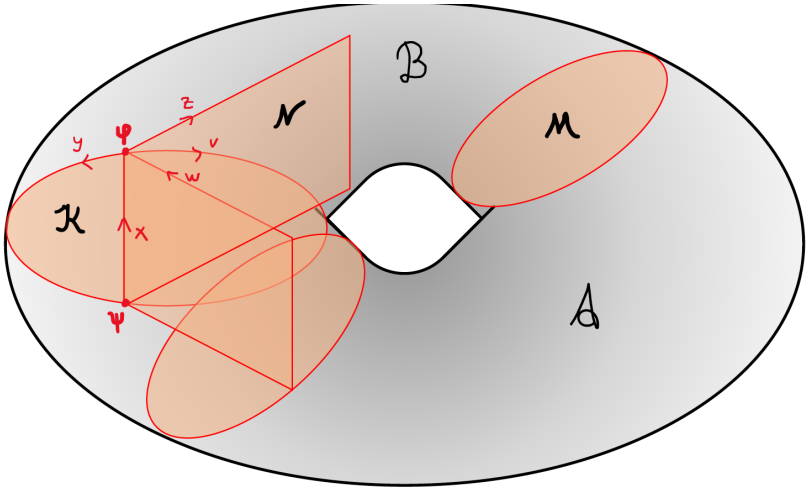
Chapter 3

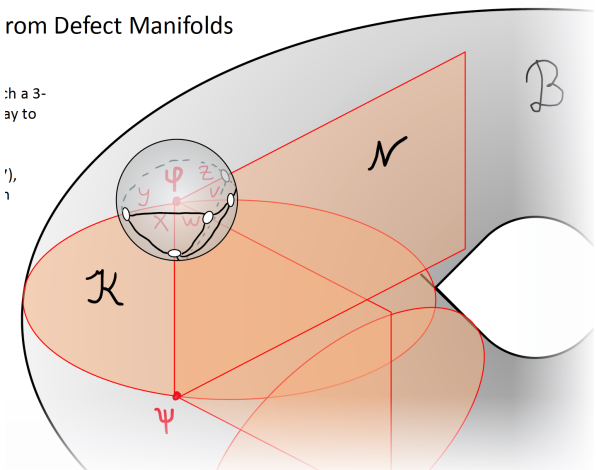
State-sum models with defects

State-sum models with defects



Closed oriented 3-manifold with skeleton

$\mathcal{A}, \mathcal{B}$ spherical fusion categories
$\mathcal{M}, \mathcal{N}$ bimodule categories
Edges: balanced Deligne products

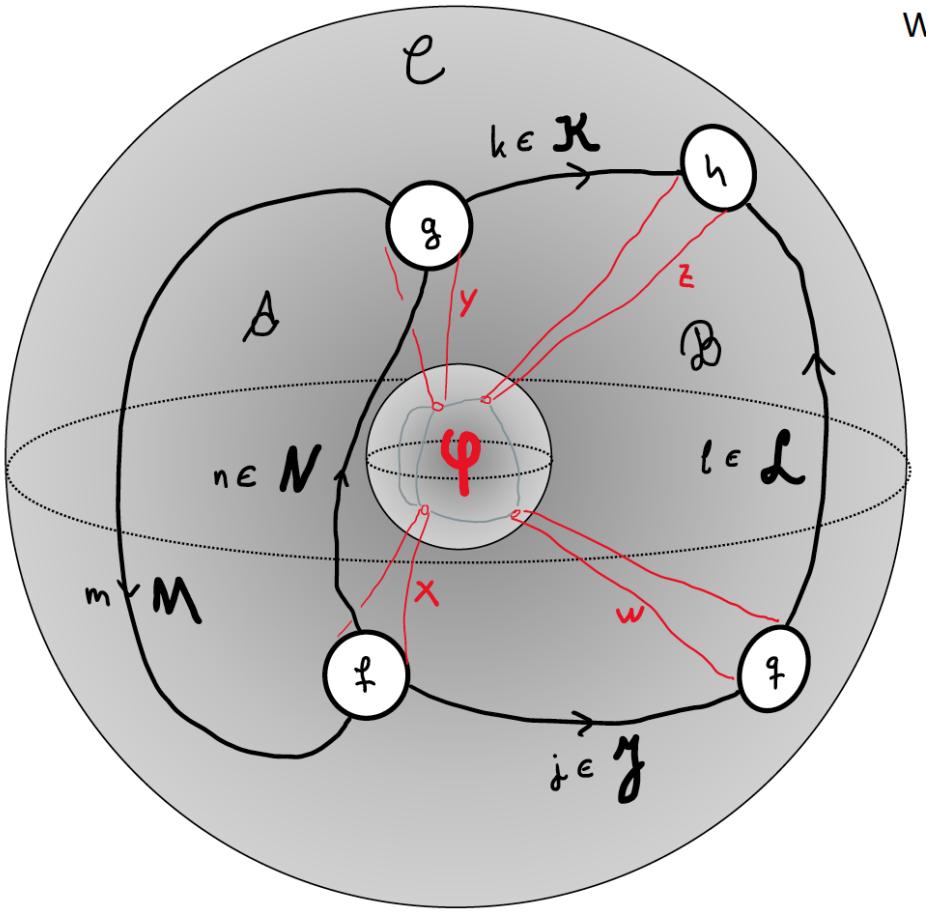


Part of the TV construction is an evaluation at vertices

Chapter 4

Extruded graphs

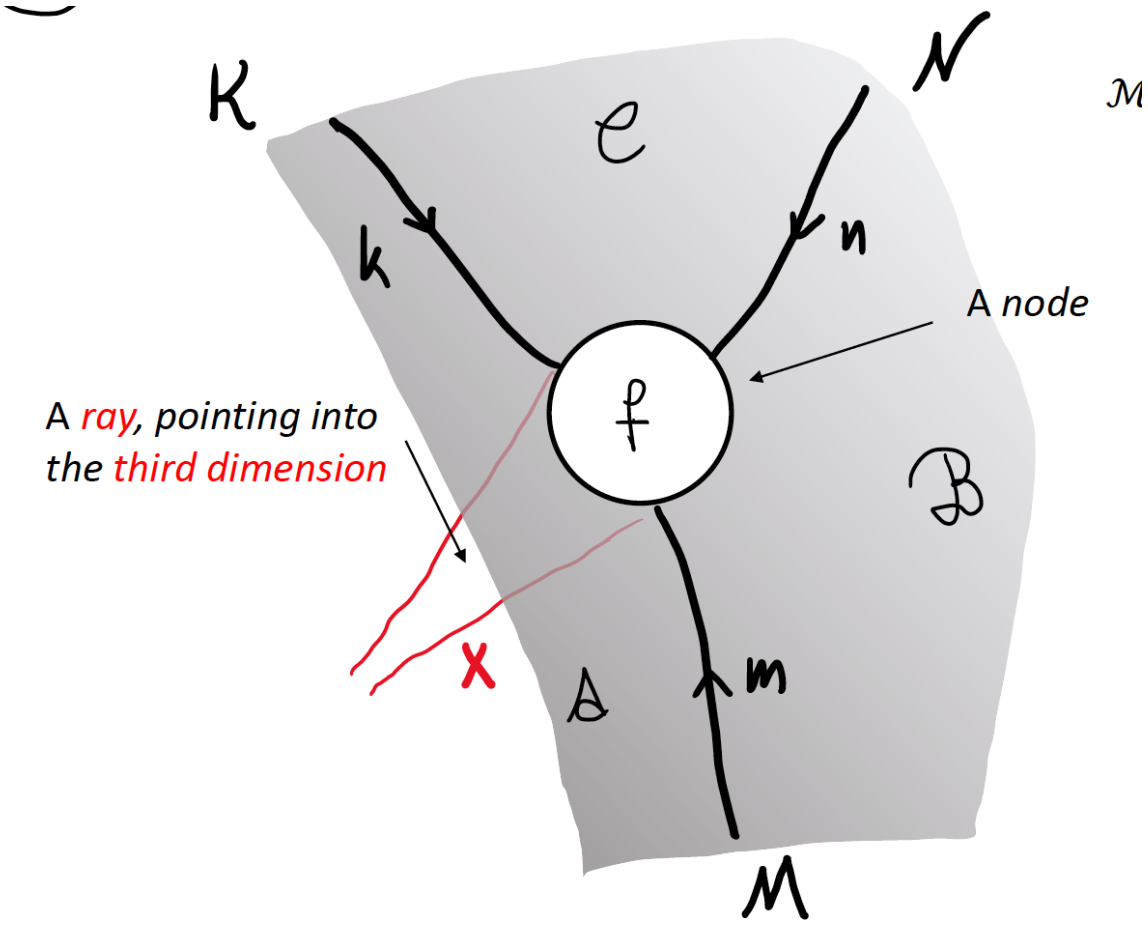
Extruded graphs



W

Artin, Theory of braids (1947):
Projection [..] which is an excellent tool for intitive investigations is a very clumsy one for proofs. This has lead me to abandon projections altogether.

Node labels

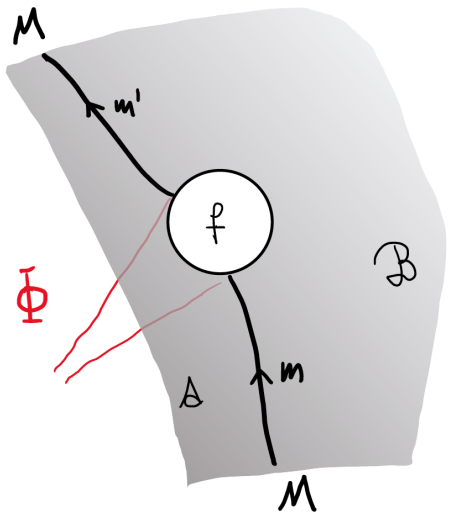


$x \in \mathcal{M} \boxtimes_{\mathcal{B}} \mathcal{N} \boxtimes_{\mathcal{C}} \mathcal{K} \boxtimes_{\mathcal{A}}$

$U(x) \in \mathcal{M} \boxtimes \mathcal{N} \boxtimes \mathcal{K}$

$f : m \boxtimes n \boxtimes n \boxtimes k \to U(x$

Sanity check: silent rays



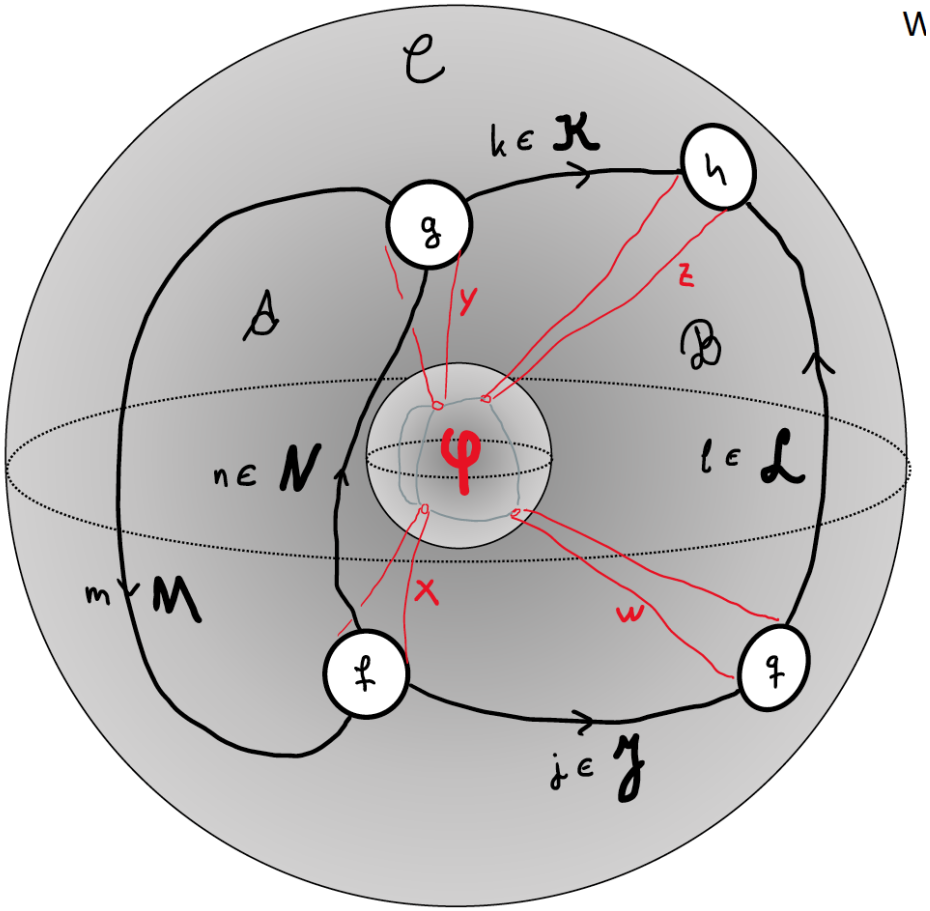
$$U(\Phi) = \bigoplus_\mu \mu \boxtimes \overline{\mu}$$

$$\mathrm{Hom}_{\mathcal{M}\boxtimes\overline{\mathcal{M}}}(m \boxtimes \overline{m'}, \bigoplus \mu \boxtimes \overline{\mu})$$

$$= \bigoplus_\mu \mathrm{Hom}_{\mathcal{M}}(m, \mu) \otimes \mathrm{Hom}_{\mathcal{M}}(\mu, m')$$

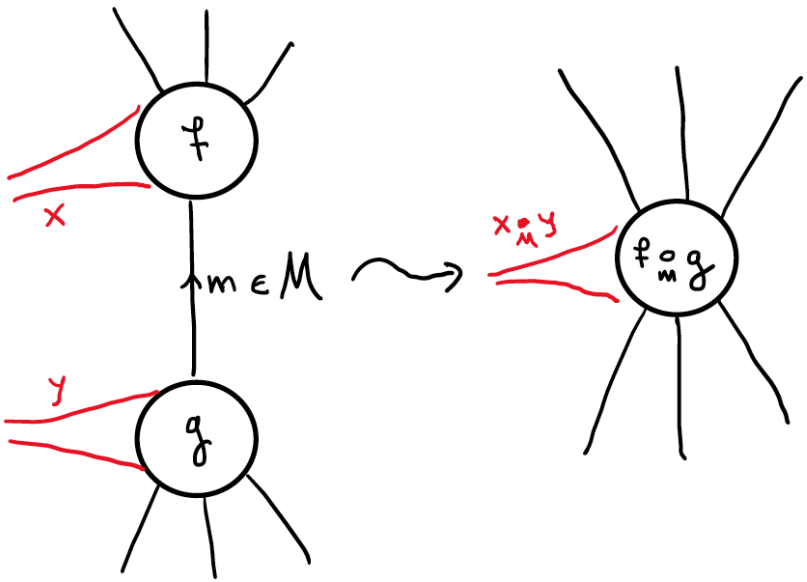
$$= \mathrm{Hom}_{\mathcal{M}}(m, m')$$

Core label

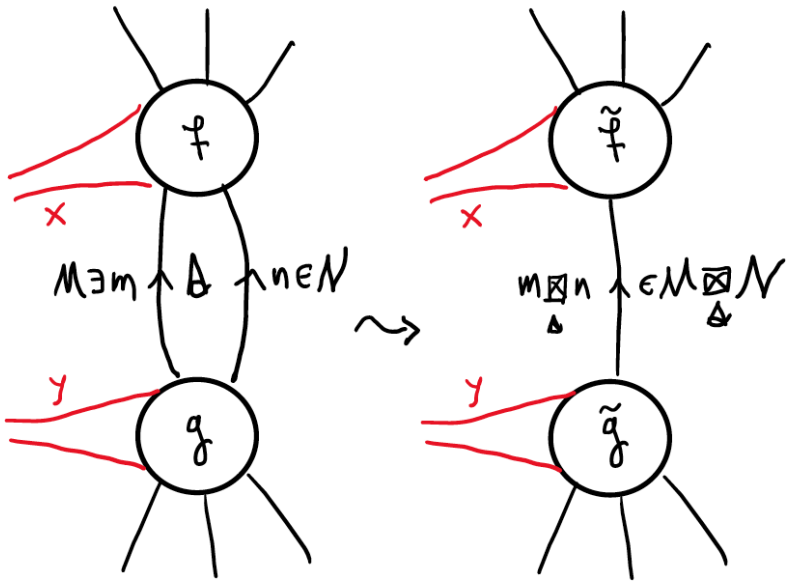

$\mathcal{C}$
$k \in \mathcal{K}$
g
h
$\mathcal{A}$
$\mathcal{B}$
y
z
φ
$n \in \mathcal{N}$
$l \in \mathcal{L}$
$m \in \mathcal{M}$
x
w
f
q
$j \in \mathcal{J}$

Local moves

The contraction move



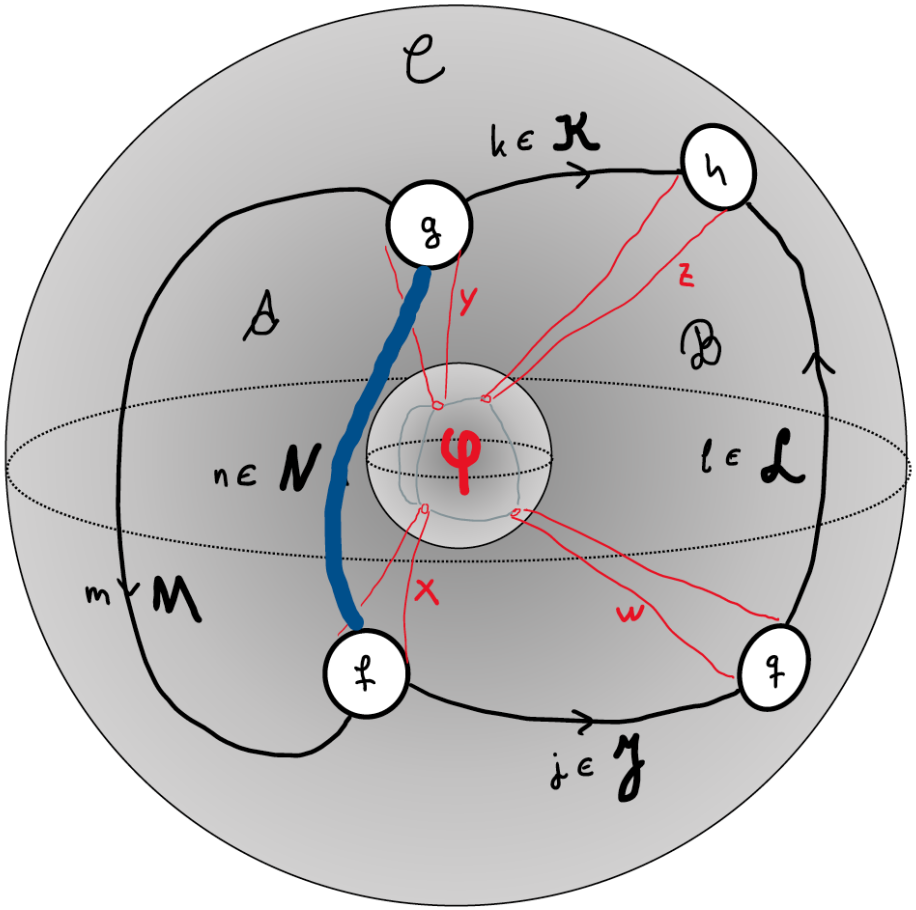
The edge fusion move



Theorem: Performing these changes locally in an extruded graph leaves the evaluation invariant.

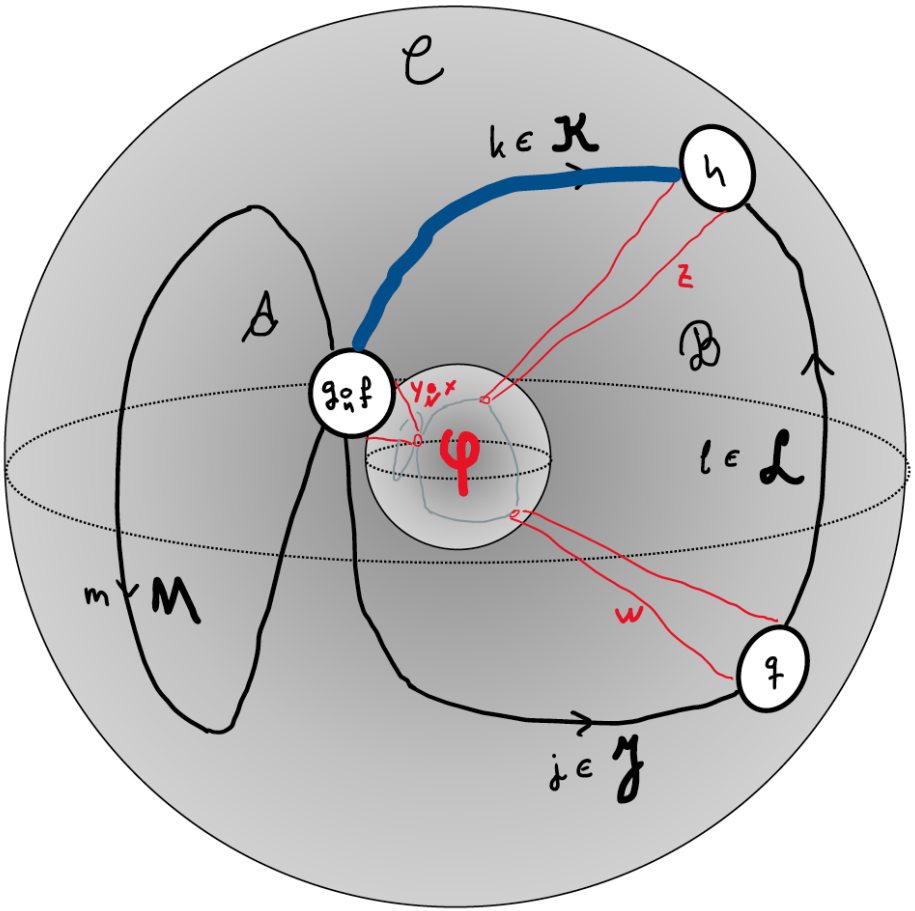
Example for an evaluation

Simplifying extruded graphs using moves



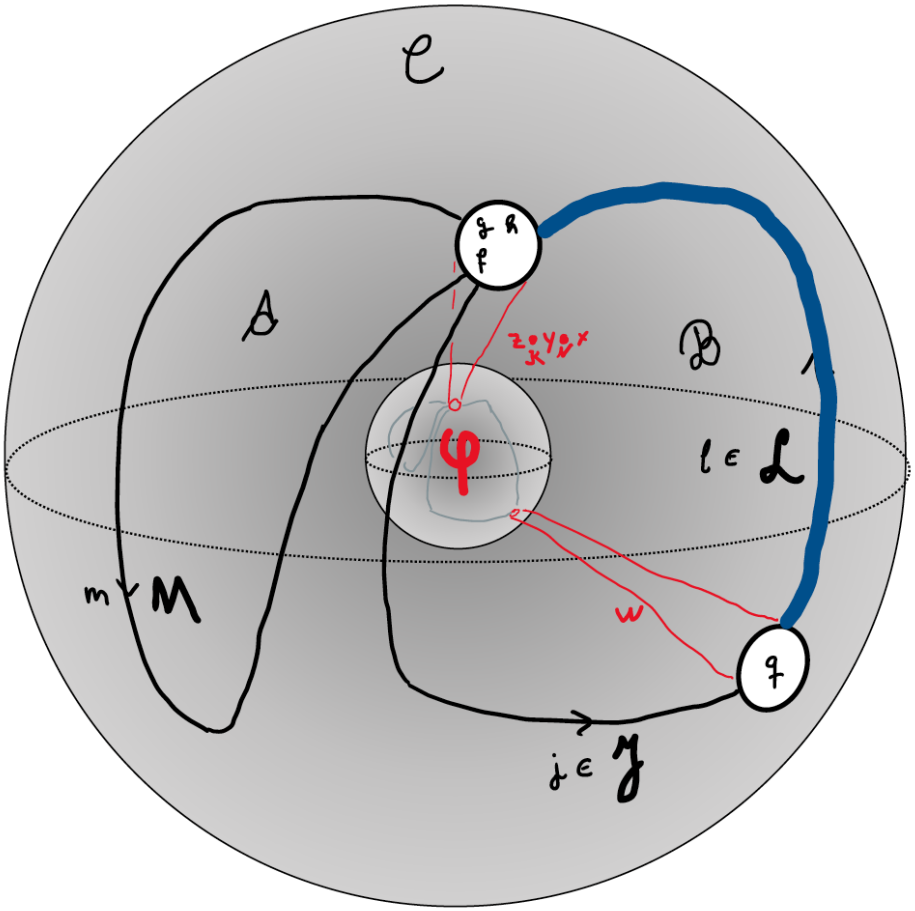
Example for an evaluation

Simplifying extruded graphs using moves



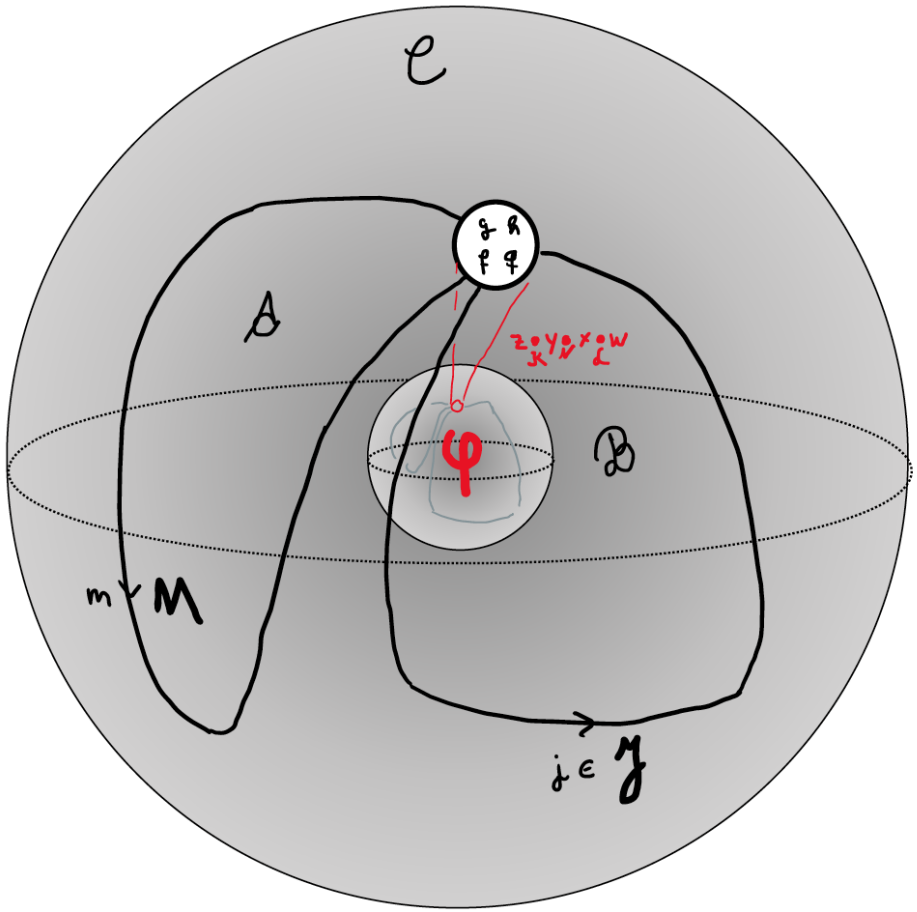
Example for an evaluation

Simplifying extruded graphs using moves



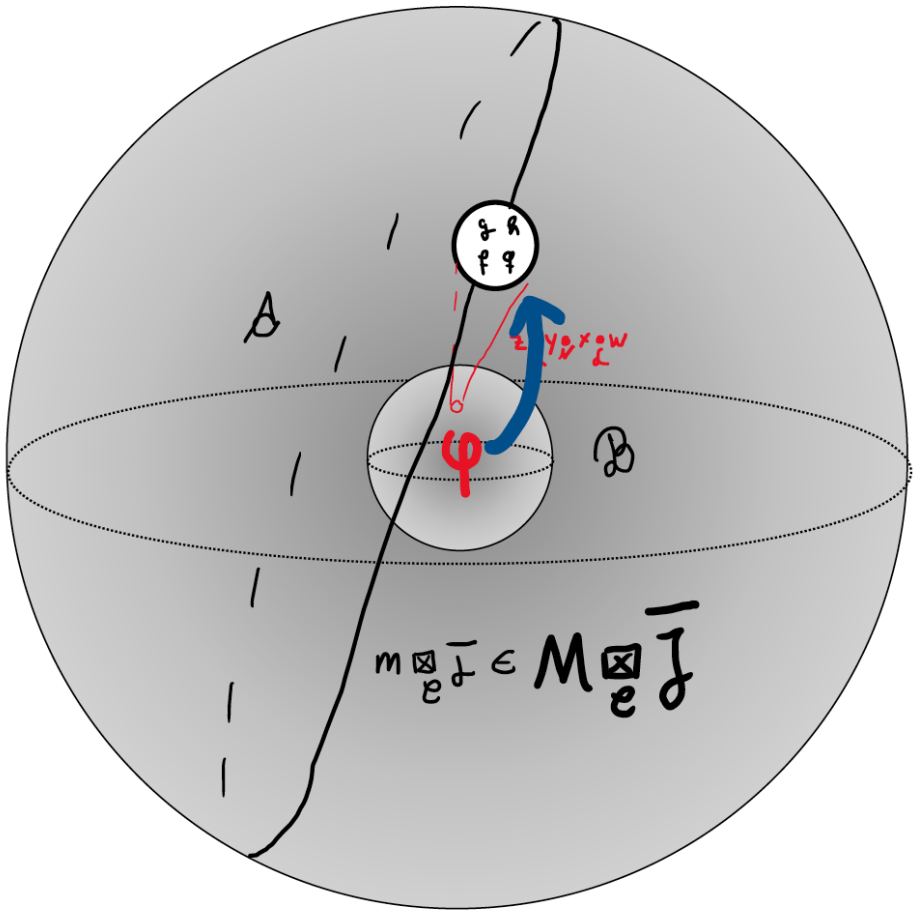
Example for an evaluation

Simplifying extruded graphs using moves



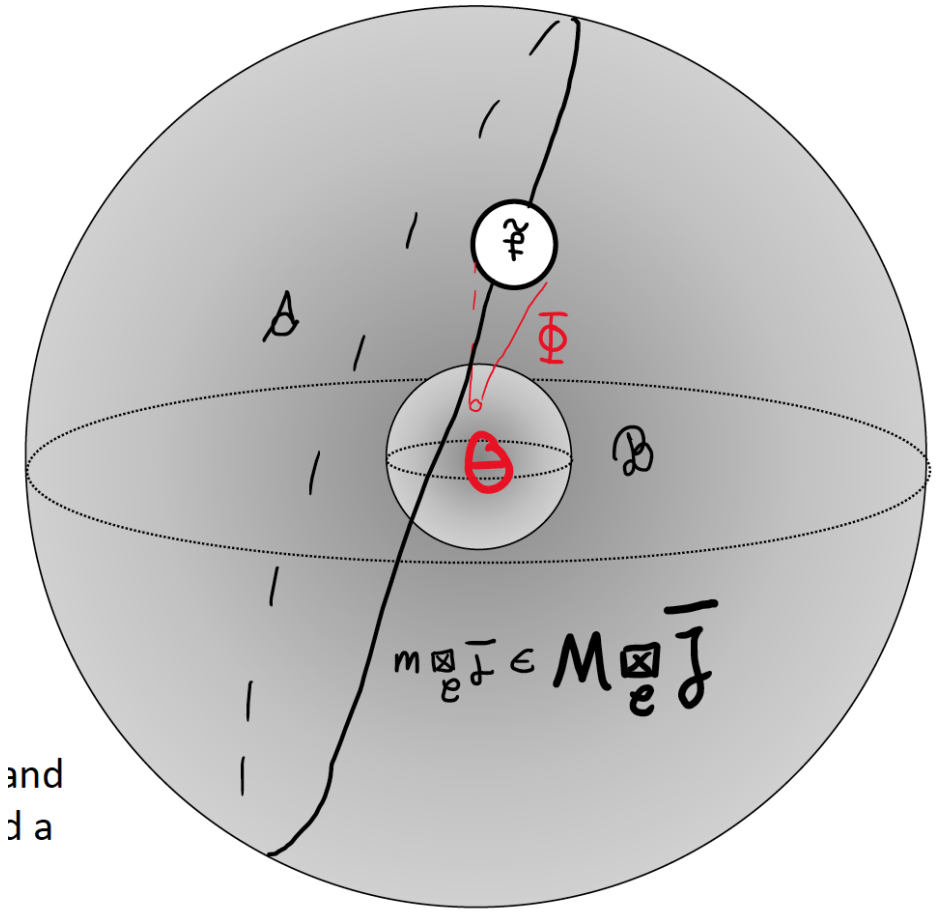
Example for an evaluation

Simplifying extruded graphs using moves



Lift the core label to the coat, leave a silent ray.

Example for an evaluation



evaluation

$\mathrm{Tr}(\tilde{f})$

and
d a