DVR 0065528

Programme on
"Mathematical Aspects of Physical Oceanography"
January 22 – March 23, 2018

organised by

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Workshop
March 12 – 16, 2018

- **Monday, March 12, 2018**

  09:00 – 09:45 *Registration*

  09:45 – 10:30 **R. Samelson (Oregon State University, USA)**
  Title: Poleward undercurrents on eastern ocean boundaries
  Abstract: A time-dependent, inviscid, linear theory for the generation of poleward undercurrent flow under upwelling conditions along mid-latitude ocean eastern boundaries is proposed. The theory relies on a conceptual separation of timescales between the rapid, coastal-trapped wave response to upwelling winds and the subsequent slow, interior quasi-geostrophic planetary wave evolution. A time-dependent coastal boundary condition on the slow-timescale interior flow is obtained from consideration of the fast-timescale, coastal-trapped response. A quasi-geostrophic potential vorticity equation is then solved to determine the interior response to this time-dependent boundary condition. Under upwelling conditions, the results show the formation of a localized region of subsurface poleward flow over the upper continental slope that is qualitatively consistent in amplitude, location, and timing with observations of poleward undercurrents on eastern boundaries.

  10:30 – 11:30 *Coffee / Tea Break*

  11:30 – 12:15 **T. Sapsis (MIT, USA)**
  Title: Quantification and prediction schemes for extreme events in turbulent dynamical systems
  Abstract: Prediction of extreme events for chaotic systems with intrinsically high-dimensional attractors is a formidable problem throughout science and engineering. Thus, a major challenge in contemporary data-driven modeling of dynamical systems is the computation of low-energy patterns or signals, which systematically precede the occurrence of these extreme transient responses. Here, we propose a variational framework for probing conditions that trigger intermittent extreme events in high-dimensional nonlinear dynamical systems. These algorithms exploit in a combined manner some basic physical properties of the chaotic attractor, as well as, stability properties of the governing equations We apply the method to two challenging problems: i) the prediction of extreme intermittent bursts of energy dissipation incompressible Navier-Stokes equations, and ii) the prediction of extreme events in a dispersive wave model for weak turbulence. We assess the performance of the derived predictors through direct numerical simulations.

  12:15 – 13:30 *Lunch Break*

13:30 – 14:15 **K. Ide (University of Maryland, USA)**
Title: Incorporating prior knowledge in observability-based path planning for ocean sampling
Abstract: Observability-based path planning of autonomous sampling platforms for flow estimation is a technique by which candidate trajectories are evaluated based on their ability to enhance the observability of underlying flow-field parameters. We present a new approach that makes use of the underlying coherent structure and introduce a new criterion for scoring candidate trajectories.

14:15 – 15:00 **F. J. Beron-Vera (University of Miami, USA)**
Title: Lagrangian geographies
Abstract: Lagrangian ocean data (satellite-tracked surface drifters, acoustically-tracked submerged floats) of different types are used to construct Markov-chain models of the dynamics. The analysis of these models leads to probabilistic partitions of the various flow domains into weakly connected regions that form the basis for the construction of geographies which do not depend on arbitrary demarcations but rather on the characteristics of the Lagrangian circulation itself. The significance of the geographies including several Markov-chain parameters (residence times, hitting times, fluxes) is tested using independent data (chemical tracers, larval abundance, fish catches).

15:00 – 15:15 *Coffee / Tea Break*

15:15 – 16:00 **G. Broström (University of Gothenburg, Sweden)**
Title: Drift in the upper ocean
Abstract: Buoyant particles rise through water toward the sea surface, while turbulence mix these particles deeper. Observations aiming at determine the content of the particles (e.g. number of plastic particles, fish eggs, oil droplets etc.) are often taken at discrete depths leaving us to estimate the total number using some model for the vertical distribution. The vertical distribution is also important for the drift velocity and direction of the particles. The role of the vertical structure for estimates on horizontal transport will also be discussed, here focusing on the role of strong shear in Eulerian current and Stokes drift in vicinity of the sea surface. In this talk I will in practice consider particles with a high rise velocity that resides in the surface layer. The role of the Stokes drift for transports in the upper ocean will be outlined, and exemplified using cod eggs, oil, and buoyant plastic particles. The region covered by the talk will be along Norwegian and Swedish coast (say drift during 1-5 weeks), as well as long tern drift patters in North Atlantic, and global ocean.

16:00 – 16:45 **M. Serra (Harvard University, USA)**
Title: Material Transport Barriers in Geophysical Flows
Abstract: We present a variational theory of Objective Coherent Structure (OCSs) in two-dimensional non- autonomous dynamical systems, such as turbulent fluid flows. OCSs uncover the hidden material skeleton of the overall dynamical system, acting as theoretical centerpieces of trajectory patterns over a finite or infinitesimally short time. We apply our results to satellite-derived ocean velocity data, sub-mesoscale ocean velocity fields reconstructed from High-Frequency-Radar measurements, and stratospheric velocity fields from ECMWF global reanalysis.

- **Tuesday, March 13, 2018**

  09:30 – 10:15 **G. Haller (ETH Zürich, Switzerland)**
  Title: Material barriers to diffusive and stochastic transport
  Abstract: Observations of tracer transport in fluids generally reveal highly complex patterns shaped by an intricate network of transport barriers. The elements of this network appear to be universal for small diffusivities, independent of the tracer and its initial distribution. In this talk, I discuss a mathematical theory for weakly diffusive tracers that predicts transport barriers and enhancers solely from the flow velocity, without reliance on diffusive simulations. The theory also extends to particle motion under uncertainties, eliminating the need for Monte-Carlo simulations in detecting stochastic transport barriers. I illustrate the results on simple flow models and on satellite-inferred ocean velocity data.

  10:15 – 11:00 **T. Tél (Loránd Eötvös University, Hungary)**
  Title: Experimental evidence for the water-holding property of three-dimensional vortices
  Abstract: When injecting dye into a vertical vortex generated by a commercial magnetic stirrer, one finds that the dye remains captured around the vortex core over minutes, while it becomes mixed with the water outside this region rather rapidly. Thus, considering its horizontal motion, the dye becomes trapped within a critical radius, despite the fact that the vortex (and the dyed region) is being perturbed by a random-like motion due to the oscillations of the position of the stirring bar. According to a recent paper by G. Haller and coworkers, three-dimensional vortices should be defined as rotating, material-holding regions of the fluid. The observed vortices generated by magnetic stirrers appear to demonstrate this material-holding property since dye remains accumulated in a cylindrical region over a long period of time, before it becomes washed out from this cylinder by the downwelling at the core of the vortices. Our measurements indicate that the radius of the dyed cylinders, i.e. the horizontal extent of the vortex, hardly depends on the rotational frequency of the stirrer bar, at least in the range investigated, but increases with the length of the bar. Joint work by I. M. Janosi, L. Kadi, T. Tél, M. Vincze.

  11:00 – 11:30 *Coffee / Tea Break*

  11:30 – 12:15 **M. Vincze (Loránd Eötvös University, Hungary)**
  Title: Wind-induced resonant thickening of the oceanic Ekman-layers: an experimental analysis
  Abstract: One of the main open problems related to the energetics of global ocean circulation is the nature of the processes that mix the deep ocean. According to the classic theory of Ekman (1905), constant wind stress at the water surface yields significant mixing of the seawater. This effect is confined only to an approximately 100 m thick Ekman layer at the top of the water column, as long as only stationary wind stress is considered. However, very recent theoretical and numerical results have demonstrated that for time-dependent wind forcing a certain resonance can develop between the Coriolis force-driven inertial motion of the fluid parcels and the wind bursts. Then the Ekman layer can penetrate orders of magnitudes larger depths than in the classic case. We have carried out a systematic experimental study of this resonance phenomenon at the LEGI Coriolis platform (Grenoble, France). The results clearly demonstrate that the resonant thickening of Ekman layers is indeed present in an ocean-like configuration. Joint work by M. Vincze, Y. Ashkenazy, N. Fenyvesi and J. Sommeria.

  12:15 – 13:00 **M. J. Olascoaga (University of Miami, USA)**
  Title: Lagrangian climatologies
  Abstract: In this talk I will present results from recent efforts aimed at synthesizing the Lagrangian information contained in large amounts of Eulerian data. Performing appropriate averages we extract quasi-Lagrangian coherent structures that are seen to organize transport on long (e.g., seasonal) timescales. We exemplify the construction of such “Lagrangian climatologies” using various long records of velocity data and support the results on independent observations in the Gulf of Mexico and the Argentine Sea. Main collaborators in these efforts are F. J. Beron-Vera, R. Duran, M. Gough, and N. Bodnariuk.

  18:30 – **Social Event: Workshop Dinner (restaurant Porzellan, Servitengasse 2, 1090 Vienna)**

- **Wednesday, March 14, 2018**

  09:30 – 10:15 **R. S. Johnson (Newcastle University, UK)**
  Title: Ekman-type flows as a shallow-water approximation of the Navier-Stokes equation for a spherical, rotating Earth
  Abstract: In this talk, we briefly describe the classical Ekman spiral, and then discuss how it can be presented as an asymptotic solution of the Navier-Stokes equation. This involves a suitable non-dimensionalisation, the introduction of a shallow-water parameter and the construction of a solution that takes this as the (only) small parameter. We first find a vorticity equation for the horizontal motion alone (which is generated by following the standard Ekman model: no vertical motion and the vertical structure is partially suppressed). This lays the foundations for seeking solutions, including the Ekman spiral, and provides the first term of a consistent (asymptotic) solution of the full system of equations, valid in the neighbourhood of a point on the surface of the sphere. We show that this enables the complete structure of the classical Ekman flow to be described, and introduces some generalisations. We conclude with a few comments on the further developments of this approach.

  10:15 – 11:00 **A. Abrashkin (National Research University, Nizhny Novgorod, Russia)**
  Title: Wind-generated equatorial Gerstner-type waves
  Abstract: The motion of a homogeneous incompressible fluid in a rotating frame of reference is studied. The effect of the wind is modeled by an inhomogeneous and non-stationary pressure distribution on the free surface of the fluid. We consider the mathematical formulation of the problem in Lagrangian variables. The expressions for the integrals of motion in a uniformly rotating fluid extending the Cauchy invariants to a non-rotating fluid are found. Their connection with the theorems of Kelvin and Ertel is shown. A class of non-stationary surface gravity waves propagating in the zonal direction in the equatorial region is descrided in the -plane approximation. These waves are described by exact solutions of the equations of hydrodynamics and are generalizations of Gerstner waves. The wave shape and pressure distribution on a free surface depend on two arbitrary functions. The trajectories of fluid particles are circumferences. The solutions admit a variable meridional current. Examples of possible wave motions are discussed

  11:00 – 11:30 *Coffee / Tea Break*

  11:30 – 12:15 **E. Vanden-Eijnden (New York University, USA)**
  Title: Extreme events, tail statistics, and large deviation theory in geophysical flows
  Abstract: In recent years several analytical and numerical methods have been introduced to characterize the pathway, rate, and likelihood of rare but important events. These methods build on large deviation theory, which indicates that the way such events occur is often predictable and offers way to compute them via solution of an optimization problem for their most likely path. In this talk, I will discuss the applicability of these techniques to geophysical flows, for example to quantify the probability and mechanism of appearance of rogue waves or explain transitions between metastable patterns in atmospheric flows

  12:15 – 13:00 **D. Giannakis (New York University, USA)**
  Title: Extraction and prediction of coherent patterns in incompressible flows through space-time Koopman analysis
  Abstract: We discuss a data-driven, spectral method for detecting and predicting the evolution of coherent spatiotemporal patterns in incompressible fluid flows driven by ergodic dynamical systems. The approach is based on a representation of the group of Koopman operators governing the evolution of observables of skew-product dynamical systems in a basis learned from velocity field data through the diffusion maps algorithm. Using this representation, we construct a Galerkin method for the eigenvalue problem for the generator of the Koopman group, and employ the resulting eigenfunctions to identify coherent spatiotemporal patterns. Moreover, we discuss a technique for approximating the evolution of observables and probability densities under the flow based on an exponentiation of the generator. We present applications in Gaussian vortex flows and chaotic flows generated by Lorenz 96 systems.

- **Thursday, March 15, 2018**

  09:00 – 10:00 **D. Crowdy (Imperial College London, UK)**
  Title: New analytical solutions for vortical flows on the plane and the surface of a sphere
  Abstract: This talk will survey several general mathematical techniques devised by the speaker over recent years for the construction of analytical solutions to the Euler equations. Given the focus of the workshop, special attention will be given to solutions involving vortical flows that model situations of potential geophysical and astrophysical significance. Vortical flows on both the plane and on the surface of a sphere will be considered.

  10:00 – 10:30 *Coffee / Tea Break*

  10:30 – 11:30 **K. Khusnutdinova (Loughborough University, UK)**
  Title: Ring waves in a stratified fluid over a depth-dependent parallel shear flow
  Abstract: We study long weakly-nonlinear ring waves in a stratified fluid in the presence of a depth-dependent parallel shear flow (e.g., oceanic internal waves generated in narrow straits and river-sea interaction zones) within the scope of the full set of Euler equations with the boundary conditions appropriate for oceanographic applications. Our results generalise the results obtained by Johnson (1980, 1990) and Lipovskii (1985). We show that despite the clashing geometries of the waves and the shear flow, there exists a linear modal decomposition (separation of variables) in the set of Euler equations describing the waves, more complicated than the known decomposition for the plane waves. We use it to describe the wavefronts of surface and internal waves, and to derive a 2+1D weakly-nonlinear model for the amplitudes of the waves. The general theory is applied to a two-layer fluid with a piecewise-constant current. The distortion of the wavefronts is described by two branches of the singular solution (envelope of the general solution) of a nonlinear first-order differential equation. The wavefronts of surface and interfacial waves propagating over the same piecewise-constant current look strikingly different. This is a joint work with Xizheng Zhang.

  11:30 – 12:15 **M. Branicki (University of Edinburgh, UK)**
  Title: Path-based and probabilistic measures of expansion rates in stochastic flows and their applications to Lagrangian transport
  Abstract: We present an information-theoretic framework for path-based characterisation of transport and mixing in stochastic dynamical systems which are non-autonomous and known over a finite time interval. This work is motivated by the desire to quantify uncertainty in time-dependent transport and to improve Lagrangian predictions in multi-scale systems based on simplified models with errors affecting path-based predictions in a non-local fashion. As a first step towards the Lagrangian uncertainty quantification we provide a link between trajectory-based notions of expansion rates and 'metrics' which utilise a class of divergencies between evolving probability measures. Uncertainty bounds on path-based functionals in the general stochastic setting will also be discussed.

  12:15 – 13:00 **B. Arbic (Michigan University, Ann Arbor, USA)**
  Title: Global modeling of the oceanic internal gravity wave spectrum
  Abstract: In this talk I will discuss modeling of the internal gravity wave spectrum in two state-of-the-art global ocean models. Internal gravity waves are waves for which gravity is the restoring force and which exist on the interfaces between fluid layers of different densities – hence the name "internal". There are many motivations for studying internal gravity waves. Like waves on a beach, internal waves break. This breaking underlies most of the mixing in the interior of the ocean, and internal-wave driven mixing has significant impacts on ocean biology, ocean circulation, and oceanic temperatures and salinities. Internal waves also have a significant signal in datasets constructed from satellite remote sensing of the ocean. Finally, internal waves have a large impact on acoustics and other operational oceanography considerations. It has been recognized just in the last few years that global models that are forced by both atmospheric fields and the astronomical tidal potential are able to resolve some of the internal gravity wave continuum spectrum – the so-called Garrett-Munk spectrum. In this talk I will discuss the technical aspects of such global models, comparisons of such models with observations, and open questions about such models, in particular, their potential use in understanding the development of the Garrett-Munk spectrum. It is this latter question that, I believe, could benefit from conversations with mathematicians.

  15:00 – 18:00 **Social Event: Visit of Schönbrunn Palace**

- **Friday, March 16, 2018**

  09:30 – 10:15 **M. Farazmand (MIT, USA)**
  Title: Optimal initial condition of passive tracers for their maximal mixing in finite time
  Abstract: The efficiency of fluid flow for mixing passive tracers is often limited by fundamental laws and/or design constraints, such that a perfectly homogeneous mixture cannot be obtained in finite time. Here we address the natural corollary question: Given a fluid flow, what is the optimal initial tracer pattern that leads to the most homogeneous mixture after a prescribed finite time? I will describe several aspects of this problem. In particular, I introduce a constrained optimization problem whose solutions coincide with the optimal initial condition of the passive tracer. I also discuss a simplified case where we assume that there is a small length-scale threshold under which the tracer blobs are considered, for all practical purposes, completely mixed. Under this assumption, I show that the optimal initial condition coincides with a right singular vector (corresponding to the smallest singular value) of a suitably truncated Perron-Frobenius (PF) operator.

  10:15 – 11:00 **P. Koltai (Free University of Berlin, Germany)**
  Title: From large deviations to transport semidistances: coherence analysis for finite Lagrangian data
  Abstract: In a quantitative, set-oriented approach to transport, finite time coherent sets play an important role. These are time-parametrized families of sets with unlikely transport to and from their surroundings under small random perturbations of the dynamics. Here we propose, as a measure of transport, (semi)distances that arise under vanishing perturbations in the sense of large deviations. Analogously, for given finite Lagrangian trajectory data we derive a discrete-time and space semidistance that comes from the "best" approximation of the randomly perturbed process conditioned on this limited information on the deterministic flow. It can be computed as shortest path in a graph with time-dependent weights. Furthermore, we argue that coherent sets are regions of maximal farness in terms of transport, hence they occur as extremal regions on a spanning structure of the state space under this semidistance – in fact, under any distance measure arising from the physical notion of transport. Based on this notion we develop a tool to analyze the state space (or the finite trajectory data at hand) and identify coherent regions. We validate our approach on idealized prototypical examples and well-studied standard cases.

  11:00 – 11:30 *Coffee / Tea Break*

  11:30 – 12:15 **D. Karrasch (Technical University of Munich, Germany)**
  Title: A geometric Lagrangian perspective on advection-diffusion and coherent structures
  Abstract: In my talk, I give a geometric description of incompressible advection-diffusion in a Lagrangian frame. In this frame, the advection-diffusion equation is of diffusion-only type, and one may associate a "geometry of mixing" to the time-averaged diffusion operator. Lagrangian coherent structures may then be viewed as metastable sets under the heat flow of the geometry of mixing.

  12:15 – 13:00 **P. Dellar (Oxford University, UK)**
  Title: Noncanonical Hamiltonian structure of the shallow water equations with complete Coriolis force
  Abstract: The shallow water equations are extended to include the non-traditional part of the Coriolis force due to the locally horizontal components of the Earth's rotation vector. This is accomplished by introducing the velocity with respect to an inertial frame as the canonical velocity, then making a gauge transformation to eliminate the vertical component. The resulting equations are a non-canonical Hamiltonian system in Eulerian variables, with the natural Lie–Poisson bracket for a momentum vector and a materially transported height field. This structure is unchanged by the inclusion of the non-traditional terms, only the relation between fluid particle velocity and the canonical velocity. Salmon (2004, J. Atmos. Sci.) showed that the energy- and potential enstrophy-conserving Arakawa–Lamb finite difference scheme for the traditional shallow water equations is a particular spatial discretisation of this non-canonical Hamiltonian structure. We use this insight to extend the Arakawa–Lamb scheme to the non-traditional shallow water equations. This is joint work with Andrew Stewart and Rick Salmon.

  13:00 – 13:45 **S. Monismith (Stanford University, USA)**
  Title: Salinity intrusion in Northern San Francisco Bay: Observations and models
  Abstract: The problem of predicting how the salinity field in estuaries responds to freshwater inflows is one that draws attention from both physical oceanographers and hydraulic engineers since it has both scientific and practical dimensions. In Northern San Francisco Bay, examination of 20+ years of data spanning the estuary shows that the overall structure of the salt field can be described using a single parameter, X2, the distance in km measured from

the Golden Gate Bridge to where the salinity on the bottom is 2. Analysis of long-term monitoring of biological data (e.g. fish abundance) shows that much of ecological functioning of the estuary depends on X2 and so regulations have been developed specifying X2 position depending on time of year and hydrologic conditions. Because these regulations can require substantial amounts of water, it is necessary to efficiently predict the behavior of X2 with some accuracy so as to help manage the competing demands for California's limited water supply. In this talk, using several data sets including one that goes back to ca. 1960, I will discuss the observed behavior of X2 and how it responds to flow, Q. In general, the tendency of freshwater flows to carry salt out of the estuary is balanced by the tendency of dispersion to move salt upstream. A surprising aspect of the X2-Q relation in Northern San Francisco Bay is that it is much weaker than would be inferred from classical estuarine circulation theory, behavior that we attribute to the effects of stratification on the turbulent flows that support upstream salt flux. I will present a rigorously derived albeit simplified integral model of salinity dynamics that can be used to understand this behavior and that can be used to create a dynamically based (rather than purely empirical) model of unsteady salinity intrusion. Finally, examination of the relevant data also suggests that inability to accurately measure freshwater flows during relatively dry periods may be a bigger limitation on accurate predictions of low-flow behavior than is choice of model structure.

All talks take place at ESI, Boltzmann Lecture Hall!