The propagation of fermions on curved backgrounds of matrix models

Emmanuele Battista

Department of Physics, University of Vienna

OUTLINE

THE GEOMETRICAL FRAMEWORK (1)

- **IKKT matrix model, defined by the action**

$$S[T,\Psi] = \frac{1}{g^2}\mathrm{Tr}\Big([T^A,T^B][T_A,T_B] \ + \overline{\Psi}\Gamma_A[T^A,\Psi]\Big)$$

where $T^A \ \ (A=0,1,\ldots,9)$ **are hermitian matrices and** Ψ **fermionic matrices of** $SO(9,1)$

- **Propagation of fermions on some given background** $\{T^A\}$ **in the semiclassical regime, where**

$$T^A : \mathscr{M} \hookrightarrow \mathbb{R}^{9,1}$$
$$[\ \cdot\ ,\ \cdot\ ] \sim i\{\ \cdot\ ,\ \cdot\ \}$$

- **3+1-dimensional spacetime brane** $\mathscr{M}^{3,1}$ **embedded along the first 3+1 matrix directions labelled by** $a=0,\ldots,3$**, setting the remaining matrices to zero (i.e,** $T^a \neq 0 \ (a=0,\ldots,3)$ **and** $T^A=0 \ (A=4,\ldots,9)$**)**

THE GEOMETRICAL FRAMEWORK (2)

- **The effective metric $G^{\mu\nu}$ on $\mathscr{M}^{3,1}$ is defined in terms of an auxiliary metric $\gamma^{\mu\nu}$ in the following way:**

$$\gamma^{\mu\nu} = E^{a\mu}E^{b\nu}\eta_{ab}, \qquad G^{\mu\nu} := \frac{1}{\rho^2}\gamma^{\mu\nu}$$

$\alpha, \beta, \ldots = 0,\ldots,3$: **coordinate indices**
$a, b, \ldots = \hat{0}, \ldots, \hat{3}$: **tetrad indices**

$E^{a\mu} = \{T^a, y^\mu\}$: **"Poisson" frame in local coordinates y^μ**

$\rho^2 = \rho_M\sqrt{|\gamma^{\mu\nu}|}$: **conformal factor or dilation**

ρ_M: **symplectic density on $\mathscr{M}^{3,1}$**

- **Effective frame**

$$\mathcal{E}^{a\mu} = \rho^{-1}E^{a\mu} \longrightarrow G^{\mu\nu} = \mathcal{E}^{a\mu}\mathcal{E}^{b\nu}\eta_{ab}$$

inverse frame

$$\mathcal{E}^{a}{}_{\mu}\mathcal{E}_{b}{}^{\mu} = \delta^a_b \longrightarrow G_{\mu\nu} = \eta_{ab}\mathcal{E}^{a}{}_{\mu}\mathcal{E}^{b}{}_{\nu}$$

THE GEOMETRICAL FRAMEWORK (3)

- **Levi-Civita connection** $\Gamma^{(G)}{}_{\mu\nu}{}^{\sigma}$ **of the effective metric** $G_{\mu\nu}$**:**

$$\Gamma^{(G)}{}_{\mu\nu}{}^{\sigma} = \frac{1}{2} G^{\sigma\rho} \left(\partial_\mu G_{\rho\nu} + \partial_\nu G_{\rho\mu} - \partial_\rho G_{\mu\nu} \right)$$

$$\nabla^{(G)}_{\mu} V^{\nu} = \partial_\mu V^{\nu} + \Gamma^{(G)}{}_{\mu\rho}{}^{\nu} V^{\rho}$$

- **Weitzenböck connection** $\tilde{\Gamma}_{\mu\nu}{}^{\sigma}$ **of the effective frame** $\mathcal{E}_a{}^{\mu}$ **:**

$$\tilde{\nabla}_{\nu} \mathcal{E}_a{}^{\mu} = 0$$

$$\tilde{\nabla}_{\mu} V^{\sigma} = \nabla^{(G)}_{\mu} V^{\sigma} + \mathcal{K}_{\mu\kappa}{}^{\sigma} V^{\kappa}$$

$$\mathcal{T}_{\mu\nu}{}^{\sigma} = \tilde{\Gamma}_{\mu\nu}{}^{\sigma} - \tilde{\Gamma}_{\nu\mu}{}^{\sigma}$$

Torsion tensor

$$\Gamma^{(G)}{}_{\mu\nu}{}^{\sigma} = \tilde{\Gamma}_{\mu\nu}{}^{\sigma} - \mathcal{K}_{\mu\nu}{}^{\sigma}$$

$$\mathcal{K}_{\mu\nu}{}^{\sigma} = \frac{1}{2} \left(\mathcal{T}_{\mu\nu}{}^{\sigma} + \mathcal{T}^{\sigma}{}_{\mu\nu} - \mathcal{T}_{\nu}{}^{\sigma}{}_{\mu} \right)$$

Contorsion tensor

Link between the two connections

FERMIONS IN IKKT MODEL (1)

- **Semiclassical Dirac-like action for fermions in the IKKT matrix model on a generic curved background and in arbitrary local coordinates y^μ:**

$$S = \mathrm{Tr}\overline{\Psi}\gamma_a[T^a, \Psi] \sim \int d^4y\, \rho_M(y)\, \overline{\Psi} i\gamma_a E^{a\mu}\partial_\mu\Psi$$

$$\overline{\Psi} = \Psi^\dagger\gamma^{\hat{0}}$$

γ^a $(a = \hat{0}, \hat{1}, \hat{2}, \hat{3})$
flat-space Dirac matrices

- **Semiclassical Lagrangian:**

$$\mathcal{L} = \frac{i}{2}\rho_M\left[\overline{\Psi}\gamma^a E_a{}^\mu\partial_\mu\Psi - (\partial_\mu\overline{\Psi})\gamma^a E_a{}^\mu\Psi\right] + i\rho\rho_M m\overline{\Psi}\Psi$$

Mass term from the six "transversal" matrices T^A $(A = 4,5,\ldots,9)$

Spin connection is "missing"!

- **Rewrite the Lagrangian in terms of the covariant derivative $\hat{D}_\mu$ for spinors**

$$\hat{D}_\mu\Psi = \left(\partial_\mu - \frac{i}{2}\hat{\omega}_\mu{}^{bc}\Sigma_{bc}\right)\Psi$$

$$\hat{\omega}_\mu{}^{ab} = \mathcal{E}^{a\nu}\nabla_\mu^{(G)}\mathcal{E}^b{}_\nu$$

Torsion-free Levi-Civita spin connection associated with $G_{\mu\nu}$

$$\Sigma_{ab} = \frac{i}{4}[\gamma_a, \gamma_b]$$

Spinor representation of the generators of the Lorentz group

FERMIONS IN IKKT MODEL (2)

we obtain

$$\mathcal{L} = \frac{\mathcal{E}}{\rho}\left[\frac{i}{2}\left(\overline{\Psi}\gamma^\mu \widehat{D}_\mu \Psi - (\widehat{D}_\mu \overline{\Psi})\gamma^\mu \Psi\right) + im\overline{\Psi}\Psi - \frac{i}{4}\mathcal{K}_{[\alpha\beta\gamma]}\,\overline{\Psi}\gamma^\alpha\gamma^\beta\gamma^\gamma\Psi\right]$$

$$\gamma^\mu := \mathcal{E}_a{}^\mu \gamma^a$$

$$\mathcal{E} := \det\left(\mathcal{E}^a{}_\mu\right) = \sqrt{-G} = \rho_M \rho^2$$

The same, up to $1/\rho$, as the Dirac Lagrangian in Riemann-Cartan spacetime

- **Introduce a general frame $e^a{}_\mu$ via**

$$\eta_{ab} e^a{}_\mu e^b{}_\nu = G_{\mu\nu}$$

Ψ is allowed to transform as usual under local Lorentz transformations

- **Introduce the spin connection $\tilde{\omega}_\mu^{ab}$ via**

$$\widehat{\omega}_\mu{}^{ab} = e^{a\nu}\nabla^{(G)}_\mu e^b{}_\nu$$

Levi-Civita spin connection associated with $e^a{}_\mu$

$$\widetilde{\omega}_\mu{}^{ab} = e^{a\nu}\widetilde{\nabla}_\mu e^b{}_\nu = \widehat{\omega}_\mu{}^{ab} - \mathcal{K}_\mu{}^{ab}$$

$$\mathcal{K}_\mu{}^{ab}$$

Contorsion tensor of the Weitzenböck connection $\tilde{\Gamma}_{\mu\nu}{}^\sigma$

Note that $\tilde{\omega}_\mu^{ab} = 0$ if $e^a{}_\mu \to \mathcal{E}^a{}_\mu$

FERMIONS IN IKKT MODEL (3)

- **In the general frame $e^a_{\ \mu}$, the Dirac Lagrangian becomes**

$$\mathcal{L} = \frac{e}{\rho}\left[\frac{i}{2}\left(\overline{\Psi}\gamma^\mu \widetilde{D}_\mu\Psi - (\widetilde{D}_\mu\overline{\Psi})\gamma^\mu\Psi\right) + im\overline{\Psi}\Psi\right]$$
$$= \frac{e}{\rho}\left[\frac{i}{2}\left(\overline{\Psi}\gamma^\mu\partial_\mu\Psi - (\partial_\mu\overline{\Psi})\gamma^\mu\Psi\right) + im\overline{\Psi}\Psi + \frac{i}{4}\widetilde{\omega}_{[\alpha\beta\gamma]}\,\overline{\Psi}\gamma^\alpha\gamma^\beta\gamma^\gamma\Psi\right]$$

$$\gamma^\mu := e_a^{\ \mu}\gamma^a,$$
$$e := \det\left(e^a_{\ \mu}\right) = \sqrt{-G}$$

$$\widetilde{D}_\mu\Psi = \left(\partial_\mu - \frac{i}{2}\widetilde{\omega}_\mu^{\ ab}\,\Sigma_{ab}\right)\Psi = \widehat{D}_\mu\Psi + \frac{i}{2}\mathcal{K}_\mu^{\ ab}\Sigma_{ab}\Psi$$

Dirac equation

$$\gamma^\mu\widehat{D}_\mu\Psi + m\Psi - \frac{1}{4}\mathcal{K}_{[\alpha\beta\gamma]}\gamma^\alpha\gamma^\beta\gamma^\gamma\Psi + \frac{\rho}{2}\left(\partial_\mu\rho^{-1}\right)\gamma^\mu\Psi = 0$$

Local Lorentz invariance is broken on nontrivial background

PARTICLE LIMIT OF THE DIRAC FIELD (1)

- **Particle limit of the Dirac field by applying the JWKB approximation**

$$\Psi(x) = \exp\left(-\frac{i}{\hbar}W(x)\right)\sum_{n=0}^{\infty}\hbar^n\psi^{(n)}(x)$$

↓

Leading order

$$(i\gamma^\mu\partial_\mu W - m)\,\psi^{(0)} = 0,$$

Next-to-leading order

$$(i\gamma^\mu\partial_\mu W - m)\,\psi^{(1)} = \gamma^\mu\widehat{D}_\mu\psi^{(0)} - \frac{1}{4}\mathcal{K}_{[\alpha\beta\gamma]}\gamma^\alpha\gamma^\beta\gamma^\gamma\psi^{(0)} + \frac{\rho}{2}\left(\partial_\mu\rho^{-1}\right)\gamma^\mu\psi^{(0)}$$

PARTICLE LIMIT OF THE DIRAC FIELD (2)

- **The leading-order result leads to the Hamilton-Jacobi equation for a relativistic nonspinning particle**

$$G^{\mu\nu} p_\mu p_\nu = -m^2$$

where

$$p_\mu = -\partial_\mu W$$

Four-momentum

$$u_\alpha = \frac{-\partial_\alpha W}{|G^{\mu\nu}\partial_\mu W \partial_\nu W|^{1/2}} = \frac{1}{m} p_\alpha$$

$$G^{\mu\nu} u_\mu u_\nu = -1$$

Four-velocity orthogonal to $W = const$

$$u^\alpha \nabla_\alpha^{(G)} u^\beta = 0$$

Geodesic motion

PARTICLE LIMIT OF THE DIRAC FIELD (3)

The spinor $\psi^{(0)}$ describes the positive-energy solutions of the flat-space Dirac equation

$$\boxed{\psi^{(0)}(x) = \beta_1(x)u^{(1)}(x) + \beta_2(x)u^{(2)}(x), \qquad \beta_1(x), \beta_2(x) \in \mathbb{C},}$$

the spin-up and spin-down spinors being

$$u^{(1)} = \left(\frac{p^{\hat{0}} + m}{2m}\right)^{1/2} \begin{bmatrix} 1 \\ 0 \\ p^{\hat{3}}/(p^{\hat{0}} + m) \\ (p^{\hat{1}} + ip^{\hat{2}})/(p^{\hat{0}} + m) \end{bmatrix},$$

$$u^{(2)} = \left(\frac{p^{\hat{0}} + m}{2m}\right)^{1/2} \begin{bmatrix} 0 \\ 1 \\ (p^{\hat{1}} - ip^{\hat{2}})/(p^{\hat{0}} + m) \\ -p^{\hat{3}}/(p^{\hat{0}} + m) \end{bmatrix},$$

$$\boxed{p^a = e^a{}_\mu p^\mu}$$

PARTICLE LIMIT OF THE DIRAC FIELD (4)

- **The** **next-to-leading-order** **result implies the following propagation equation for $\psi^{(0)}$:**

$$u^\alpha \widetilde{D}_\alpha \psi^{(0)} = -\frac{\widehat{\theta}}{2}\psi^{(0)} - \frac{i}{2}\mathcal{K}_{[\alpha\beta]\gamma}\sigma^{\alpha\beta}u^\gamma\psi^{(0)} - \frac{\rho}{2}\left(\partial_\mu \rho^{-1}\right) u^\mu \psi^{(0)}$$

$\widehat{\theta} = \nabla^{(G)}_\beta u^\beta$ **Expansion scalar of the geodesic congruence** $\sigma^{\alpha\beta} = 2\Sigma^{\alpha\beta}$

If we introduce the normalized spinor $b^{(0)}(x)$ via the relations

$\psi^{(0)}(x) = f(x) b^{(0)}(x)$ $\quad i\bar{b}^{(0)} b^{(0)} = 1$ **with** $f^2(x) = |\beta_1(x)|^2 + |\beta_2(x)|^2$

then we obtain

$$u^\alpha \widetilde{D}_\alpha b^{(0)} = -\frac{i}{2}\mathcal{K}_{[\alpha\beta]\gamma}\sigma^{\alpha\beta}u^\gamma b^{(0)} \qquad u^\alpha \widetilde{D}_\alpha \bar{b}^{(0)} = \frac{i}{2}\mathcal{K}_{[\alpha\beta]\gamma}\bar{b}^{(0)}\sigma^{\alpha\beta}u^\gamma$$

PARTICLE LIMIT OF THE DIRAC FIELD (5)

- **It will prove to be useful the introduction of a new connection. Let us define the new affinities**

$$\overset{*}{\Gamma}_{\mu\nu}{}^{\lambda} = \widetilde{\Gamma}_{\mu\nu}{}^{\lambda} + 2\mathcal{K}_{[\nu\epsilon]\mu}G^{\epsilon\lambda} = \Gamma^{(G)}{}_{\mu\nu}{}^{\lambda} + 3\mathcal{K}_{[\mu\nu\epsilon]}G^{\epsilon\lambda},$$
$$\overset{*}{\omega}_{\mu}{}^{ab} = \widetilde{\omega}_{\mu}{}^{ab} - 2\mathcal{K}^{[ab]}{}_{\mu} = \widehat{\omega}_{\mu}{}^{ab} - 3\mathcal{K}^{[ab\epsilon]}G_{\epsilon\mu},$$

$$\overset{*}{\Gamma}_{\mu\nu}{}^{\lambda} = e_{a}{}^{\lambda}\overset{*}{D}_{\mu}e^{a}{}_{\nu} = e_{a}{}^{\lambda}\left(\partial_{\mu}e^{a}{}_{\nu} + \overset{*}{\omega}_{\mu}{}^{a}{}_{b}e^{b}{}_{\nu}\right),$$
$$\overset{*}{\omega}_{\mu}{}^{ab} = e^{a\nu}\overset{*}{\nabla}_{\mu}e^{b}{}_{\nu} = e^{a\nu}\left(\partial_{\mu}e^{b}{}_{\nu} - \overset{*}{\Gamma}_{\mu\nu}{}^{\lambda}e^{b}{}_{\lambda}\right)$$

$$\overset{*}{\nabla}_{\alpha}G_{\mu\nu} = 0$$

Compatible with the effective metric

Then we obtain

$$u^{\alpha}\overset{*}{D}_{\alpha}b^{(0)} = 0$$
$$u^{\alpha}\overset{*}{D}_{\alpha}\overline{b}^{(0)} = 0$$

The normalized spinor is parallelly propagated along the geodesic path

SPIN PRECESSION EQUATION (1)

- **The spin vector of the Dirac particle can be written via the JWKB approximation as**

$$S^{\alpha} = S^{\alpha}_{(0)} + \mathrm{O}\left(\hbar\right),$$

where

$$S^{\alpha}_{(0)} = \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}u_{\beta}\overline{b}^{(0)}\sigma_{\gamma\delta}b^{(0)}$$

Lowest-order correction

$$\varepsilon^{\alpha\beta\gamma\delta} = e_{a}{}^{\alpha}e_{b}{}^{\beta}e_{c}{}^{\gamma}e_{d}{}^{\delta}\epsilon^{abcd}$$

ϵ^{abcd}**:Levi-Civita symbol**

- $S^{\alpha}_{(0)}$ **satisfies**

$$u_{\alpha}S^{\alpha}_{(0)} = 0$$

$$G_{\alpha\beta}S^{\alpha}_{(0)}S^{\beta}_{(0)} = 1$$

SPIN PRECESSION EQUATION (2)

- $S^{\alpha}_{(0)}$ is characterized by propagation equation

$$u^{\mu}\overset{*}{\nabla}_{\mu}S^{\alpha}_{(0)} = 0$$

The lowest-order spin vector is parallelly transported along the particle's classical geodesic trajectory

Spin precession equation in terms of the Levi-Civita connection of $G_{\mu\nu}$

$$u^{\rho}\nabla^{(G)}_{\rho}S^{\mu}_{(0)} = 3\varepsilon^{\mu\alpha\lambda\epsilon}\mathcal{A}_{\alpha}S_{(0)\lambda}u_{\epsilon}$$

$$\mathcal{A}^{\mu} = \frac{1}{6}\varepsilon^{\mu\alpha\beta\gamma}\mathcal{K}_{[\alpha\beta\gamma]}$$

Axial-vector part of the contorsion tensor

TRANSLATIONAL MOTION (1)

- It follows from the Dirac equation that the effective Dirac current $\rho^{-1}J^\mu$ is conserved, i.e.,

$$\boxed{\nabla^{(G)}_\mu\left(\rho^{-1}J^\mu\right)=0,}$$

where $J^\mu := \bar{\Psi}\gamma^\mu\Psi$

- Gordon decomposition of the effective Dirac current

$$\boxed{\rho^{-1}J^\mu \equiv \mathcal{J}^\mu_{\mathrm{M}}+\mathcal{J}^\mu_{\mathrm{C}}}$$

where

$$\mathcal{J}^\mu_{\mathrm{M}}=\frac{i\hbar}{2m\rho}\left[\widehat{D}_\nu\left(\overline{\Psi}\sigma^{\mu\nu}\Psi\right)+\rho\left(\partial_\nu\rho^{-1}\right)\overline{\Psi}\sigma^{\mu\nu}\Psi\right]=\frac{i\hbar}{2m}\widehat{D}_\nu\left(\frac{\overline{\Psi}\sigma^{\mu\nu}\Psi}{\rho}\right),$$

$$\mathcal{J}^\mu_{\mathrm{C}}=\frac{\hbar}{2m\rho}\left[\left(\widehat{D}^\mu\overline{\Psi}\right)\Psi-\overline{\Psi}\widehat{D}^\mu\Psi-\frac{3i}{2}\mathcal{K}_{[\alpha\beta\gamma]}\overline{\Psi}\sigma^{\alpha\beta}G^{\gamma\mu}\Psi\right]=\frac{\hbar}{2m\rho}\left[\left(\overset{*}{D}{}^\mu\overline{\Psi}\right)\Psi-\overline{\Psi}\overset{*}{D}{}^\mu\Psi\right].$$

Magnetization current

Convection current

TRANSLATIONAL MOTION (2)

Conserved quantities

$$\nabla_\mu^{(G)} \mathcal{J}_{\mathrm{M}}^\mu = 0,$$
$$\nabla_\mu^{(G)} \mathcal{J}_{\mathrm{C}}^\mu = 0.$$

- **Four-velocity for the translational motion**

$$v^\alpha = \frac{\mathcal{J}_{\mathrm{C}}^\alpha}{\sqrt{-G_{\mu\nu}\mathcal{J}_{\mathrm{C}}^\mu \mathcal{J}_{\mathrm{C}}^\nu}},$$

$$v^\mu = u^\mu + \frac{\hbar}{2m}\left[\left(\overset{*}{D}{}^\mu \bar{b}^{(0)}\right) b^{(0)} - \bar{b}^{(0)} \overset{*}{D}{}^\mu b^{(0)}\right] + \mathrm{O}\left(\hbar^2\right)$$

The spin forces the particle to follow a quantum corrected trajectory which deviates from the classical geodesic motion

TRANSLATIONAL MOTION (3)

- **Four-acceleration of the translational motion**

$$a_\alpha = v^\beta \nabla_\beta^{(G)} v_\alpha = -\frac{i}{2}\left(\frac{\hbar}{2m}\right) \overset{*}{R}_{\alpha\beta\mu\nu} u^\beta\, \bar{b}^{(0)} \sigma^{\mu\nu} b^{(0)} + \mathrm{O}\left(\hbar^2\right)$$

where, in our conventions,

$$\overset{*}{R}_{\mu\nu}{}^{\lambda}{}_{\sigma} = e_a{}^{\lambda} e^b{}_{\sigma} \overset{*}{R}_{\mu\nu}{}^{a}{}_{b} = \partial_\mu \overset{*}{\Gamma}_{\nu\sigma}{}^{\lambda} - \partial_\nu \overset{*}{\Gamma}_{\mu\sigma}{}^{\lambda} + \overset{*}{\Gamma}_{\mu\rho}{}^{\lambda} \overset{*}{\Gamma}_{\nu\sigma}{}^{\rho} - \overset{*}{\Gamma}_{\nu\rho}{}^{\lambda} \overset{*}{\Gamma}_{\mu\sigma}{}^{\rho}$$

$$\overset{*}{R}_{\mu\nu}{}^{ab} = \partial_\mu \overset{*}{\omega}_{\nu}{}^{ab} - \partial_\nu \overset{*}{\omega}_{\mu}{}^{ab} + \overset{*}{\omega}_{\mu}{}^{ac} \overset{*}{\omega}_{\nu c}{}^{b} - \overset{*}{\omega}_{\nu}{}^{ac} \overset{*}{\omega}_{\mu c}{}^{b}$$

Our model predicts a spin precession equation and translational motion having the same form as in standard Einstein-Cartan theory

APPLICATION TO THE FLRW BACKGROUND (1)

- **In the FLRW spacetime and adopting Cartesian coordinates x^μ, the $SO(3,1)$-invariant effective metric reads as**

$$ds_G^2 = G_{\mu\nu}dx^\mu dx^\nu = -R^2|\sinh\eta|^3 d\eta^2 + R^2|\sinh\eta|\cosh^2\eta\, d\Sigma^2 \;\; = -dt^2 + a^2(t)d\Sigma^2$$

$$d\Sigma^2 = d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)$$

Invariant length element on the spacelike hyperboloids H^3

- **Frames**

$$E_a{}^\mu = (\sinh\eta)\,\delta_\mu^a\,,$$
$$\mathcal{E}^a{}_\mu = \rho E^a{}_\mu\,.$$

- **Dilaton**

$$\rho^2 = |\sinh\eta|^3$$

APPLICATION TO THE FLRW BACKGROUND (2)

- **Dirac equation**

$$\gamma^\mu \widehat{D}_\mu \Psi + m\Psi + \frac{3}{4}\frac{\tau_\mu}{\rho^2 R^2}\gamma^\mu \Psi = 0,$$

where $\tau = a(t)\partial_t$ is the $SO(3,1)$-invariant cosmic timelike vector field which is responsible for the breaking of the local Lorentz invariance

- **The analysis of the propagation of fermions is greatly simplified as**

$$\mathcal{K}_{[\alpha\beta\gamma]} = 0$$

SUMMARY (1)

- We have examined the evolution of a Dirac particle on a generic curved 3+1-dimensional background brane within the IKKT matrix model

- The fermionic action resembles the one given in Einstein-Cartan theory and differs from the one given in general relativity only through a coupling to the totally antisymmetric part of the Weitzenböck contorsion

- Both the spin precession and the translation motion assume the same form as in Einstein-Cartan theory

- We have considered the particular case of the FLRW cosmological background

SUMMARY (2)

- **Further details can be found in:**

Emmanuele Battista and Harold C. Steinacker, "*Fermions on curved backgrounds of matrix models*", *Phys. Rev. D* ***107****, 046021* (2023), *arXiv*: 2212.08611

...and the best is yet to come!