Programme on
"Modern Maximal Monotone Operator Theory: From Nonsmooth Optimization to Differential Inclusions"
January 28 – March 8, 2019

organized by

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Workshop 1 on
"Nonsmooth and Variational Analysis"
January 28 – February 1, 2019

Abstracts

Samir Adly (University of Limoges)

Nonconvex degenerate sweeping process

Abstract: The main concern of this talk is the study of degenerate sweeping process involving uniform prox-regular sets via an unconstrained differential inclusion by showing that the sets of solutions of the two problems coincide. This principle of reduction to unconstrained evolution problem can be seen as a penalization of the subdifferential of the distance function. Using this reduction technique, an existence and uniqueness result of a Lipschitz perturbed version of the degenerate sweeping process is proved in the finite dimensional setting. An application is given to quasistatic unilateral dynamics in nonsmooth mechanics where the moving set is described by a finite number of inequalities. We provide sufficient verifiable conditions ensuring both the prox-regularity and the Lipschitz continuity with respect to the Hausdorff distance of the moving set.

Didier Aussel (University of Perpignan)

New existence results for quasivariational inequalities on product spaces: why, what and how?

Abstract: Our aim in this talk is to present some new existence results for quasivariational inequalities defined on products of Banach spaces. We will first explain why to consider such problems. The main motivation comes from macroeconomics. Then we will bring to the fore that the "classical" existence results for quasivariational inequalities cannot apply in this case. Finally we will develop new existence results and the associated new concept of "net" lower-sign continuity.

Vincenzo Basco (University of Rome Tor Vergata)

Necessary conditions for infinite horizon optimal control problems under state constraints and Hamilton-Jacobi-Bellman equations

Abstract: In this talk I will discuss sufficient conditions for Lipschitz regularity of the value function for an infinite horizon optimal control problem subject to state constraints. I focus on problems with cost functional admitting a discount rate factor and allow time dependent dynamics and Lagrangian. Furthermore, state constraints may be unbounded and may have a nonsmooth boundary. Lipschitz regularity is recovered as a consequence of estimates on the distance of a given trajectory of control system from the set of all its viable (feasible) trajectories, provided the discount rate is sufficiently large (cf. [2]). I will talk about first order necessary optimality conditions: a constrained maximum principle and sensitivity relations involving generalized gradients of the value function (cf. [1]). Finally, I will address nonautonomous Hamilton-Jacobi-Bellman equations, with time-measurable data, and their weak solutions: an existence and uniqueness result for solutions to the H-J-B equation associated with an infinite horizon control problem is discussed (cf. [3]).

References

[1] V. Basco, P. Cannarsa, H. Frankowska, *Necessary conditions for infinite horizon optimal control problems with state constraints*, Mathematical Control & Related Fields, 8:535–555, 2018.

[2] V. Basco, H. Frankowska, *Lipschitz continuity of the value function for the infinite horizon optimal control problem under state constraints*, to appear.

[3] V. Basco, H. Frankowska, *Hamilton-Jacobi-Bellman equations with time-measurable data and infinite horizon*, submitted for publication.

Terence Bayen (University of Montpellier)

About the minimal time crisis problem

Abstract: This presentation is about the so-called minimal time crisis problem, which amounts to minimize the total time spent by trajectories of a controlled system outside a given subset K. Such problem arises in the context of viability theory and is of particular interest when the initial condition is not in the viability kernel, when K is not a viability domain or when it is a repeller. We will first give first order optimality conditions that rely on a transverse assumption on optimal trajectories at each crossing time of the set K. We will also study a regularization scheme (based on Moreau-Yosida approximation) that does not require this transverse assumption. Finally, we give an example showing that the optimal strategy of the time crisis problem can be better than the minimum time strategy to reach the viability kernel.

References

[1] J.-P. Aubin, *Viability Theory*, Birkhäuser, 1991.

[2] T. Bayen, A. Rapaport, *About Moreau-Yosida regularization of the minimal time crisis problem*, Journal of Convex Analysis 23:263–290, 2016.

[3] T. Bayen, A. Rapaport, *Minimal time crisis versus minimum time to reach a viability kernel: a case study in the prey-predator model*, Optimal Control and Applications Methods, DOI: 10.1002/oca.2484.

[4] L. Doyen, P. Saint-Pierre, *Scale of viability and minimal time of crisis*, Set-Valued Analysis 5:227–246, 1997.

Ewa Bednarczuk (Warsaw University of Technology)

On dynamical system related to a class of proximal primal-dual best approximation method

Abstract: We derive first order projected dynamical system (PDS) which is the limit of a proximal primal-dual (PP-D) best approximation method for solving convex optimization problems. This (PDS) exhibits an approximate equivalence to (PP-D) best approximation method and can be used as a tool for the analysis. An important element of the approach presented is the investigation of sufficient conditions for local Lipschitzness of projections onto moving family of sets. In this investigation we exhibit the importance of the relaxed constant rank constraint qualification introduced recently by Minchenko and Stakhovski.

The results are obtained together with Krzysztof Rutkowski.

Piernicola Bettiol (University of Western Brittany)

A necessary condition in the calculus of variations and some applications

Abstract: We consider the classical problem of the Calculus of Variations, and we establish the validity of a new Weierstrass-type condition for a $W^{1,m}$ local minimizer of the reference problem. This necessary condition allows to derive important properties of minimizers for a broad class of problems involving a non-autonomous Lagrangian. A first consequence is the Erdmann-Du Bois-Reymond necessary condition expressed in terms of classical tools of convex analysis (e.g., Dini derivatives or convex subdifferentials), and in terms of limiting subdifferentials. If the Lagrangian satisfies an additional growth condition (less restrictive than the classical coercivity or superlinearity), this Weierstrass-type condition yields also the Lipschitz regularity of the minimizers.

Joint work with Carlo Mariconda.

Orestes Bueno (University of the Pacific)

Remarks on p-cyclically monotone operators

Abstract: In this talk, we deal with some aspects of p-cyclically monotone operators. First, we introduce a notion of monotone polar adapted for p-cyclically monotone operators, and use it to construct maximal extensions of certain finite p-cyclically monotone operators.

We also deal with linear operators and provide characterizations of p-cyclic monotonicity and maximal p-cyclic monotonicity. Finally we show that the Brézis-Browder theorem preserves p-cyclic monotonicity in reflexive Banach spaces.

Aris Daniilidis (University of Chile)

Linear structure of Lipschitz functions that saturate their Clarke subdifferential

Abstract: A real-valued Lipschitz continuous function is said to saturate its Clarke subdifferential, whenever this latter is maximal at every point (namely, equal to kB^*, where k is the Lipschitz constant of the function). In this

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work, we show that the set of Clarke-saturated functions, defined on an open convex domain of a finite dimensional space, contains a linear subspace of uncountable dimension. This result goes in the line of a previous result by J. Borwein and X. Wang that establishes genericity of k-saturated, in the (complete) metric space of k-Lipschitz functions under the topology of the uniform convergence. Contrary to that result, our approach is constructive and does not depend on the uniform convergence.

Joint work with Gonzalo Flores (University of Chile).

Asen Dontchev (University of Michigan)

Uniform strong regularity, Newton's method, and model predictive control

Abstract: We consider a finite-horizon continuous-time optimal control problem with nonlinear dynamics, an integral cost, control constraints and a parameter which represents uncertainty. After a discretization of the problem, we employ a model predictive control (MPC) algorithm to obtain a piecewise constant function in time as control. We consider the question under what conditions the MPC-generated control is a good approximation of an optimal feedback control of the continuous-time system. An answer to that question is found which is based on the concept of uniform strong regularity. It allows us to establish uniform Lipschitz stability of the discretized problem, uniform convergence of the Newton method employed, as well as an estimate of the difference between the MPC-generated control and the optimal feedback control.

Hélène Frankowska (Pierre and Marie Curie University of Paris)

Domain invariance for local solutions of semilinear evolution equations in Hilbert spaces

Abstract: A closed set K of a Hilbert space H is said to be invariant under the evolution equation

$$X'(t) = AX(t) + f\big(t, X(t)\big) \qquad (t > 0)$$

whenever all solutions starting from a point of K, at any time $t_0 \geqslant 0$, remain in K as long as they exist. For a self-adjoint strictly dissipative operator A, perturbed by a (possibly unbounded) nonlinear term f, we give necessary and sufficient conditions for the invariance of K, formulated in terms of A, f, and the distance function from K. Then, we also give sufficient conditions for the viability of K under a semilinear control system

$$X'(t) = AX(t) + f\big(t, X(t), u(t)\big) \qquad (t > 0, \ u(t) \in U).$$

In collaboration with P. Cannarsa and G. Da Prato.

Rafal Goebel (Loyola University Chicago)

Lyapunov-like tools for pointwise asymptotic stability of a set of equilibria

Abstract: Steepest descent for a convex function, and related dynamics in continuous time and discrete time, often render each equilibrium Lyapunov stable and every solution convergent to some equilibrium. This stability and attractivity property of the set of equilibria, referred to as pointwise asymptotic stability or semistability, is the topic of the talk. Examples, including some of recent interest in the control engineering literature, and Lyapunov-like

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techniques for the analysis of pointwise asymptotic stability and of its robustness to perturbations will be presented.

René Henrion (Weierstrass Institute Berlin)

Robust control of a sweeping process with probabilistic end-point constraints

Abstract: We consider a sweeping process defined by the controlled movement of a halfspace. The process is constrained by an end-point condition, requiring that the trajectory finally hits a given target set. We shall assume that our control is subjected to random perturbations, so that the end-point becomes random too. Therefore, it is reasonable to introduce a probabilistic end-point constraint which makes the optimal control robust in the sense of guaranteeing that the resulting random trajectories reach the target set with high probability. Alternatively, we will be looking for the most robust control, i.e., the one which independent of any costs yields the largest probability of finally hitting the target. Existence results and algorithmic approaches will be discussed.

Michael Hintermüller (Weierstrass Institute Berlin)

A function space framework for structural total variation regularization with applications in inverse problems

Abstract: We introduce a function space setting for a wide class of structural/weighted total variation (TV) regularization methods motivated by their applications in inverse problems. In particular, we consider a regularizer that is the appropriate lower semi-continuous envelope (relaxation) of a suitable total variation type functional initially defined for sufficiently smooth functions. We study examples where this relaxation can be expressed explicitly, and we also provide refinements for weighted total variation for a wide range of weights. Since an integral characterization of the relaxation in function space is, in general, not always available, we show that, for a rather general linear inverse problems setting, instead of the classical Tikhonov regularization problem, one can equivalently solve a saddle-point problem where no a priori knowledge of an explicit formulation of the structural-TV functional is needed. In particular, motivated by concrete applications, we deduce corresponding results for linear inverse problems with norm and Poisson log-likelihood data discrepancy terms. Finally, we provide proof-of-concept numerical examples where we solve the saddle-point problem for weighted TV-denoising as well as for MR guided PET image reconstruction.

Josef Hofbauer (University of Vienna)

Monotone operators and ODEs - some examples

Abstract: I plan to present some examples of ODEs related to monotone operator theory, from game theory and biomathematics.

Diethard Klatte (University of Zurich)

On extensions of Newton's method for equations and inclusions

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Abstract: We present approaches to extensions of Newton's method in the framework of generalized equations

$$0 \in f(x) + M(x),$$

where f is a function and M is a multifunction. The Newton steps are defined by suitable approximations g of f and the solutions of $0 \in g(x) + M(x)$. We analyze superlinear local convergence analysis of such methods, in particular we extend convergence results via Newton maps from equations to generalized equations both for linear and nonlinear approximations g. Moreover, we present relations between semi-smoothness, Newton maps and directional differentiability of f.

The talk is based on joint work with Bernd Kummer (Humboldt University of Berlin).

References

[1] D. Klatte, B. Kummer, *Approximations and generalized Newton methods*, Mathematical Programming 168:673–716, 2018.

Wojciech Kryszewski (Nicolaus Copernicus University Torun)

Semigroup and resolvent invariance of convex sets

Abstract: When studying on $\Omega \subset \mathbb{R}^N$ the existence of solutions to elliptic problems of the form $Au = f(u)$, or the corresponding parabolic problems of the form $u_t + Au = f(u)$, where A is an elliptic (in general vector valued) differential operator and $f : \mathbb{R}^M \to \mathbb{R}^M$ is a continuous map, such that $u(x) \in K$ for a.a. $x \in \Omega$, where $K \subset \mathbb{R}^M$ is a given closed convex set of state constraints, an important hypothesis concern the so-called resolvent invariance of K, i.e. $(I + \lambda A)^{-1}(\mathbf{K}) \subset \mathbf{K}$ for sufficiently small $\lambda > 0$, where $\mathbf{K} := \{u \in L^2(\Omega, \mathbb{R}^M) \mid u(x) \in K, \text{for a.a. } x \in \Omega\}$. We will discuss the sufficient and necessary conditions for the invariance stated in terms of the coefficients of A. This topic is strictly related to the study of the viability and invariance questions for partial differential equations.

Szilárd László (Technical University of Cluj-Napoca)

A second order dynamical approach with variable coefficients to nonconvex smooth minimization

Abstract: We investigate a second order dynamical system with variable coefficients in connection with the minimization of a nonconvex differentiable function. The dynamical system is formulated in the spirit of the differential equation which models Nesterov's accelerated convex gradient method. We show that the generated trajectory converges to a critical point, if a regularization of the objective function satisfies the Kurdyka-Łojasiewicz property. We also provide convergence rates for the trajectory formulated in terms of the Łojasiewicz exponent.

Shu Lu (University of North Carolina at Chapel Hill)

Statistical inference for piecewise normal distributions and application to stochastic variational inequalities

Abstract: Confidence intervals for the mean of a normal distribution with a known covariance matrix can be computed using closed-form formulas. In this talk, we consider a distribution that is the image of a normal distribution

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under a piecewise linear function, and provide a formula for computing confidence intervals for the mean of that distribution given a sample point under certain conditions. We then apply this method to compute confidence intervals for the true solution of a stochastic variational inequality. This method is based on a closed-form formula and the computation is very easy.

D. Russell Luke (University of Göttingen)

Variational analysis of random function iterations

Abstract: We recently proposed a framework for the analysis of fixed point algorithms encountered in nonsmooth and nonconvex optimization in [1]. We present the extension of this framework to the analysis of iterated random functions for stochastic fixed point problems in several different settings, under decreasingly restrictive regularity assumptions of the fixed point mappings. We contrast consistent and inconsistent stochastic feasibility problems, focusing mostly on the inconsistent case. Convergence of such random function iterations (when such a concept can be reasonably defined) is most appropriately in terms of measures. Existence of invariant measures is established for continuous Markov operators associated with fixed point iterations inspired by first-order proximal methods in optimization (e.g. the Douglas-Rachford, and proximal-point algorithms). Convergence of sequences of iterated random functions is established for ergodic nonexpansive mappings and nonergodic sequences under the assumption that the mappings are averaged.

References

[1] D.R. Luke, N.H. Thao, M.K. Tam, *Quantitative convergence analysis of iterated expansive, set-valued mappings*, Mathematics of Operations Research 43:1143–1176, 2018.

Elsa Maria Marchini (Polytechnic University of Milan)

Second order necessary conditions for infinite dimensional control problems

Abstract: We derive second order necessary optimality conditions for a Mayer type optimal control problem with state and final-point constraints. We use a separation theorem, and our conditions do not need the validity of maximum principles in normal form, allowing quite general assumptions on the data. The setting is infinite dimensional, we work with abstract semigroups on a separable Banach space. Our results apply to control problems involving PDEs.

Juan-Enrique Martínez-Legaz (Autonomous University of Barcelona)

On the structure of higher order Voronoi cells

Abstract: The classic Voronoi cells can be generalized to a higher-order version by considering the cells of points for which a given k-element subset of the set of sites consists of the k closest sites. We study the structure of the k-order Voronoi cells and illustrate our theoretical findings with a case study of two-dimensional higher-order Voronoi cells for four points.

Joint work with Vera Roshchina and Maxim Todorov.

Boris Mordukhovich (Wayne State University)

Criticality of Lagrange multipliers in conic programming with applications to superlinear convergence of SQP

Abstract: This talk concerns the study of criticality of Lagrange multipliers in variational systems that have been recognized in both theoretical and numerical aspects of optimization and variational analysis. In contrast to the previous developments dealing with polyhedral KKT systems and the like, we now focus on general nonpolyhedral systems that are associated, in particular, with problems of conic programming. Developing a novel approach, which is mainly based on advanced techniques and tools of second-order variational analysis and generalized differentiation, allows us to overcome principal challenges of nonpolyhedrality and to establish complete characterizations on noncritical multipliers in such settings. We present applications of noncritical multipliers to deriving efficient conditions of the sequential quadratic programming method for conic programs.

Based on joint work with Ebrahim Sarabi (Miami University, Oxford, OH, USA).

Teemu Pennanen (King's College London)

Convex duality in nonlinear optimal transport

Abstract: We study problems of optimal transport, by embedding them in a general functional analytic framework of convex optimization. This provides a unified treatment of a large class of related problems in probability theory and allows for generalizations of the classical problem formulations. General results on convex duality yield dual problems and optimality conditions for these problems. When the objective takes the form of a convex integral functional, we obtain more explicit optimality conditions and establish the existence of solutions for a relaxed formulation of the problem. This covers, in particular, the mass transportation problem and its nonlinear generalizations.

Adrian Petruşel (Babeş-Bolyai University Cluj-Napoca)

Variational analysis concepts in coincidence point theory

Abstract: In this talk, we will present some coincidence point results in metric structures using some variational analysis concepts. Existence and stability theorems will be presented and some open questions are discussed.

Georg Pflug (University of Vienna)

Stochastic constrained optimization in Hilbert spaces with applications

Abstract: We consider a constrained stochastic optimization problem in a Hilbert space and study the behavior of a projected gardient method with line search. The constraint set is assumed to be convex. We distinguish the three cases: (1) the objective function is strictly convex; (2) the objective function is convex, but not strictly; (3)

the objective function is nonconvex. The behavior of the projected stochastic gradient algorithm is different in the three cases. We discuss convergence speed in cases (1) and (2) and weak convergence to a stationary point in case (3). As an application, we apply the algorithm to the optimization of shapes in random environments or under random forces.

This is a joint work with Caroline Geiersbach.

Marc Quincampoix (University of Western Brittany)

On a multiagent Bolza control problem

Abstract: This concerns a class of optimal control problems, where a central planner aims to control a multi-agent system in order to minimize a certain cost of Bolza type. At every time and for each agent, the set of admissible velocities, describing his underlying microscopic dynamics, depends both on his position, and on the configuration of all the other agents at the same time. So the problem is naturally stated in the space of probability measures on the state space equipped with the Wasserstein distance. The problem consists in finding a well-adapted notion of subdifferentialin the Wasserstein space in order to characterize the value-function as the unique viscosity solution of a suitable Hamilton-Jacobi-Bellman Equation.

Franco Rampazzo (University of Padova)

Higher order necessary conditions for impulsive optimal control problems: a set-separation approach

Abstract: We will discuss necessary conditions for a minimum of a Mayer optimal control connected with a non-linear, control-affine, system, where the controls range on an m-dimensional Euclidean space. Let us stress that several mechanical systems with mobile constraints as controls happen to have this form. Since the allowed velocities are unbounded and no coercivity assumptions make big speeds unfavorable (as instead it happens in the classical Calculus of Variations), minimizing sequences may converge toward "impulsive", namely discontinuous, trajectories. While an approach in the sense of distributions is ruled out by the nonlinearity of the system, a robust definition of solution is recovered by embedding the system in the graph space for both controls and states. We will illustrate how the chance of using unbounded velocity perturbations makes it possible to derive –within this setting and by the use of set separation–, a Maximum Principle which includes both the usual needle variations and higher order conditions involving Lie brackets.

Carlos N. Rautenberg (Humboldt University of Berlin)

Parabolic quasi-variational inequalities with gradient and obstacle type constraints

Abstract: We consider quasi-variational inequalities (QVIs), with pointwise constraints on function values and its gradient, arising in recent mathematical models from physics and biology. A semi-discretization in time is employed for the study of the problem class and the derivation of a solution algorithm. We prove convergence of the discretization scheme, and establish existence, regularity, and the non-decreasing property of the solution. The talk is finalized with a report on numerical tests obtained involving different nonlinearities and types of constraints.

R. Tyrrell Rockafellar (University of Washington)

Decomposition algorithms for generalized equation problems with elicitable monotonicity

Abstract: Finding a zero of a sum of mappings, single-valued or set-valued, is a major target for designing algorithms that break computations down to relatively easy subproblems. Other problem structures can likewise motivate such approaches. Much of the work on this topic has focused on mappings that are maximal monotone globally. However, a newly developed procedure called the progressive decoupling algorithm can exhibit local convergence even in situations where maximal monotonicity is not available directly but is locally elicitable around a solution. Examples with a background in optimization include nonconvex minimization problems where augmented Lagrangians can produce saddle point characterizations of local primal-dual solution pairs.

Jose-Francisco Rodrigues (University of Lisbon)

On a class of nonlocal variational and quasivariational inequalities

Abstract: We extend classical results on variational inequalities with convex sets with gradient constraint to a new class of fractional partial differential equations with constraint on the distributional Riesz fractional gradient, the s-gradient ($0 < s < 1$). We prove the approximation of the solution with a family of quasilinear penalisation problems and we establish continuous dependence results on the data, including the threshold of the fractional s-gradient. Using these properties we give new results on the existence of a class of quasivariational variational inequalities of fractional gradient constraint type via compactness and via contraction arguments.

This is a joint work with Lisa Santos.

Stephen Simons (University of California Santa Barbara)

Quasidense multifunctions

Abstract: "Quasidensity" is a concept that can be applied to subsets of the product of a (nonzero real) Banach space and its dual space. Every closed quasidense monotone set is maximally monotone, but there exist maximally monotone sets that are not quasidense. The graph of the subdifferential of a proper, convex lower semicontinuous function on a Banach space is quasidense. The graphs of certain subdifferentials of certain nonconvex functions are also quasidense. (This follows from joint work with Xianfu Wang.) The closed monotone quasidense sets have a number of very desirable properties, including a sum theorem and a parallel sum theorem, and so quasidensity satisfies the ideal calculus rules. We know of ten conditions equivalent to the statement that a closed monotone set be quasidense, but quasidensity seems to be the only one of the ten that extends easily to nonmonotone sets. We give examples in general Banach spaces, Hilbert spaces and finite dimensional spaces.

Ulisse Stefanelli (University of Vienna)

The WIDE variational principle for nonlinear evolution

Abstract: The weighted inertia-energy-dissipation (WIDE) principle consists in a global-in-time variational ap-

proach to a large variety nonlinear evolution equations. The problems that are have been presently tackled by this method include gradient flows, also in metric spaces, doubly nonlinear and rate-independent evolutions, and nonlinear waves. The idea is to reformulate the differential systems as convex minimization problems in order to export the machinery of the Calculus of Variations (direct method, gamma-convergence, relaxation,...) to evolutionary situations. I will present the basics of this methodology and comment on recent applications to the incompressible Navier-Stokes system and to some dynamical mechanical problems.

Michel Théra (University of Limoges)

Metric regularity and directional metric regularity of multifunctions

Abstract: My aim in this talk is to make an overview on metric regularity, a concept which plays an important role in optimization and related topics and has attracted over the recent years a large number of contributions. I intend to give some recent advances on directional metric regularity.

Fredi Tröltzsch (Technical University of Berlin)

Sparse optimal control for a semilinear heat equation with mixed control-state constraints

Abstract: A problem of sparse optimal control for a semilinear parabolic equations is considered, where pointwise bounds on the control and mixed pointwise control-state constraints are given. A quadratic objective functional is to be minimized that includes a Tikhonov regularization term and the L^1-norm of the control accounting for the sparsity. Special emphasis is laid on existence and regularity of Lagrange multipliers for the mixed control-state constraints. To this aim, the duality theory for linear programming problems in Hilbert spaces is applied. Associated necessary optimality conditions are established and discussed up to the sparsity of optimal controls.

This is joint work with Eduardo Casas.

André Uschmajew (Max Planck Institute for Mathematics in the Sciences Leipzig)

Critical points of quadratic low-rank optimization problems

Abstract: The absence of spurious local minima in certain non-convex minimization problems, e.g. in the context of recovery problems in compressed sensing, has recently triggered much interest due to its important implications on the global convergence of optimization algorithms. One example is low-rank matrix sensing under rank restricted isometry properties. It can be formulated as a minimization problem for a quadratic cost function constrained to a low-rank matrix manifold, with a positive semidefinite Hessian acting like a perturbation of identity on cones of low-rank matrices. We present an approach to show strict saddle point properties and absence of spurious local minima for such problems under improved conditions on the restricted isometry constants.

This is joint work with Bart Vandereycken.

Vladimir Veliov (Vienna University of Technology)

Metric regularity properties of the optimality mapping for ODE optimal control

Abstract: The talk will begin with a discussion of the metric regularity of the differential variational inequality representing the first order optimality conditions for coercive ODE optimal control problems. Some new results will be presented, leading, in particular to conditions for existence of a Lipschitz continuous optimal feedback control law and a certain smoothness of the associated value function. Then some recent results on metric sub-regularity and bi-metric regularity for (non-coercive) control-affine problems will be presented. These results provide the basis for error analysis of a new numerical approach, which will be briefly discussed.

Richard Vinter (Imperial College London)

Multifunctions of bounded variation and control theory applications

Abstract: Several possible generalizations of the classical theory of bounded variation functions in a single variable have been investigated. One direction of research has been to provide concepts of bounded variation functions in several variables, principally with a view to describing regularity properties of solutions to PDE's. We report on another, new, direction, concerning the properties of differential inclusions, when the multifunction involved depends on the time and the state, and has, in some sense, bounded variation in time. Special attention is given to differential inclusions when they are used to model the admissible velocity set of a control system, assumed to have bounded variation in time. Velocity sets of this kind are a natural feature of hybrid systems. Necessary conditions in optimal control can be derived for velocity sets whose time dependence is merely measurable. But when it is know that the velocity sets are Lipschitz continuous with respect to time, extra conditions pertain. We show that many of these extra conditions have analogues, when the time dependence of the velocity set is merely of bounded variation. These included conditions asserting regularity properties of minimizing state trajectories, or of the Hamiltonian evaluated along such trajectories. We discuss also the implications of bounded variation time dependence for control system sensitivity.

Xiao-qi Yang (The Hong Kong Polytechnic University)

On error bound moduli for locally Lipschitz and regular functions

Abstract: In this paper we study local error bound moduli for a locally Lipschitz and regular function via outer limiting subdifferential sets. We show that the distance from 0 to the outer limiting subdifferential of the support function of the subdifferential set, which is essentially the distance from 0 to the end set of the subdifferential set, is an upper estimate of the local error bound modulus. This upper estimate becomes tight for a convex function under some regularity conditions. We show that the distance from 0 to the outer limiting subdifferential set of a lower $\mathcal{C}^1$ function is equal to the local error bound modulus. We will also discuss the stable local error bound modulus.

Constantin Zălinescu (Alexandru Ioan Cuza University of Iaşi)

On Lagrange multipliers in convex entropy minimization

Abstract: As mentioned in [2], "entropy optimization, used for recovering a probability distribution from information on a few of its moments, is well established and ubiquitous throughout the sciences".

Generally, the Lagrange multipliers method is used in a formal way to get the optimal solutions. As pointed in [1], this approach has serious drawbacks.

Based on a characterization of the optimality of a feasible solution of a convex entropy minimization problem, we show that the feasible solutions obtained using formally the Lagrange multipliers method are indeed optimal.

References

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[2] J.M. Borwein, R. Choksi and P. Maréchal, *Probability distributions of assets inferred from option prices via the principle of maximum entropy*, SIAM Journal on Optimization 14:464–478, 2003.