Advection-Diffusion-Reaction PDE

Let $\mathbf{q}: \Omega \times D \to \mathbb{R}^d$ be the speed of advection, $\kappa: \Omega \times D \to \mathbb{R}$ be a diffusion coefficient.

We search for a solution $u{:}\,\Omega imes D imes [0,T] o \mathbb{R}$, such that

$$egin{aligned} \partial_t u + \mathbf{q} \cdot
abla u - \operatorname{div}(\kappa
abla u) = r(u), & \mathbf{x} \in D, \quad t \in (0,T] \ u(\mathbf{x},0) = f(\mathbf{x}), & \mathbf{x} \in D \ u(\mathbf{x},t) = g(\mathbf{x},t), & \mathbf{x} \in \partial D \end{aligned}$$

Examples Qol

- $\mathrm{Q}(u) = \int_{ ilde{D}} u(t) dx$
- $\mathrm{Q}(u) = \int_{\Gamma_{out}} u(t) \mathbf{q} \cdot n dx$

Motivation

- Search for a model problem with pathwise instability and apply idea from graham2019full.pdf
- MIMC for h and homotopy parameter like artificial diffusion (or Newton-steps in the nonlinear case; or penalty term for DG discretization)

 \rightarrow Problem: in all cases parameters are coupled with h

Benchmark problem for <u>UM-Bridge</u>?
 → First step: Only use containerized deterministic part of Mpp

Other ideas

• MIMCMC: Truncation of time series data and corresponding simulation time as one MI dimension?

Example configuration

- Deterministic: $\kappa\equiv 0.01$, $r(u)\equiv 0$
- Random field: $\mathbf{q} : \Omega imes D o \mathbb{R}^d$ as a random Darcy flux

Sample Computations







Instability in convection dominant case - 0.00001



Discretization

Search for $u_h(t) \in V_h$

$$(M_h\partial_t u_h + A_h u_h, v_h)_\Omega = (R(u_h), v_h)_\Omega \quad orall v_h \in ilde V_h$$

with artificial diffusion $ilde{V}_h = \{v_h + \delta_K \mathbf{q} \cdot
abla v_h, \, v_h \in V_h\}$ via Petrov-Galerkin

alternatively Penalized DG-discretization

Benchmarks

Current state:

- Some forward models and Bayesian posteriors already defined
- Portability / reproducibility through containers and automated testing
- Online documentation

Next steps:

- Add more benchmark problems to cover other aspects of UQ (forward problems, model type, dimensionality, run time, smoothness, ...) → Decisions by community
- Publish benchmark suite (maybe combined with method review?)
- Add PyMC3 integration for UM-Bridge