Seminar

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Structural stability of quasilinear flows via evolutionary Γ-convergence

Wednesday, June 29, 2016

at 14:00 h

ESI, Boltzmann Lecture Hall

Abstract: After the seminal works of Brezis–Ekeland [BE], Nayroles [N] and Fitzpatrick [F], for any maximal monotone operator $\alpha : V \to \mathcal{P}(V')$ (V being a Banach space) the flow

$$D_t u + \alpha(u) \ni h \qquad \text{in } V', \text{ a.e. in }]0,T[, \qquad u(0) = u^0 \tag{1}$$

may be given a variational representation, even if α is not a cyclically monotone. This leads one to formulate a so-called *null-minimization problem.*

This talk deals with the stability of this problem w.r.t. arbitrary perturbations not only of the data h, u^0 but also of operator α. This is achieved by

(i) noticing that the operator $D_t + \alpha$ is maximal monotone in a space of functions $]0,T[\to V$,

(ii) introducing a nonlinear topology in the space $L^2(0,T;V \times V')$,

(iii) defining a notion of *evolutionary* Γ*-convergence* w.r.t. that topology (this extends the classical Γ-convergence of De Giorgi).

These results are applied to (1), and to doubly-nonlinear parabolic flows of the form

$$D_t\gamma(u) + \alpha(u) \ni h \qquad \text{in } V', \qquad u(0) = u^0, \tag{2}$$

γ being a cyclical maximal monotone operator $V \to \mathcal{P}(V')$.

Details may be found in [V] and in a paper in preparation.

References

[BE] H. Brezis, I. Ekeland: *Un principe variationnel associé à certaines équations paraboliques. I. Le cas indépendant du temps* and *II. Le cas dépendant du temps.* C. R. Acad. Sci. Paris Sér. A-B **282** (1976) 971–974, and ibid. 1197–1198

[F] S. Fitzpatrick: *Representing monotone operators by convex functions.* Workshop/Miniconference on Functional Analysis and Optimization (Canberra, 1988), 59–65, Proc. Centre Math. Anal. Austral. Nat. Univ., 20, Austral. Nat. Univ., Canberra, 1988

[N] B. Nayroles: *Deux théorèmes de minimum pour certains systèmes dissipatifs.* C. R. Acad. Sci. Paris Sér. A-B **282** (1976) A1035–A1038

[V] A. Visintin: *Variational formulation and structural stability of monotone equations.* Calc. Var. Partial Differential Equations **47** (2013) 273–317

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