Erwin Schrödinger International Institute
for Mathematics and Physics
The Landscape versus the Swampland
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F-theory: Landscape of Particle Physics Models

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I. The program:

Globally consistent F-theory compactifications with the gauge symmetry and matter spectrum of the Standard Model

Key building blocks: gauge symmetry; matter spectrum; global conditions

Upenn-centric

II. Status of constructions: via toric geometry techniques

a) First globally consistent three-family Standard Model

[M.C., Klevers, Peña, Oehlmann, Reuter 1503.02068;
M.C., Lin, Liu, Oehlmann 1807.01320]

b) Landscape of three-family Standard Models

[M.C., Jim Halverson, Ling Lin, Muyang Liu, Jiahua Tian,
"A Quadrillion Standard Models from F-theory" 1903.00009, PRL]

III. Further analysis:

Time permitting

a) Constraints on moduli stabilization scenarios

[M.C., Cody Long, J. Halverson,L. Lin 2004.00630]

b) Toward charged vector matter pairs

[Martin Bies, M.C., Ron. Donagi, L. Lin, M. Liu, Fabian Rühle 2007.0009]
[Bies,M.C. Donagi, Liu 2102.10115, 2104.08297]
[Bies,M.C. Donagi, Marielle Ong 2205.00008, 2307.02535]

Outline:

I. F-theory: Geometric Approach

Appearance of non-Abelian gauge symmetry, matter and Yukawa couplings

Appearance of Abelian continuous and discrete symmetries, global constraints

II. Construction of Particle physics Models

Highlight

Building blocks for consistent models via toric techniques

Landscape of three-family Standard Models

III. Further Analysis:

Time permitting

Constraints on moduli stabilization scenarios for Standard Model constructions; toward vector-pair matter calculation

IV. Outlook: work in progress & open issues

F-theory?

Type II String

- back-reacted D-branes
- regions with large g_s on non-CY space

g_s –string coupling

F-theory?

F-theory	=	Type II String
• Coupling g_s part of geometry (12dim)		• back-reacted D-branes
• Torus fibered Calabi-Yau manifold		• regions with large g_s on non-CY space

g_s –string coupling

F-theory?

M-theory (11dim SG)

on S^1

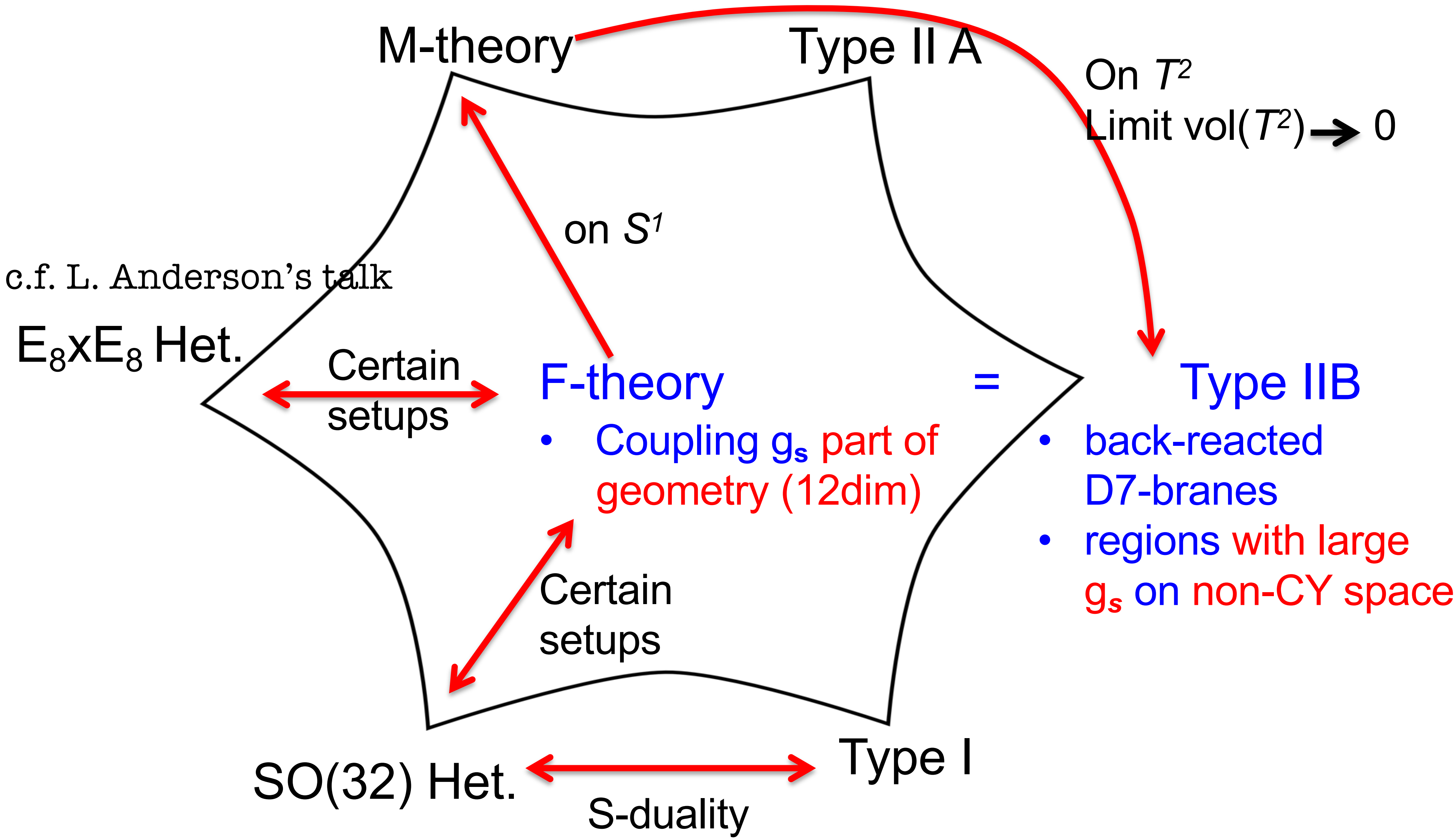
F-theory = Type IIB

- Coupling g_s part of geometry (12dim)

- back-reacted D7-branes
- regions with large g_s on non-CY space

g_s –string coupling

F-theory?


M-theory
Type II A
On T^2
Limit vol(T^2) → 0
on S^1
c.f. L. Anderson's talk
E_8xE_8 Het.
Certain setups
F-theory
• Coupling g_s part of geometry (12dim)
=
Type IIB
• back-reacted D7-branes
• regions with large g_s on non-CY space
Certain setups
SO(32) Het.
Type I
S-duality

I. F-theory basic ingredients

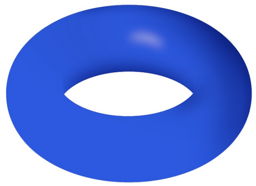
Type IIB string perspective

F-theory compactification

[Vafa'96], [Morrison,Vafa'96]

review [Weigand 1806.01854]

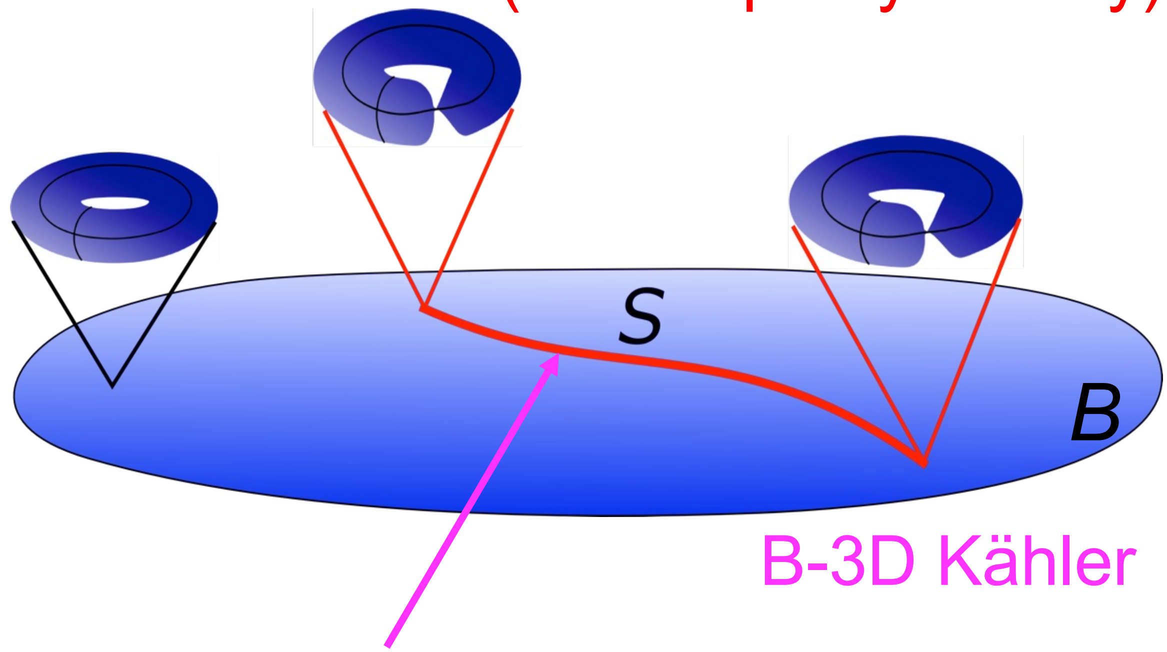
Singular torus fibered Calabi-Yau manifold X (N=1 supersymmetry)

To B add torus: 

Modular parameter of torus (elliptic curve)

$$\tau \equiv C_0 + i g_s^{-1}$$

(SL(2,Z) of Type IIB string)



divisor

Weierstrass normal form for torus (elliptic) fibration of *X*

$$y^2 = x^3 + f x z^4 + g z^6$$

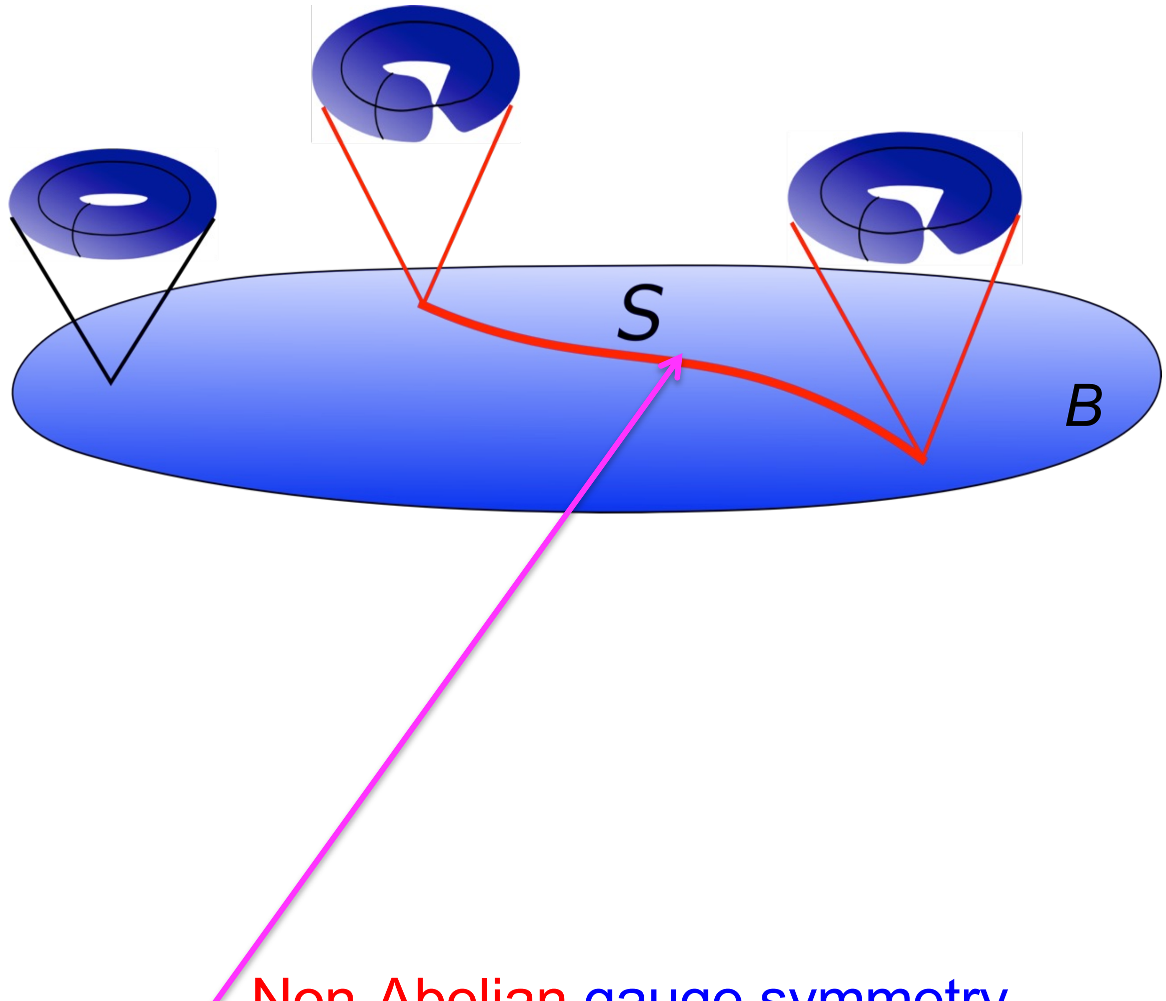
[z:x:y] - homogeneous coordinates on $\mathbf{P}^2(1,2,3)$ $(x,y,z) \simeq (\lambda^2 x, \lambda^3 y, \lambda z)$
weighted projective space

f, g – sections of $\overline{\mathcal{K}}_B{}^6$ and $\overline{\mathcal{K}}_B{}^4$ on ***B*** $\overline{\mathcal{K}}_B$ -anti-canonical bundle on ***B***

F-theory compactification

Singular elliptically fibered Calabi-Yau manifold X

Modular parameter of two-torus
(elliptic curve)
$\tau \equiv C_0 + i g_s^{-1}$



divisor- singular elliptic-fibration
$g_s \rightarrow \infty$ location of (p,q) 7-branes

Non-Abelian gauge symmetry
(co-dim 1) – ADE singularities

I.a Non-Abelian Gauge Symmetry

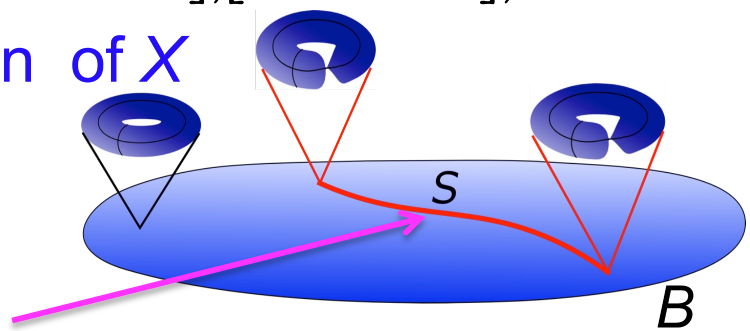
Standard Model has $SU(2)_L$ x $SU(3)_C$

Non-Abelian Gauge Symmetry

[Kodaira],[Tate], [Vafa], [Morrison,Vafa],…[Esole,Yau],
[Hayashi,Lawrie,Schäfer-Nameki],[Morrison], …

- Weierstrass normal form for elliptic fibration of *X*

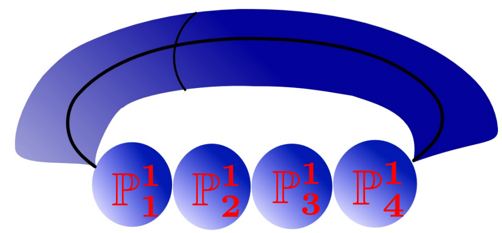
$$y^2 = x^3 + fxz^4 + gz^6$$



- Severity of singularity along divisor S in B
  specified by $[ord_S(f), ord_S(g), ord_S(\Delta)]$

- Resolution: structure of a tree of $\mathbb{P}^1$'s over *S*

  Resolved I_n-singularity ←→ SU(n) Dynkin diagram



Kodaira classification (ADE classification)

Cartan gauge bosons: supported by (1,1) form $\omega_i \leftrightarrow \mathbb{P}^1_i$ on resolved X

(via M-theory Kaluza-Klein reduction of C_3 potential $C_3 \supset A^i \omega_i$)

Non-Abelian gauge bosons: light M2-brane excitations on $\mathbb{P}^1$'s

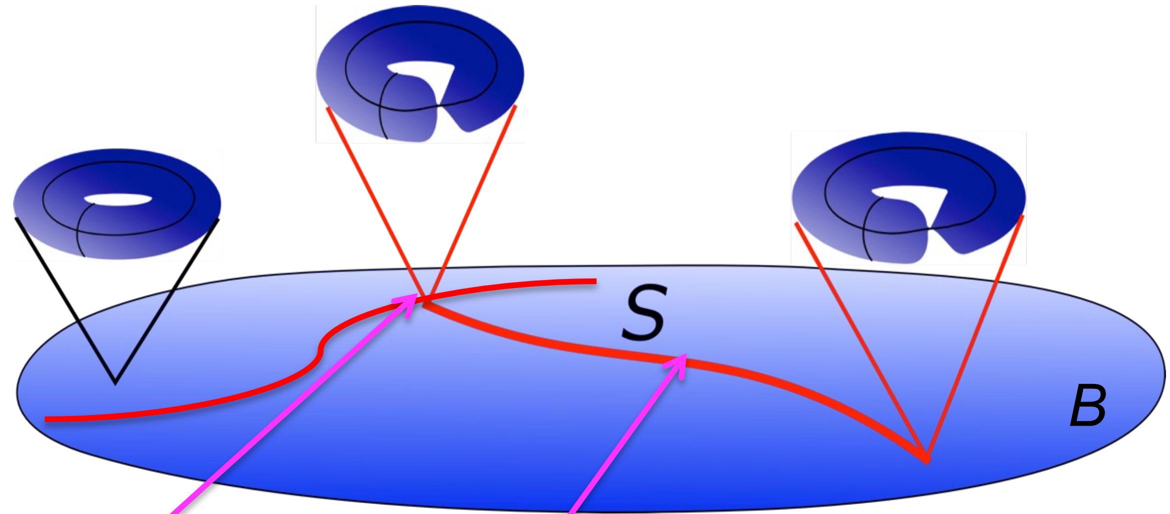
[Witten]

F-theory compactification

Singular elliptically fibered Calabi-Yau manifold X

Modular parameter of two-torus (elliptic curve)

$\tau \equiv C_0 + i g_s^{-1}$



Matter (co-dim 2)

divisor- singular elliptic-fibration
$g_s \rightarrow \infty$ location of (p,q) 7-branes

Non-Abelian gauge symmetry (co-dim 1) – ADE singularities

Abelian symmetries different

→ More later

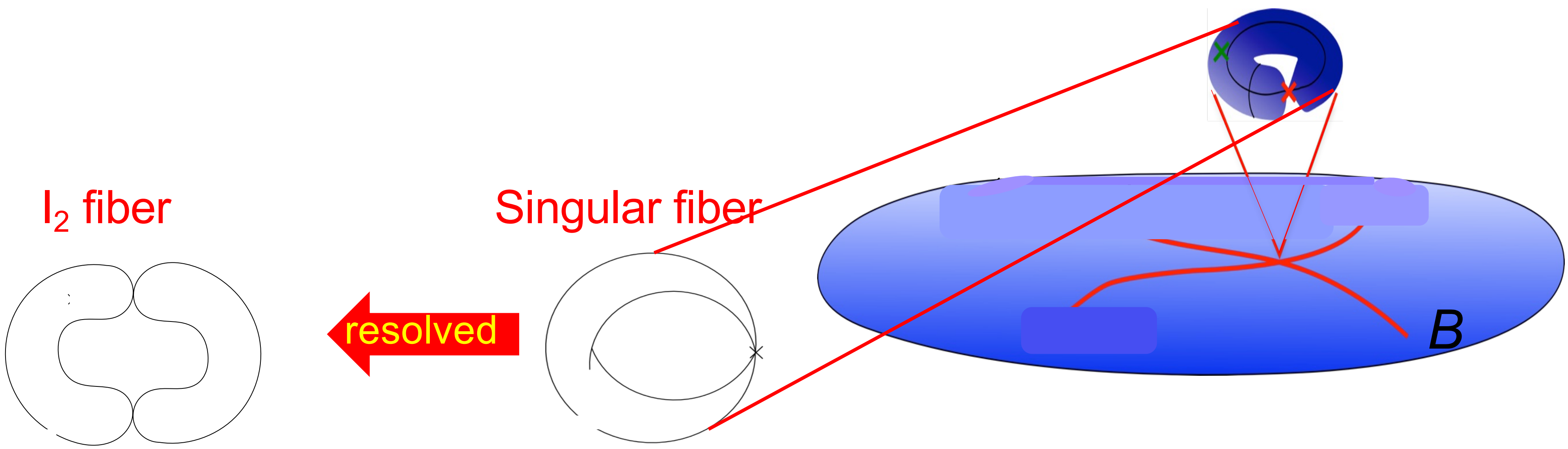
Matter

Standard Model $SU(2)_L \times SU(3)_C$ has

quarks Q ~ (**2** , **3**)

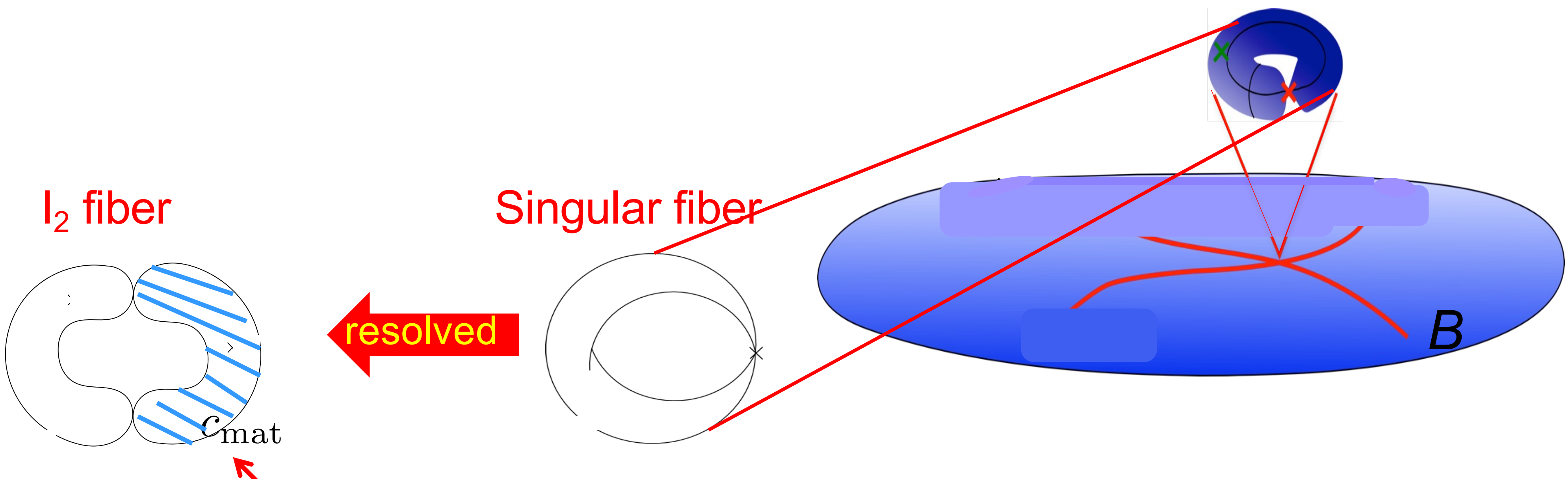
Matter

Singularity at codimension-two in B:



Matter

Singularity at codimension-two in B:



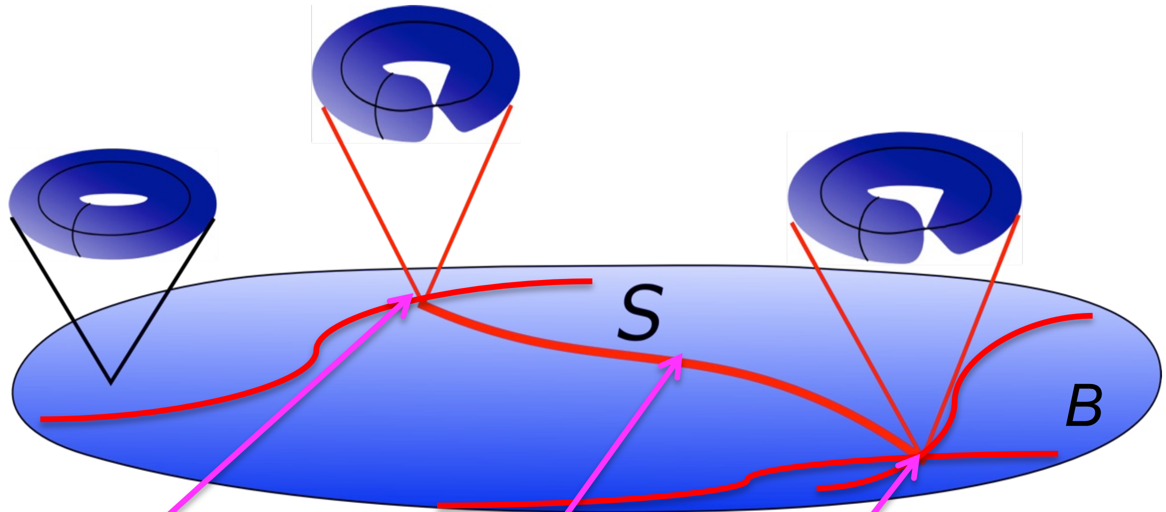
w/isolated (M2-matter) curve wrapping $\mathbb{P}^1$ → charged matter
(determine rep. via intersection theory)

F-theory compactification

Singular elliptically fibered Calabi-Yau manifold X

Modular parameter of two-torus (elliptic curve)

$\tau \equiv C_0 + i g_s^{-1}$



Matter (co-dim 2)

Chirality: Should add G_4-flux → More later

c.f., [Lin, Mayrhofer, Till,Weigand, 1508.00162]
[M.C., Grassi, Klevers, Piragua, 1306.3987]

Yukawa couplings (co-dim 3) → No time

Divisor- singular elliptic-fibration
$g_s \rightarrow \infty$ location of (p,q) 7-branes

Non-Abelian gauge symmetry (co-dim 1) – ADE singularities

Abelian symmetries different → More later

I.b Abelian Gauge Symmetry - U(1)

Standard Model has $SU(2)_L$ x $SU(3)_C$ x $U(1)_Y$

I.b Abelian Gauge Symmetry - U(1)

Different: (1,1) forms ω_m, supporting U(1) gauge bosons, isolated & associated with I_1-fibers, only

[Morrison,Vafa'96]

(1,1) - form ω_m ⟺ rational section of elliptic fibration

Abelian Gauge Symmetry & Mordell-Weil Group

rational sections of elliptic fibr. ⟺ rational points of elliptic curve

Abelian Gauge Symmetry & Mordell-Weil Group

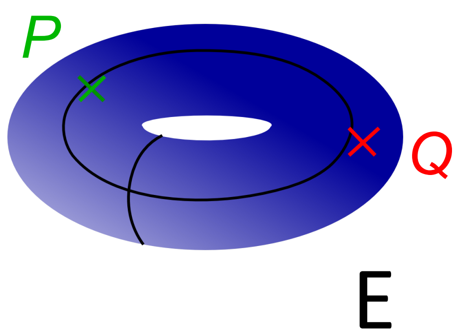
rational sections of elliptic fibr. ⟷ rational points of elliptic curve

Rational point Q on elliptic curve E with zero point P

- is solution (x_Q, y_Q, z_Q) in field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

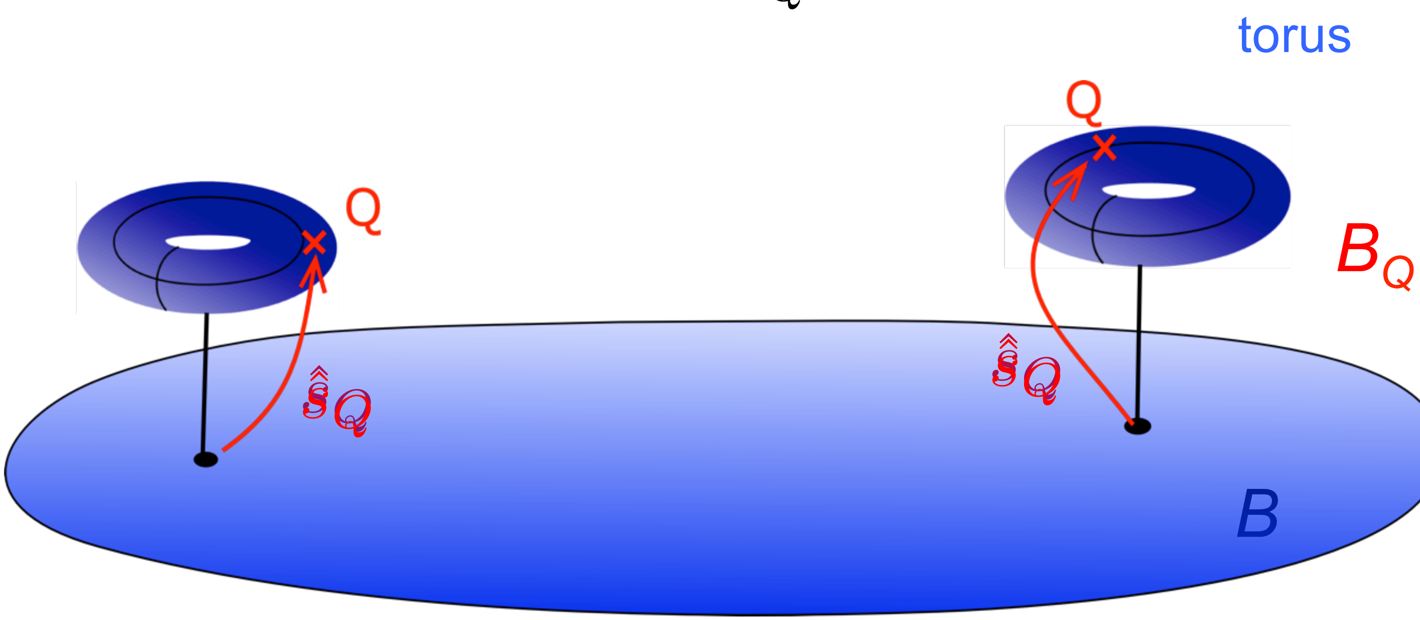
- Rational points form group (addition) on E



➡ Mordell-Weil group of rational points

U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point **Q** *E* induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration

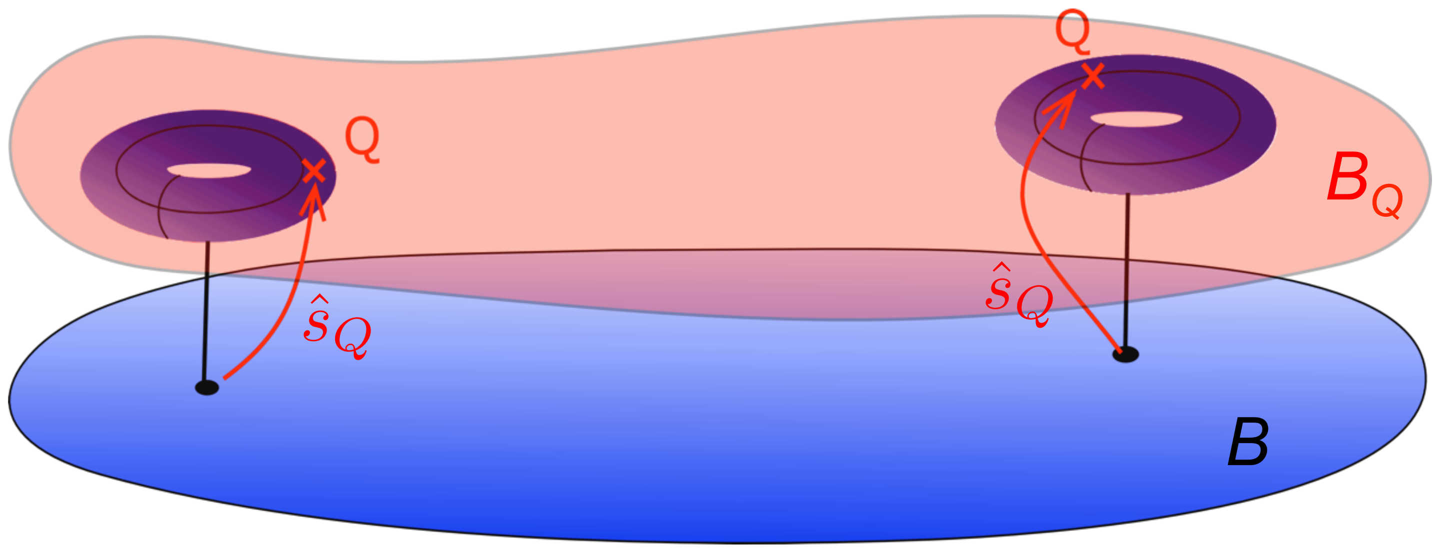


➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point **Q** *E* induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration



➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X* ➡ Construct (1,1) form ω_m from B_Q

Shioda map of $\hat{s}_Q$, complementary to $\mathcal{B}_{\mathcal{P}}$ - zero section & $\mathcal{E}_i$ - Cartan divisors:

$$\sigma\left(\hat{s}_Q\right) = B_Q - B_P - \sum_i l_i E_i + \cdots$$

[M.C., Lin, 1706.08521]

Implications for global constraints on gauge symmetries→ shortly

Earlier work: [Grimm,Weigand 1006.0226]...[Grassi,Perduca 1201.0930]
[M.C.,Grimm,Klevers 1210.6034]...

Explicit Examples: (n+1)-rational sections – $U(1)^n$

[Deligne]

[via line bundle constr. on elliptic curve E- *CY in (blow-up) of* $W\mathbb{P}^m$]

n=0: with P - generic CY in $\mathbb{P}^2(1,2,3)$ (Tate form)

n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park 1208.2695]...

Explicit Examples: (n+1)-rational sections – $U(1)^n$

n=0: with P - generic CY in $\mathbb{P}^2(1,2,3)$ (Tate form)

n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park 1208.2695]...

n=2: with *P, Q, R* - specific example: generic CY in dP_2

[Borchmann,Mayerhofer,Palti,Weigand 1303.54054,1307.2902]
[M.C.,Klevers,Piragua 1303.6970,1307.6425]
[M.C.,Grassi,Klevers,Piragua 1306.0236]

generalization to nongeneric cubic in $\mathbb{P}^2[u:v:w]$ →collisions of P,Q,R→
→ gauge enhancement & higher index representations….
[M.C.,Klevers,Piragua,Taylor 1507.05954]

n=3: with *P, Q, R, S* - CICY in $\mathrm{Bl}_3\mathbb{P}^3$
n=4: determinantal variety in $\mathbb{P}^4$ [M.C.,Klevers,Piragua,Song 1310.0463]

higher *n,* not clear…

No time → c.f.,[M.C. Lin,1809.00012] TASI review

[M.C. and Ling Lin 1706.08521]

c.f., review [M.C. Lin,1809.00012]

I.b U(1) & Non-Abelian Gauge symmetry

Shioda map of section $\hat{s}_Q$ more involved than $\mathcal{B}_Q$:

a map onto divisor complementary to $\mathcal{B}_\mathcal{P}$ divisor of zero section $\hat{s}_P$

& $\mathcal{E}_i$ – resolution (Cartan) divisors of non-Abelian gauge symmetry

$$\sigma\left(\hat{s}_Q\right) = B_Q - B_P - \sum_i l_i E_i + \cdots$$

Ensures proper F-theory interpretation of U(1)
(via M-theory/F-theory duality)

$l_i = C_{ij}^{-1}(B_Q - B_P) \, . \, \mathbb{P}_j^1$ - fractional # always an integer κ s.t. $\forall i : \kappa \, l_i \in \mathbb{Z}$

↑ Cartan matrix

↑ Fiber of divisor E_j

Construct non-trivial central element of U(1) × G:

Employing (a) $q_{\mathfrak{u}(1)} = \frac{n}{\kappa}$, $n \in \mathbb{Z}$ & (b) $l_i \mathbf{w}_i = l_i \mathbf{v}_i \mod \mathbb{Z} := L(\mathcal{R}_{\mathfrak{g}}^{(i)})$

w$_i$, **v**$_i$ – rep. weights

$C(\mathbf{w}) := [e^{2\pi i q(\mathbf{w})} \otimes (e^{-2\pi i \, l_i \, \mathbf{w}_i} \times \mathbb{1})] \, \mathbf{w} \overset{(b)}{=} [e^{2\pi i q(\mathbf{w})} \otimes (e^{-2\pi i \, L(\mathcal{R}_{\mathfrak{g}})} \times \mathbb{1})] \, \mathbf{w}$
defines element in centre of $U(1) \times G$; (a) $\Rightarrow C^{\kappa} = 1$.

& $C(\mathbf{w}) = \exp(2\pi i \underbrace{\xi(\mathbf{w})}_{\in \mathbb{Z}}) \, \mathbf{w} = \mathbf{w}$.

$\xi(w) = (B_Q - B_P) \, . \, \mathbb{P}^1 \in \mathbb{Z}$

$$G_{\mathrm{global}} = \frac{U(1) \times G}{\langle C \rangle} \cong \frac{U(1) \times G}{\mathbb{Z}_{\kappa}}$$

Global Constraint on Gauge Symmetry:

$$G_{\mathsf{global}} = \frac{U(1) \times G}{\langle C \rangle} \cong \frac{U(1) \times G}{\mathbb{Z}_\kappa}$$

Exemplify for SU(5) GUT's and Standard Model constructions

Including for globally consistent three family SM

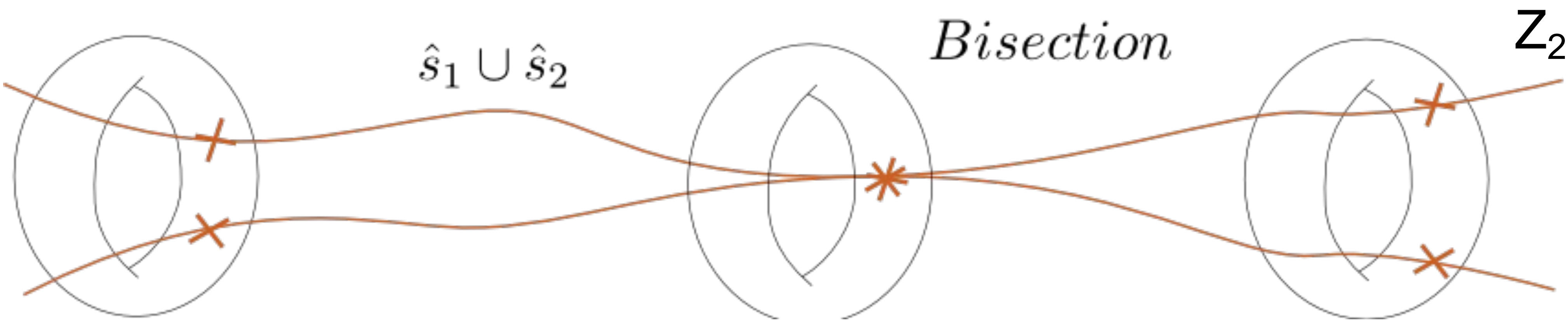
More later

I.c Discrete Abelian Gauge Symmetries Z_n

In Standard Model one often introduces Z_2 (R-parity) to prevent baryon number violating couplings

I.c Discrete Abelian Gauge Symmetries - Z_n

Geometric origin: torus fibrations that do not admit a section, but a multi-section



Earlier work: [Witten; deBoer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi;...]

Recent extensive efforts'14-'16: [Braun, Morrison; Morrison, Taylor; Klevers, Mayorga-Pena, Oehlmann, Piragua, Reuter; Anderson,Garcia-Etxebarria, Grimm; Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand; M.C., Donagi, Klevers, Piragua, Poretschkin; Grimm, Pugh, Regalado; M.C., Grassi, Poretschkin;...]

Standard Model with Z_2 matter parity [M.C., Lin, Liu, Oehlmann 1807.01320]

Transition between continuous and discrete symmetry: $U(1) \rightarrow Z_2$ example

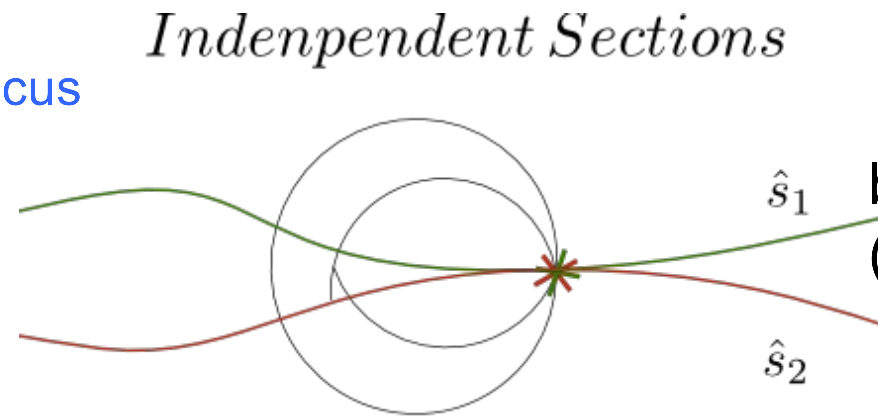
Singular codim-2 locus
I_2 -fiber

U(1)

Massless field
ϕ with charge 2

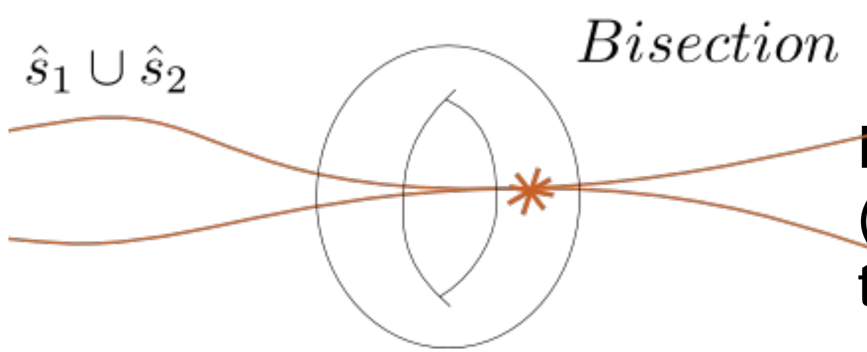
ϕ acquires VEV
<ϕ>≠0

Z_2



blow-down
(P^1 in the geometry w/
multiple sections collapses)

deformation



Deformation
(S^3 glues several sections
to a multi-section)

Field theory

Geometry

I.c Discrete Abelian Gauge Symmetries

Geometries with n-section ⟷ Tate-Shafarevich Group Z_n

Z_2 [Anderson,Garcia-Etxebarria, Grimm;
Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand'14]

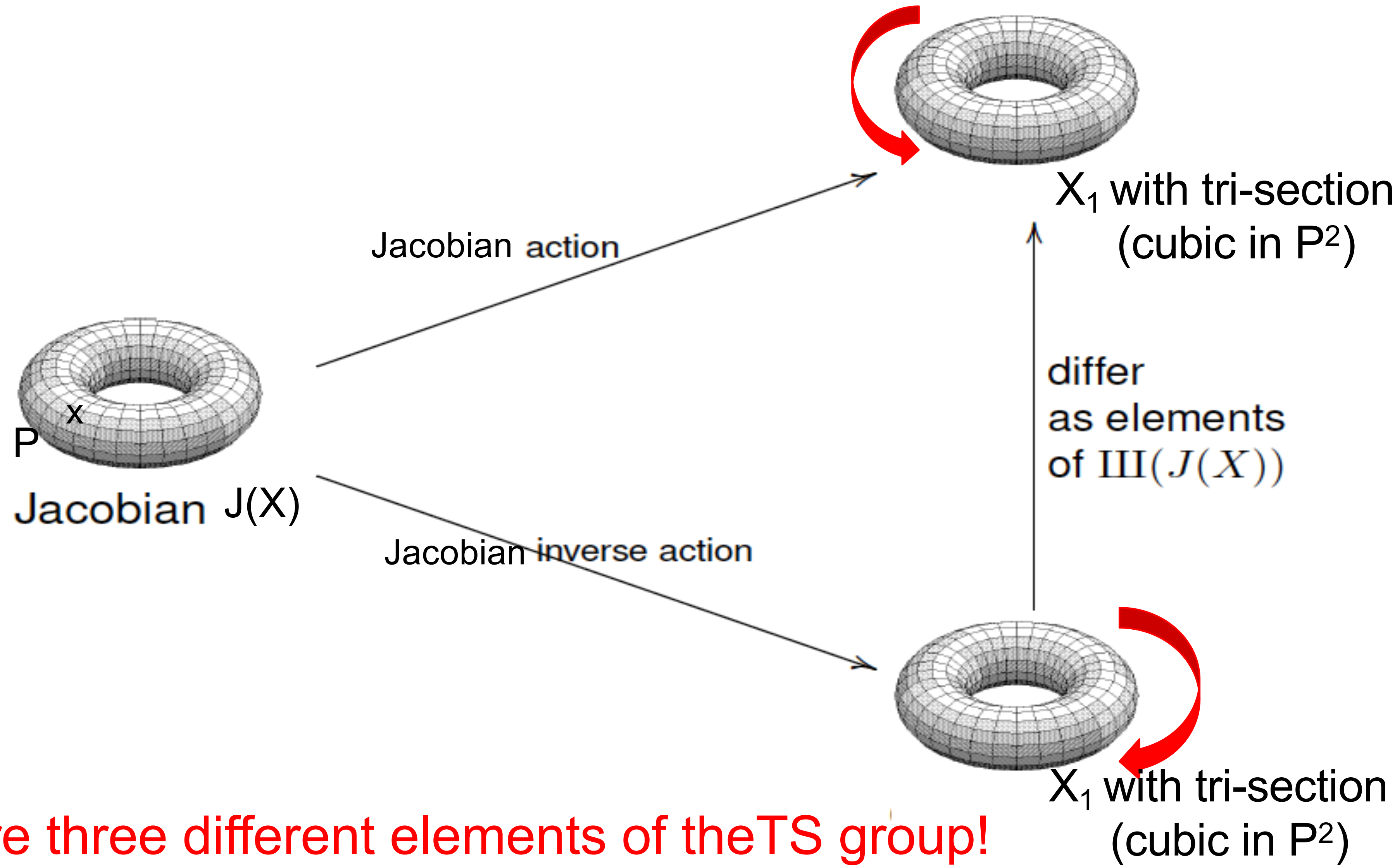
Z_3 [M.C.,Donagi,Klevers,Piragua,Poretschkin 1502.06953]

No time, but

Tate-Shafarevich group and Z_3

[M.C., Donagi, Klevers, Piragua, Poretschkin 1502.06953]

Only two geometries: X_1 w/ trisection and Jacobian $J(X_1)$



There are three different elements of theTS group!

Shown to be in one-to-one correspondence with three M-theory vacua.

II. Particle physics Constructions

Initial focus: F-theory with SU(5) Grand Unification

[Donagi,Wijnholt'08][Beasley,Heckman,Vafa'08]...

Model Constructions:

local [Donagi,Wijnholt'09-10]...[Marsano,Schäfer-Nameki,Saulina'09-11]...

Review: [Heckman]

global

[Blumehagen,Grimm,Jurke,Weigand'09][M.C., Garcia-Etxebarria,Halverson'10]...
[Marsano,Schäfer-Nameki'11-12]...[Clemens,Marsano,Pantev,Raby,Tseng '12]...

Progress on Standard Model:

Standard Model building blocks (via toric techniques) [Lin,Weigand'14]

II. Particle physics Constructions

Globally consistent models via torique techniques

Construction of elliptically fibered Calabi-Yau manifold

i. Elliptic curve E

Examples of constructions via toric techniques:

E_{F_i} as a hypersurface in the two-dimensional toric variety $\mathbb{P}_{F_i}$

(generalized weighted projective spaces, associated with16 reflexive polytopes F_i):

c.f., [Klevers, Pena, Oehlmann, Piragua, Reuter '14]

$$E_{F_i} = \{p_{F_i} = 0\} \text{ in } \mathbb{P}_{F_i}$$

ii. Elliptically fibered Calabi-Yau space: X_{F_i}

Impose Calabi-Yau condition:
coordinates in $\mathbb{P}_{F_i}$ and coeffs. of E_{F_i} lifted to
sections of specific line-bundles on B

$$\begin{array}{ccc} E_{F_i} \subset \mathbb{P}_{F_i} & \longrightarrow & X_{F_i} \\ & & \downarrow \\ & & B \end{array}$$

Fibration depends only on the anti-canonical divisor $\overline{\mathcal{K}}$
& two additional S_7 and S_9 divisor classes

iii. Chiral index for D=4 matter:

Standard Model with three families of quarks and leptons

iii. Chiral index for D=4 matter:

$$\chi(\mathbf{R}) = \int_{\mathcal{C}^w_{\mathbf{R}}} G_4$$

$$c_{mat} \longrightarrow \boxed{\mathcal{C}_{\mathbf{R}}} \longrightarrow \Sigma_{\mathbf{R}}$$

a) construct G_4 (=dC_3) flux by computing $H_V^{(2,2)}(\hat{X})$
[so-called vertical fluxes – do not induce Gukov-Vafa-Witten potential]

b) determine matter surface $\mathcal{C}_{\mathbf{R}}$ (via resultant techniques)

iv. Global consistency – D3 tadpole cancellation:

$$\frac{\chi(X)}{24} = n_{\mathrm{D3}} + \frac{1}{2}\int_X G_4 \wedge G_4$$

a) satisfied for integer and positive n_{D3}

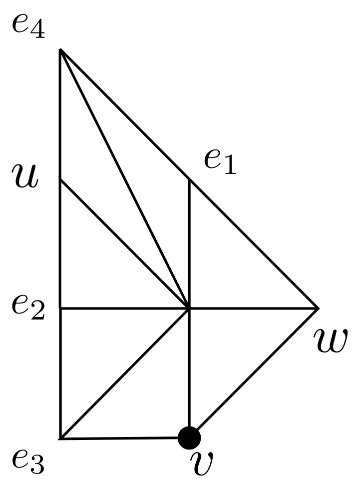
b) constraint on integer valued flux G_4 $\qquad G_4 + \frac{1}{2}c_2(X) \in H^4(\mathbb{Z}, \hat{X})$

C.f., Standard Model building blocks (via toric techniques) initiated in [Lin, Weigand'14] ; SM x U(1) [1604.04292]

Standard Model

[M.C., Klevers, Peña, Oehlmann, Reuter, 1503.02068]

F_{11} polytope



P_{F11}

$\mathbb{P}^2$ [u:v:w] with four non-generic blow-ups [e_1:e_2:e_3:e_4]

Elliptic curve:

$$p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 u v^2 + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2$$

Hypersurface constraint in P_{F11}

Construction of Calabi-Yau four-fold

Construction of Calabi-Yau four-fold

$$E_{F_{11}} \subset \mathbb{P}_{F_{11}} \longrightarrow X_{F_{11}}$$
$$\downarrow$$
$$B$$

Coordinates and s_i → sections of line-bundles of the base B

[Toric techniques via Stanley-Reisner ideal] →

Section	Line Bundle
u	$\mathcal{O}(H - E_1 - E_2 - E_4 + \mathcal{S}_9 + [K_B])$
v	$\mathcal{O}(H - E_2 - E_3 + \mathcal{S}_9 - \mathcal{S}_7)$
w	$\mathcal{O}(H - E_1)$
e_1	$\mathcal{O}(E_1 - E_4)$
e_2	$\mathcal{O}(E_2 - E_3)$
e_3	$\mathcal{O}(E_3)$
e_4	$\mathcal{O}(E_4)$

section	Line Bundle
s_1	$\mathcal{O}_B(3[K_B^{-1}] - \mathcal{S}_7 - \mathcal{S}_9)$
s_2	$\mathcal{O}_B(2[K_B^{-1}] - \mathcal{S}_9)$
s_3	$\mathcal{O}_B([K_B^{-1}] + \mathcal{S}_7 - \mathcal{S}_9)$
s_4	$\mathcal{O}_B(2\mathcal{S}_7 - \mathcal{S}_9)$
s_5	$\mathcal{O}_B(2[K_B^{-1}] - \mathcal{S}_7)$
s_6	K_B^{-1}
s_7	$\mathcal{O}_B(\mathcal{S}_7)$
s_8	$\mathcal{O}_B([K_B^{-1}] + \mathcal{S}_9 - \mathcal{S}_7)$
s_9	$\mathcal{O}_B(\mathcal{S}_9)$
s_{10}	$\mathcal{O}_B(2\mathcal{S}_9 - \mathcal{S}_7)$

H - hyperplane divisor;

K_B^{-1} - anti-canonical divisor $\overline{\mathcal{K}}$

Fibration depends only on additional S_7 and S_9 divisor classes.

Construction of Calabi-Yau four-fold → Divisors

$$\begin{array}{ccc} E_{F_{11}} \subset \mathbb{P}_{F_{11}} & \longrightarrow & X_{F_{11}} \\ & & \downarrow \\ & & B \end{array}$$

Over the locus $s_3 = 0$ → fiber degenerates to I_2 - fiber → SU(2)

Over the locus $s_9 = 0$ → fiber degenerates to I_3 - fiber → SU(3)

Cartan divisors of these gauge groups:

$$\mathrm{E}_1^{\mathrm{SU}(2)} = [e_1]\,, \quad \mathrm{E}_1^{\mathrm{SU}(3)} = [e_2] \quad \mathrm{E}_2^{\mathrm{SU}(3)} = [u]$$

Two rational sections:

$[u : v : w : e_1 : e_2 : e_3 : e_4]$

$\hat{s}_0 = X_{F_{11}} \cap \{v = 0\}$: $[1 : 0 : s_1 : 1 : 1 : -s_5 : 1]$ - zero section

$\hat{s}_1 = X_{F_{11}} \cap \{e_4 = 0\}$: $[s_9 : 1 : 1 : -s_3 : 1 : 1 : 0]$ - section associated with U(1)

Standard Model gauge symmetry: SU(3) x SU(2) x U(1)

Global Standard Model Gauge Symmetry & Matter Reps.

gauge algebra $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$

[M.C., Lin, 1706.08521]

Shioda map:

C-central element

$$\sigma(\hat{s}_1) = S_1 - S_0 + \frac{1}{2} E_1^{\mathfrak{su}(2)} + \frac{1}{3}(2\, E_1^{\mathfrak{su}(3)} + E_2^{\mathfrak{su}(3)}) \Rightarrow C^6 = 1,$$

$$G_{\text{global}} = [SU(3) \times SU(2) \times U(1)]/\langle C \rangle \cong [SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6.$$

Matter (at co-dim 2 singularities):

$$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}\,,\quad (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}\,,\quad (\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}\,,\quad (\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}\,,\quad (\mathbf{1},\mathbf{1})_{1}$$

Compatible with the Z_6 global constraint

Construct G_4 for chiral index & D3-tadpole constraint

Standard Model:

Hyperplane divisor class

$H = 4\overline{\mathcal{K}}$

Base B = $\mathbb{P}^3$ Divisors in the base:

$$\mathcal{S}_7 = n_7 H$$
$$\mathcal{S}_9 = n_9 H$$

$$n_7, n_9 \in \mathbb{Z}$$

Solutions (#(families);n_{D3}) for allowed (n_7,n_9):

$n_7 \backslash n_9$	1	2	3	4	5	6	7
7	–	(27; 16)	–	–			
6	–	(12; 81)	(21; 42)	–	–		
5	–	–	(12; 57)	(30; 8)	–	(3; 46)	
4	(42; 4)	–	(30; 32)	–	–	–	–
3	–	(21; 72)	–	–	–	(15; 30)	
2	(45; 16)	(24; 79)	(21; 66)	(24; 44)	(3; 64)		
1	–	–	–	–			
0	–	–	(12; 112)				
-1	(36; 91)	(33; 74)					
-2	–						

Tip of the Iceberg?

II. Landscape of Standard Models

Toric analysis

[M.C., J. Halverson, L. Lin, M. Liu and J. Tian, 1903.0009]

a) Take the same toric elliptic fibration as before:

hyperplane constraint in 2D reflexive polytope F_{11}

Gauge symmetry: $$\frac{\mathbf{SU(3)\times SU(2)\times U(1)}}{\mathbb{Z}_6}$$

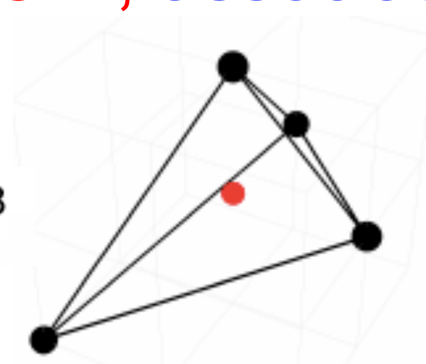
Global gauge symmetry

[M.C., Lin, 1706.08521]

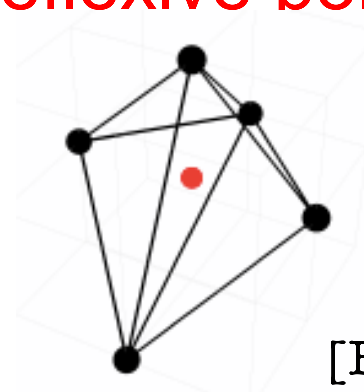
b) Take bases B, associated with 3D reflexive polytopes

E.g.,

$\mathbb{P}^3$



$\mathbb{P}^2 \times \mathbb{P}^1$



[Batyrev; Kreuzer-Skarke]

For each reflexive polytope, different bases B are associated with different fine-star-regular triangulations of a chosen polytope

[Triangulations determine intersections of divisors]

Triangulations grow exponentially with the complexity of a polytope

c) Specific choice of divisors: $S_{7,9} = \overline{\mathcal{K}}$

[anti-canonical divisor of the base B – fixed by the polytope]

SU(3) an SU (2) divisors S_9 and S_3 with class $\overline{\mathcal{K}}$ →

$$g_{3,2}^2 = 2/\mathrm{vol}(\overline{\mathcal{K}})$$

U(1) - (height-pairing) divisor volume $5\overline{\mathcal{K}}/6$ →

$$\frac{5}{3}\,g_Y^2 = \frac{2}{\mathrm{vol}(\overline{\mathcal{K}})}$$

⬇

Standard Model with gauge coupling unification!

$$g_3^2 = g_2^2 = 5/3 g_Y^2$$

Connected torically to Pati-Salam Model $SU(4)_C x SU(2)_L x SU(2)_R$

c.f., [M.C., Klevers, Peña, Oehlmann, Reuter,1503.02068]

Non-torically connected to SU(5) GUT

[Taylor, Turner 1906.11092; & Ranghuram 1912.10991]

d) Remaining conditions:

iii. 3-families of quarks and leptons (chiral index)

iv. D3-tadpole constraints

Technical, no time

- Construct G_4 flux in terms of (1,1)-forms, Poincaré dual to divisor classes c.f., [Lin, Mayrhofer, Till,Weigand, 1508.00162] [M.C., Grassi, Klevers, Piragua, 1306.3987]

- Chirality, D3 tadpole and G_4 integrality expressed in terms of intersection numbers of divisors in the base B → Geometric conditions!

- In the case $\mathcal{S}_{7,9} = \overline{\mathcal{K}}$ and n_F – families, the D3 tadopole:

$$n_{D3}(n_F, \overline{K}^3) = 12 + \frac{5\overline{K}^3}{8} - \frac{5n_F^2}{2\overline{K}^3} \in \mathbb{Z}_{\geq 0}$$

Geometrized D3-tadpole condition

Depends only on the polytope and not on triangulation → Universality of the Standard Model

Landscape count for n_F=3 families:

toric

$$12 + \frac{5}{8}\overline{\mathcal{K}}^3 - \frac{45}{2\overline{\mathcal{K}}^3} \in \mathbb{Z}_{\geq 0}$$ satisfied for $\overline{\mathcal{K}}^3 \in \{2, \boxed{6, 10, 18, 30}, 90\}$

- Out of 4319 3D reflective polytopes → 708 satisfy the constraint (many of them with a large number of lattice points).

- Triangulation of polytopes can be handled combinatorially (each corresponds to a different basis B).
  It can be implemented on computer, e.g., in SageMath:
  i) for 237 polytopes w/ < 15 lattice points →414310 MSSM models.
  ii) for 471 polytopes w/ ≥ 15 lattice points – exp. growing comp. time → counting via fine-regular triangulation of facets & estimate regular fine-star triang.

c.f., [Halverson, Tian, 1610.08864]

⬇

- Provide a bound: $7.6 \times 10^{13} \lesssim N_{\mathrm{SM}}^{\mathrm{toric}} \lesssim 1.6 \times 10^{16}$

Summary

Globally consistent F-theory Standard Models
(Toric techniques w/elliptic fibration: hypersurface in F_{11})

⬇

First three family Standard Models

Anticipated: tip of the iceberg

⬇ Indeed, geometric advances

Landscape of globally consistent Standard Models
w/ exact chiral spectrum of three-families of quarks &leptons
& gauge coupling unification > quadrillion models

III. Further Analysis

III.a Moduli stabilization

Related to issues of supersymmetry breaking, cosmological implications, dark matter candidates…

⬇

Moduli Stabilization for quadrillion Standard Models

[M.C., Long, Halverson, Lin, 2004.00630]

Under which conditions moduli stabilization can be pursued via effective field theory techniques w/ g_s perturbative (à la KKLT or Large Volume Scenario):

i) gauge coupling constraint: $\alpha_{1,2,3} = \alpha_{GUT} = (g_s \ell_s^4)/\text{Vol}(\bar{\text{K}}) \sim 1/25$

→ $\text{Vol}(\overline{\mathcal{K}}) \lesssim \mathcal{O}(100)$,

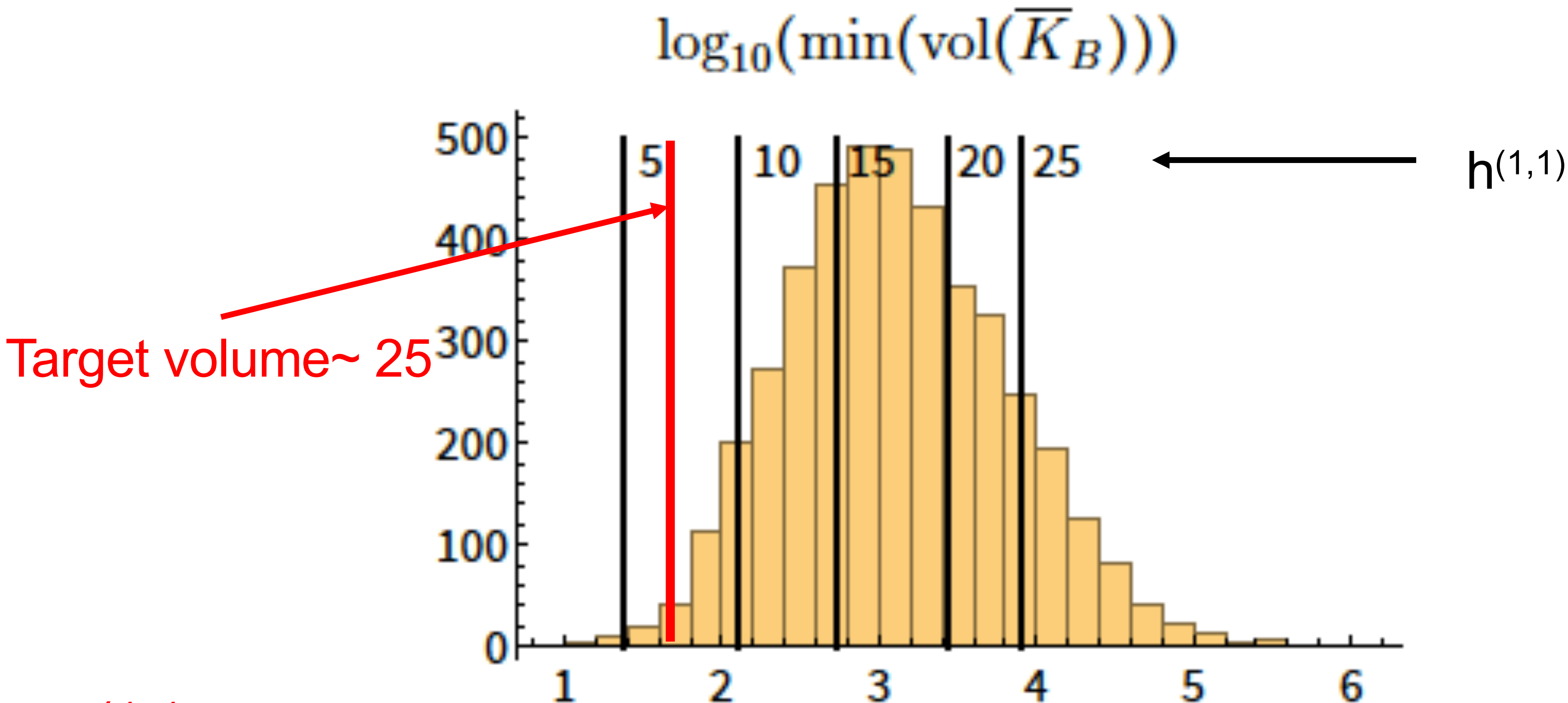
ii) all divisor and curves w/ $\text{vol}(C_a) \geq 1$ (in string units) in order to suppress world-sheet and ED3 instanton contributions, c.f., $e^{-2\pi n \text{vol}(c)} \ll 1$.

→ Stretched Kähler cone

- where all divisors & curves w/volumes >1
- Since $\overline{\mathcal{K}} = -\sum D_i$ → Vol($\overline{\mathcal{K}}$) expected to be typically large

Distribution of *Min*(Vol($\overline{\mathcal{K}}$))

[one triangulation per each of 4319 polytopes]



$h^{(1,1)}<7$ →~10^4 out of quadrillion models satisfy constraints

Comments:

- Moduli stabilization scenarios, based on effective theory & perturbative g_s (KKLT,LVS), significantly reduces the number of viable Standard Models with gauge coupling unification.

- Moduli stabilization could take place in a regime where effective theory & perturbative string theory approaches fail. → poorly explored/difficult to explore.

- Could abandon to have only Standard Model and/or gauge coupling unification → typically leads to additional D7-(p,q) sectors w/ interesting dark gauge sector implications.

c.f., Halverson, Long, Nelson, Salinas 1909.05257

⬇

Further exploration of other Standard Model constructions

III.a Counting of vector matter pairs

[...Donagi,Wijnholt '08,...,Bies,Mayrhofer, Pehle, Weigand '14,'17]

Depends on C_3 potential, encoded in intermediate the Jacobian of Y_4.

When restricted to the matter curve $\mathcal{C}$, C_3 defines a line bundle $\mathcal{L}$ w/

massless chiral modes $\subset H^0(\mathcal{C};\mathcal{L})$
massless anti-chiral modes $\subset H^1(\mathcal{C};\mathcal{L})$

[chiral index $\chi = h^0 - h^1$ topological invariant (depends on $G_4 = dC_3$)]

$H^i(\mathcal{C};\mathcal{L})$ – computation via algorithm implemented in computer algebra system CAP [Bies '17; Bies, Posur '19]

Counting of vector matter pairs

For the quadrillion Standard Models the analysis difficult due to the complexity of the construction and high genera of matter curves.

$$g = 1 + 9/2\overline{K}^3$$

Goal: to determine the range of complex structure moduli of the F-theory compactification, for which we have the Minimal Supersymmetric Standard Model (one Higgs doublet pair, no other vector pair exotics).

Making progress...

[Martin Bies, M.C., Ron. Donagi, L. Lin, M. Liu, Fabian Rühle, 2007.0009]
[Bies,M.C. Donagi, Liu, 2102.10115, 2104.08297]
[Bies,M.C. Donagi, Marielle Ong, 2205.00008, 2307.02535]

Outlook

Particle physics models in F-theory compactifications have come a long way, but there is much more to go.

- Technical advances, to be pursued:

  Exact matter spectrum for quadrillion Standard Models

  Yukawa couplings (some progress for a toy model)

  [M.C., Lin, Liu, Zoccarato, Zhang 1906.10119]

  Systematic exploration of other particle physics models (possibly beyond toric techniques) →

  c.f., W. Taylor's et al. talk(s)

Thank you!